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ELEMENTARY SCHOOLS.

THE LATE MASTER OF TRINITY.

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As the basis of all real progress in Mathematics, boys ought to acquire a good knowledge of Arithmetic and a habit of performing the common operations of Arithmetic, and of applying the rules in a correct and intelligent manner. This acquirement appears to be often neglected at our most eminent classical schools. Such a neglect is much to be regretted; for the want of this acquirement is a great practical misfortune, and is often severely felt in after-life. Many persons who are supposed to have received the best education which the country affords, are, in all matters of numerical calculation, ignorant and helpless, in a manner which places them, in this respect, far below the members of the middle class, educated as they usually are. Arithmetic is a matter of habit, and can be learnt only by long-continued practice. *For some years of boyhood there ought to be a daily appropriation of time to this object.*

A SHILLING
ARITHMETIC

DESIGNED FOR THE USE OF

ELEMENTARY SCHOOLS.

BY

JOHN WILLIAM COLENSO, D.D.

BISHOP OF NATAL.

BIBLIOTHECA·BODLEIANA·

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NEW-STREET SQUARE

PREFACE.

THE OBJECT of this little Work is to provide Elementary Schools with a Text-Book of Arithmetic, accompanied by a Series of easy and amusing examples, arranged progressively, in order of difficulty, and sufficiently numerous to give complete practice in the different rules of Arithmetic—beginning from the very simplest steps of the science. The Author has been led to prepare it, mainly from the want he has felt of such a work in his own experience as a Teacher of Arithmetic in his own village and adult schools. To this want he is disposed to ascribe much of that ignorance and helplessness in matters of Arithmetic, which is known so generally to prevail in these schools.

The method of using the Work will be obvious. The Teacher should first thoroughly explain the Rule in any case to the class, and then set the pupils down to work the Exercises.

ADVERTISEMENT.

THIS Shilling Arithmetic is a new edition of Bishop Celenso's Text-Book of Elementary Arithmetic, in which the Exercises have been incorporated with the Text; but the original Edition, in which the Text-Book is published separately from the Examples, is still kept on sale. The work has been carefully revised throughout, and in a few instances alterations have been made with the view to improve the explanatory portion of the book.

December 1866.

ARITHMETICAL TABLES.

MULTIPLICATION TABLE.

1	2	3	4	5	6	7	8	9	10	11	12
2	4	6	8	10	12	14	16	18	20	22	24
3	6	9	12	15	18	21	24	27	30	33	36
4	8	12	16	20	24	28	32	36	40	44	48
5	10	15	20	25	30	35	40	45	50	55	60
6	12	18	24	30	36	42	48	54	60	66	72
7	14	21	28	35	42	49	56	63	70	77	84
8	16	24	32	40	48	56	64	72	80	88	96
9	18	27	36	45	54	63	72	81	90	99	108
10	20	30	40	50	60	70	80	90	100	110	120
11	22	33	44	55	66	77	88	99	110	121	132
12	24	36	48	60	72	84	96	108	120	132	144

I. MONEY.

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12 Pence . . . 1 Shilling (*s.*)
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A Guinea = 21 Shillings.

d.		*s.*	*d.*	*d.*		*s.*	*d.*
12	make	1	0	80	make	6	8
20	. .	1	8	84	. .	7	0
24	. .	2	0	90	. .	7	6
30	. .	2	6	96	. .	8	0
36	. .	3	0	100	. .	8	4
40	. .	3	4	108	. .	9	0
48	. .	4	0	110	. .	9	2
50	. .	4	2	120	. .	10	0
60	. .	5	0	130	. .	10	10
70	. .	5	10	132	. .	11	0
72	. .	6	0	140	. .	11	8

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16 Drams make 1 Ounce (*oz.*)
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III. LENGTH.

4 Inches	make	1 Hand.	
12 Inches	. . .	1 Foot.	
3 Feet .	. . .	1 Yard.	
6 Feet .	. . .	1 Fathom.	
5½ Yards	. . .	1 Rod, Pole, or Perch.	
40 Poles (220 yds.)	.	1 Furlong.	
8 Furlongs (1760 yds.)		1 Mile.	
3 Miles	. . .	1 League.	
2¼ Inches	. . .	1 Nail	*Cloth Measure.*
4 Nails	. . .	1 Quarter	
4 Quarters .	. . .	1 Yard	
5 Quarters .	. . .	1 Ell	
22 Yards (4 Poles)	.	1 Chain	*Land Measure.*
10 Chains	. . .	1 Furlong	

IV. SURFACE.

144 Square Inches	make	1 Square Foot.
9 Square Feet	. .	1 Square Yard.
30¼ Square Yards	. .	1 Square Rod, Pole, or Perch (*P.*)
40 Perches	. . .	1 Rood (*R.*)
4 Roods (4840 sq. yds.)	.	1 Acre (*A.*)
640 Acres	. . . .	1 Square Mile.

V. SOLIDITY.

1728 Cubic Inches	make	1 Cubic Foot.
27 Cubic Feet .	. .	1 Cubic Yard.

VI. CAPACITY.

4 Gills or Noggins	make	1 Pint.
2 Pints	. . . .	1 Quart.
2 Quarts .	. . . .	1 Pottle.
4 Quarts .	. . . .	1 Gallon.
2 Gallons .	. . . .	1 Peck.
4 Pecks	. . . .	1 Bushel.
4 Bushels	. . .	1 Coom.
8 Bushels	. . .	1 Quarter.
5 Quarters	. . .	1 Load.
3 Bushels	. . .	1 Sack.
12 Sacks	. . . .	1 Chaldron.
A Barrel of *Beer*	contains	36 Gallons.
A Hogshead	. . .	54 Gallons.
A Hogshead of *Wine*	.	63 Gallons.
A Pipe	. . . .	2 Hogsheads.

VII. TIME.

60 Seconds	make	1 Minute.
60 Minutes	. . .	1 Hour.
24 Hours	. . .	1 Day.
7 Days .	. . .	1 Week.
4 Weeks	. . .	1 Lunar Month.
365 Days .	. . .	1 Year.

Since 52 Weeks, or 13 Lunar Months, contain 364 Days, these are often reckoned as a Year.

The Year is also divided into 12 Months, called *Calendar Months*, which contain unequal numbers of Days—

January . .	31	May . . .	31	September .	30
February . .	28	June . . .	30	October . .	31
March . . .	31	July . . .	31	November .	30
April . . .	30	August . .	31	December . .	31

Of these all contain 31 days, except February, which has 28, and those mentioned in the following rhymes, which have 30:—

Thirty days have September,
April, June, and November.

Every fourth year contains 366 Days, and is called Leap-Year; and in such a year February has 29 Days.

ARITHMETIC.

PART I.

1. Arithmetic teaches about *numbers*—about the way of writing them down by means of figures, and about the manner of reckoning with them, in the various dealings of people with one another, in the common course of business, and in the different concerns of life.

2. The number *one* is called *unity*; and an *integer*, or *whole number*, is a collection of *ones*, *unities*, or *units*.

3. Notation is the art of writing down any number in figures.

4. The figures 1, 2, 3, 4, 5, 6, 7, 8, 9 are used to express the whole numbers, *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*, respectively; and the figure 0 represents *nought* or *nothing*.

5. By means of the above ten figures, which are called the *nine digits and zero*, we are able to express any number whatever.

This is effected thus:

A figure standing *by itself*, or on the *right hand* of other figures, is used with its own proper value, to express so many *units*;

A figure standing in the *second* place from the right (that is, with *one* figure after it) is used to express so many *tens of units*;

A figure standing in the *third* place from the right (that is, with *two* figures after it) is used to express so many *tens of tens*, or *hundreds of units*;

A figure standing in the *fourth* place from the right (that is, with *three* figures after it) is used to express so many *tens of hundreds*, or *thousands of units*;

And so on, according to the following Table, which is called

THE NUMERATION TABLE.

7	9	8	7	1	3	5	4	6	2	3	0	7	2	4	6	8	9	7	1
&c.	Trillions.	Hundreds of T. of Billions.	Tens of T. of Billions.	Thousands of Billions.	Hundreds of Billions.	Tens of Billions.	Billions.	Hundreds of T. of Millions.	Tens of T. of Millions.	Thousands of Millions.	Hundreds of Millions.	Tens of Millions.	Millions.	Hundreds of Thousands.	Tens of Thousands.	Thousands.	Hundreds.	Tens.	Units.

and so on to *trillions*, *quadrillions*, &c. if necessary.

6. Let it be noted, then, that the value of any figure is increased *ten-fold* by its being moved *one place to the left.*

Thus 7 means *seven:* but, in 73, the 7 stands for *ten times seven*, or *seventy;* in 731, the 7 stands for *ten times seventy*, or *seven hundred;* in 7031, the 7 stands for *ten times seven hundred*, or *seven thousand;* and so on.

Ex. Express in figures the following number:—*Seven millions four hundred and seventy thousand three hundred and eighteen.*

Ans. 7,470,318.

Ex. 1.

Express in figures the following numbers:

(1.) Twelve, fourteen, nineteen, twenty-seven.

(2.) Thirty, thirty-nine, forty-seven, fifty.

(3.) Sixty-one, seventy, eighty-eight, ninety-two.

(4.) One hundred and one, one hundred and ten, one hundred and thirteen, one hundred and fifty-five.

(5.) Two hundred, two hundred and two, two hundred and twelve, three hundred and thirty.

(6.) Five hundred and seven, six hundred and sixty-three, seven hundred and seventeen, eight hundred and eighty.

(7.) One thousand and one, one thousand and ten, one thousand one hundred, one thousand and eleven.

(8.) One thousand one hundred and one, one thousand one hundred and ten, one thousand one hundred and eleven, one thousand six hundred and five.

(9.) One thousand and seventy, two thousand three hundred and three, three thousand and thirty, four thousand and fourteen.

(10.) Three thousand two hundred and twelve, four thousand one hundred and eighteen, five thousand six hundred and seven, seven thousand one hundred and ninety.

(11.) One thousand six hundred, two thousand and eleven, eight thousand and fifty, nine thousand and one.

(12.) One thousand and fourteen, three thousand, seven thousand and six, eight thousand five hundred and twenty-three.

(13.) Twenty thousand, forty-three thousand and fifty-nine, sixty-eight thousand and seventy, eighty-nine thousand six hundred and five.

(14.) Sixty-one thousand, seventy thousand and eight, ninety-four thousand six hundred and twenty, thirty thousand two hundred and eleven.

(15.) Thirteen thousand and thirty-one, thirty thousand and thirteen, thirty-one thousand three hundred and one, fifty-two thousand and seventy-three.

(16.) Seventy-eight thousand and eleven, ninety thousand seven hundred and four, twelve thousand and five, twenty thousand one hundred and two.

(17.) Five hundred thousand and two, six hundred thousand seven hundred and thirteen, eight hundred and ten thousand one hundred and one, nine hundred and five thousand and fifty.

(18.) Three millions five hundred thousand and eighteen, fourteen millions two hundred and seven thousand five hundred and seventy-three, one hundred and seven millions two hundred and four thousand and three, one thousand two hundred and thirty-four millions five hundred and sixty-seven thousand eight hundred and ninety.

---

7. **Numeration** is the art of reading, or writing down in words, the number which is expressed by any set of figures.

Ex. Write down in words the number expressed by 5,402,679.

Ans. Five millions four hundred and two thousand six hundred and seventy-nine.

Ex. 2.

Read (or write down) in words the following numbers:

(1.) 10, 13, 17, 23.
(2.) 35, 40, 55, 73.
(3.) 100, 101, 110, 111.
(4.) 276, 317, 371, 401.
(5.) 410, 444, 505, 611.
(6.) 717, 880, 909, 971.

(7.) 1000, 1001, 1010, 1100. (8.) 1110, 1101, 1011, 1111.
(9.) 2032, 3105, 4004, 5130. (10.) 6006, 7607, 8118, 9090.
(11.) 7000, 6005, 8030, 9300. (12.) 4760, 3907, 2081, 6789.
(13.) 12345, 40340, 60045, 312147.
(14.) 70717, 82010, 97531, 605103.
(15.) 728007, 801011, 910001, 1023714.
(16.) 450160, 207703, 560602, 2103756.
(17.) 3046302, 4471107, 5610098, 7234010.
(18.) 9876543, 98765432, 987654321, 9876543210.

---

8. For the numbering of the Psalms in the Prayer-Book, or for the numbers upon the face of a school-clock, it is likely that there will be another set of figures used. These are called the *Roman* Numerals, while the others are called the *Arabic*, because we got the one set from the Romans, and the other from the Arabians. The Arabic system of Notation is far more convenient than the Roman, especially for expressing large numbers, and is universally employed, except for such purposes as those above mentioned. We will, however, shortly explain the Roman system, and give one Exercise upon it.

9. The Romans used I for *one*, V for *five*, L for *fifty*, C for a *hundred*, D for *five hundred*, and M for a *thousand*.

When any one of these letters was put *after* one of equal or greater value, it was meant to be *added* to it; but, when put *before* one of greater value, it was meant to be *subtracted*, or taken away from it.

Thus the numbers 1, 2, 3, &c., up to 12, were expressed by

I, II, III, IV, V, VI, VII, VIII, IX, X, XI, XII;

and so XXX stood for 30, XL for 40, LX for 60, LXX for 70, XC for 90, CX for 110, &c.

For 400 they used CCCC, *not* CD; and this is the only instance where they allowed *four* letters of the same kind to come together in expressing a number.

A line drawn above a character increased its value a thousand-fold.

Thus $\overline{\mathrm{V}}$ stood for 5,000, $\overline{\mathrm{L}}$ for 50,000, $\overline{\mathrm{C}}$ for 100,000, $\overline{\mathrm{M}}$ for a *million*.

Ex. Express 1962 in Roman Numerals.
Ans. MDCCCCLXII.

Ex. 3.

Express in Roman numerals:

(1.) 13, 14, 15, 16.
(2.) 17, 18, 19, 20.
(3.) 25, 29, 34, 37.
(4.) 46, 49, 55, 58.
(5.) 63, 67, 70, 76.
(6.) 84, 88, 91, 99.
(7.) 103, 154, 214, 227.
(8.) 338, 383, 427, 472.
(9.) 504, 560, 616, 677.
(10.) 789, 798, 815, 851.
(11.) 909, 944, 1037, 1054.
(12.) 1234, 1567, 1853, 1999.

---

10. The following signs are much used in Arithmetic:

+ (*plus*) shows that the number *before* which it is placed is to be *added;*

− (*minus*) shows that the number *before* which it is placed is to be *subtracted;*

× (*into*) shows that the number which stands *before* it is to be *multiplied* by the number which comes *after* it;

÷ (*by*) shows that the number which stands *before* it is to be *divided* by the number which comes *after* it;

= (*equals*) shows that the number which stands *before* it is *equal* to the number which comes *after* it.

11. Addition. When any numbers are taken together, or *added*, the resulting number is called their *sum.*

Ex. *Add* 94163
21954
7812
593
35647
4895
165064

In order to add whole numbers together, we must first place them under one another, with their units-figures in the same vertical line. We then add the units-column thus: 5 *and* 7 *are* 12, *and* 3 *are* 15, *and* 2 *are* 17, *and* 4 *are* 21, *and* 3 *are* 24, that is, 24 *units*, or 2 *tens and* 4 *units:* we set the 4 under the units-column, to be the units-figure of the result, and *carry* the 2 *tens* to be added to the second or tens-column.

We next add the tens-column in the same manner, beginning with the 2 carried, thus: 2 *and* 9 *are* 11, *and* 4 *are* 15, &c.; and we find the sum of this column to be 36, that is, 36 *tens*, or 3 *tens of tens and* 6 *tens*—that is, 3 *hundreds and* 6 *tens:* we set the 6 under the tens-column, to be the tens-figure of the result, and *carry* the 3 *hundreds* to be added to the third or hundreds-column.

We go on to add the hundreds-column in the same manner, beginning with the 3 carried, and we find the sum of this column to be 40, that is, 40 *hundreds*, or 4 *tens of hundreds and* 0 *hundreds*, that is, 4 *thousands and* 0 *hundreds:* we set the 0 under the hundreds-column, to be the hundreds-figure of the result, and *carry* the 4 *thousands;* and so on.

12. The *Answer* of any sum in Addition may be *proved* to be correct by adding it over again, beginning with the *top* line, and going downwards, when the result will be the same as before, if the sum be worked correctly.

Ex. 4.

(1.)	(2.)	(3.)	(4.)	(5.)	(6.)
23	15	34	46	13	27
42	22	25	24	35	46
14	43	32	45	57	68

(7.)	(8.)	(9.)	(10.)	(11.)	(12.)
123	135	352	547	159	167
234	246	463	321	862	284
456	357	571	789	473	359

(13.)	(14.)	(15.)	(16.)
1234	2345	3456	4567
5678	6789	7890	8901
9012	1234	1638	2435
3456	5678	2547	7968

(17.)	(18.)	(19.)	(20.)
3607	9976	7627	6549
2859	475	6854	7238
3768	89	776	470
427	7654	65	3986

(21.)	(22.)	(23.)	(24.)
13579	90423	31141	15897
24680	78514	25639	68561
69154	56186	48507	32426
37812	34975	29849	59791
95643	12639	67635	43043

(25.)	(26.)	(27.)	(28.)
876208	682475	478	6752
343672	468854	614562	784519
394625	713628	9875	22687
4713	299	589	456275
29875	51876	423765	44824
368207	2945	48567	612785

(29.)	(30.)	(31)	(32.)
371623	6817	347326	562347
85738	695737	587765	85476
654297	462385	768246	36728
8277	752	87335	6544
365685	93607	654	97328
423748	342665	363578	475564

Ex. 5.

(1.) There were seventeen slate-pencils in a bag, and I put in nineteen more. How many are there now?

(2.) In the National School there are twenty-three boys and twenty-nine girls, and in the Infant School there are seventeen boys and twenty-four girls. How many *boys* are there? how many *girls?* and how many *children*, altogether?

(3.) A farmer's wife drove a donkey to market, with two panniers of eggs. The donkey fell down by the way, and smashed fifteen eggs in one pannier, and nineteen in the other. She sold ninety-seven, and carried home again sixty-nine that were left. How many had she at first?

(4.) The Creation of the World took place B. C. 4004. How many years is it since then to the end of A. D. 1852? How many since the Deluge, B. C. 2348, to the end of A. D. 1852?

(5.) Add together all the numbers upon the face of a clock or watch.

(6.) John Brown was born in the year 1797. In what year will he be sixty-six years old?

(7.) James has seventeen marbles in his pocket, John has twenty-three, and William has nine. How many have they among them?

(8.) A turkey-merchant passed through a parish, and picked up, at one farmer's, twenty-nine young turkeys, at another's, thirty-five, and seventeen at each of two others. How many did he buy altogether?

(9.) In the first class there are five boys and eleven girls; in the second, six boys and thirteen girls; in the third, seven boys and nine girls; in the fourth, six boys and eight girls. How many children are there in the school?

(10.) A packet of pins was dropped upon the school floor, and four boys picked them up. *A* picked up thirty-seven, *B* twenty-nine, *C* twenty, and *D* thirty-eight. How many were there in the packet?

(11.) From five pear-trees were gathered, respectively, seventy-five, a hundred and fifty-seven, ninety-eight, a hundred and nine, and a hundred and nineteen, pears. How many were gathered in all?

(12.) I met three flocks of sheep going to market, and counted them. In the first there were a hundred and forty-nine, in the second, seventy-six, and in the third, a hundred and seven. How many were there altogether?

(13.) Farmer Roberts rents his land of three landlords. Of the first he rents eighty-two acres, of the second a hundred and twelve, and of the third two hundred and ninety. How many does he farm altogether?

(14.) From London to Peterborough is a hundred and two miles, from Peterborough to Newark, forty-six, from Newark to York, seventy-two, from York to Newcastle, eighty-four, from Newcastle to Berwick, sixty-five, and from Berwick to Edinburgh, fifty-nine miles. How far is it by this line from London to Edinburgh?

(15.) In a certain parish there are eighty-seven men, ninety-five women, twenty-nine young persons, thirty-seven school-children, and thirty-four infants. What is the whole population of the parish?

(16.) The Duke of Wellington's army at Waterloo consisted of the following British troops; Infantry, twenty thousand six hundred and sixty-one; Cavalry, eight thousand seven hundred and thirty-five; Artillery, six thousand eight hundred and seventy-seven: besides thirty-three thousand four hundred and thirteen Belgian and other foreign allies. How many British were there in all? and what was the whole number of his army?

(17.) Napoleon's army at Waterloo was composed of Infantry, forty-eight thousand nine hundred and fifty; Cavalry, fifteen thousand seven hundred and sixty-five; and Artillery, seven thousand seven hundred and thirty-two. How many Frenchmen were engaged in the battle?

(18.) Three boys, *A*, *B*, and *C*, were set to pump out a tank of water. *A* pumps two hundred and seventy-six gallons, *B*, three hundred and nine, and *C*, four hundred and fifteen. How many gallons did the tank contain?

(19.) The Bishop of New Zealand, in visiting his diocese, had to sail by sea one thousand one hundred and eighty miles, to walk seven hundred and sixty-two, to ride eighty-seven, and to go by boat on rivers two hundred and forty-nine. What was the entire distance he travelled?

(20.) A man and his children were picking up apples in an orchard. John took up two hundred and thirty-five, Robert, one hundred and ninety-seven, and Jane, one hundred and thirty-six; and the father took up as many as John and Jane together. How many did they pick up between them?

(21.) I saw an excursion-train pass with sixty-seven first-class passengers, a hundred and ninety-two second-class, and the number of third-class passengers was just thirty-two more than all the rest put together. How many were in the train?

(22.) In a book-case there were five shelves. On the first there were twenty-nine books, and on the second, seven more than on the first; on each of the third and fourth there were nineteen volumes, and on the fifth, as many as on the second, third, and fourth, together. How many books were there in the book-case?

(23.) *A*, *B*, and *C*, came home from nutting, and began to count their nuts. *A* had five hundred and seventy-three in his bag, and a hundred and eighty-nine in his pockets; *B* had six hundred and ninety-eight in his bag, and a hundred and sixty-seven in his pockets; *C* had none in his pockets, but as many in his bag as *A* in his bag and *B* in his pockets, together. How many nuts had they between them?

(24.) A heap of bricks is carried away in five carts. One of them takes three hundred and eighty-nine, another, two hundred and seventy-six, a third, three hundred and forty-seven, a fourth, a hundred and twenty-six, and the fifth, as many as the first, third, and fourth, together. How many bricks were there in the heap?

---

13. SUBTRACTION. When one number is *subtracted* or taken away from another, the number that is left is called the *remainder*, or the *difference* of the two numbers.

Ex. From 794327
Take 342814
451513

In order to subtract one whole number from another, we must first place the number to be subtracted under the other, with their units-figures in the same line. We then take the units-figure, 4, of the lower number from that of the other, 7; thus, 4 *from* 7, *there remains* 3, that is, 3 *units:* we place the 3 under the units-column, to be the units-figure of the result.

Then we go on to the tens-column, and say, 1 *from* 2, *there remains* 1, that is, 1 *ten*, and we set down the 1 under the tens-column.

Next we proceed to the hundreds-column, and say, 8 *from* 3 *I cannot*; but, if we take, or *borrow*, 1 *thousand* out of the 4 *thousands* in the upper line (leaving now 3 *thousands* instead of 4), and turn this thousand into *ten hundreds*, we shall thus have *thirteen hundreds* in the upper line, and we can now say, 8 *from* 13, *there remains* 5, that is, 5 *hundreds;* and we set down the 5 as the hundreds-figure of the result.

We have now to take 2 *thousands* from 3 *thousands;* or (which is just the same, but more convenient in practice), instead of supposing the *upper* figure, 4, to be *diminished*, when we borrow 1, we may suppose the *lower* figure, 2, in the same column, *increased*, that is, we may add or *carry* 1 to it, and say, 3 *from* 4, *there remains* 1, that is, 1 *thousand;* and so on.

14. The *Answer* of any sum in Subtraction may be *proved* to be correct by adding the remainder to the *lower* number, when the result will be the *upper* number, if the sum be worked correctly.

Ex. 6.

(1.) 567897 344663	(2.) 754389 430125	(3.) 978546 623124	(4.) 886179 543025
(5.) 437628 312335	(6.) 765147 343523	(7.) 842738 526275	(8.) 350644 217293
(9.) 660821 343517	(10.) 784023 252608	(11.) 892360 651427	(12.) 570356 467129
(13.) 156743 89268	(14.) 723502 267175	(15.) 280251 97814	(16.) 910310 456278
(17.) 365312 93856	(18.) 762004 85773	(19.) 160035 76783	(20.) 713000 5678

(21.) 43560127 37681549	(22.) 118016500 79492878	(23.) 83214623 64923594
(24.) 80012345 432987	(25.) 67125040 8434765	(26.) 10010203 431859
(27.) 43110123 36687194	(28.) 104777326 4889678	(29.) 54267542 26879468
(30.) 96543002 87864579	(31.) 75111654 67222387	(32.) 87654321 12345678

Ex. 7.

(1.) What sum, taken from a million, will leave six hundred and eighty-nine thousand and eighty-seven? What sum, added to seven hundred and forty-five, will make a thousand?

(2.) John gave James twenty-five nuts, and George gave him seventeen. But James gave away eleven to his little brother, William. How many nuts does James now want to make up fifty?

(3.) How many years passed between the Creation (B. C. 4004) and the Deluge (B. C. 2348)? How many between Saul's being made king (B. C. 1095) and the Babylonish Captivity (B. C. 587)?

(4.) How long is it to Christmas, 1852, since the Conquest of England by William the Norman, A. D. 1066?

(5.) Her Majesty Queen Victoria was born in 1819, and the Prince of Wales in 1841. What was the age of each at Christmas, 1852?

(6.) A farmer had ninety-one head of poultry in his yard. There were twenty-seven fowls, twenty-nine ducks, and seventeen geese, and the rest were turkeys. How many were there of these latter?

(7.) A basket contained seventy-five eggs. Forty-seven were ducks' eggs, and the rest chickens' eggs. How many were there of these?

(8.) A boy carried home his nuts in a bag. Before he started he counted them, and found that he had gathered two hundred and thirteen: but, when he got home, he discovered that there was a hole in the bag, and that he had only a hundred and seventy-nine nuts left. How many had he lost?

(9.) A brewer's vat held one thousand and one gallons of beer. Four hundred and ninety-nine gallons were drawn off, and then three hundred and forty-seven gallons more were added. How much beer was now in the vat?

(10.) Tom's grandfather was just as much older than Tom's father, as Tom's father was older than Tom. Now Tom was nine years old, and his father was thirty-four. How old was his grandfather?

(11.) From four trees I gathered a thousand pears. From one I took two hundred and seventy-seven, from another. eighty-nine *more*, and from a third, eighty-nine *less*, than this, What number did I take from the fourth tree?

(12.) A grocer has goods in his shop, worth three hundred and seventy-nine pounds, and a hundred and twenty-seven pounds' worth of debts owing to him; but thirty-nine pounds of these are *bad* debts, and he himself owes two hundred and fifty-eight pounds. How many pounds is he actually worth?

(13.) William Coleman took up seven hundred and seventy-six potatoes from his plot of ground. Two hundred and forty-three of these, he found, were diseased; and a hundred and forty-four he had to lay by for seed. How many had he for use?

(14.) There were three hundred and twenty-two apples on a tree, of which a hundred and ninety-eight were gathered, and eighty-seven were blown down by the wind. How many were left on the tree?

(15.) What is the difference between one thousand and one and one hundred and two? between one thousand and twelve and eleven hundred and one?

(16.) *A* and *B* start together from the same place, and in the *same* direction. *A* walks, the first day, twenty-nine miles, and rides, the second, seventy-six; *B* rides, the first day, sixty-seven miles, and walks, the second, nineteen. How many miles are they apart at the end of the second day? How many miles would they be apart, if they set off in *opposite* directions?

(17.) Two trains start from the opposite ends of the line from London to Edinburgh, four hundred and twenty-eight miles apart. After that one has gone a hundred and eighty-nine miles, and the other a hundred and ninety-eight, how far will they be asunder?

(18.) The Duke of Wellington was born May 1, 1769, and died Sept. 14, 1852. Napoleon Buonaparte was born Feb. 5, 1768, and died May 5, 1821. How old was each of them at the date of the battle of Waterloo, June 18, 1815? And how old was each at the time of his death?

(19.) Lord Nelson was killed at the battle of Trafalgar. 'And that happened,' said John Brookes, 'on this very same day of the month, when I was a boy of nine years old.' He said this on Oct. 21, 1852, and he was then a man of fifty-six. When did the battle of Trafalgar take place?

(20.) The postman had to deliver sixty-three letters and newspapers on Monday, forty-seven on Tuesday, the same number on Wednesday, twenty-nine on Thursday, thirty-six on Friday, and on Saturday as many as on the previous two days together. He delivered in all twenty-eight newspapers. How many were the letters?

(21.) Four old dames sat down to a cup of tea, and began to reckon up their ages. One was seventy-five, another seventy-two, another sixty-eight, and another sixty-six. 'And, if only my old man were here,' said one of them, 'we should make up just three hundred and fifty years between us.' How old was he?

(22.) A woman has four children, and there are two years between the successive birthdays of each of them. She is now fifty-two years old, and her youngest child is twenty-one. How old was she when her eldest and youngest children were born?

(23.) Adam was created B. C. 4004, and lived nine hundred and thirty years; Methuselah lived nine hundred and sixty-nine years, and died in the year of the Deluge (B. C. 2348). How many years did they live together?

(24.) Shem lived ninety-eight years before the Flood, and died at the age of six hundred; and Abraham was born B. C. 1996. How many years old was Abraham when Shem died?

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15. MULTIPLICATION is the method of finding in a short way what number would be the *sum* obtained, by adding together several of the *same* numbers.

Thus, if we add together *five sevens*, the sum is 35; and 35 is the number given in the Multiplication Table for the value of *five times seven*. By learning, therefore, and remembering this Table, we shall always be able to write down at once the value of *six sevens*, or of any other such quantity, up to *twelve times twelve;* and by means of these, as will be seen, we can multiply any whole number whatever by any other whole number.

$$\begin{array}{r}7\\7\\7\\7\\7\\\hline 35\end{array}$$

16. Since the same number, 35, is also the value of *seven fives*, or of *seven times five*, we observe that, when two numbers are multiplied together, it matters not which of the two we take as the multiplier. It is best to take the *least* of the two as multiplier.

Thus, suppose we had to multiply 2 by 3467, that is, to take the number 2 *three thousand four hundred and sixty-seven times*, or to add together *three thousand four hundred and sixty-seven twos*, it would be easier to multiply 3467 by 2 (as is done in the next example), that is, to take the number 3467 *twice*, or to add together 3467 and 3467.

17. The numbers multiplied together in any case are called *factors*, and the result is called the *product*.

Thus 5 and 7 are the *factors* of the *product* 35; and 2 and 3467 are the *factors* of the *product* 6934, as found in the following example.

Ex. $\begin{array}{r}3467\\2\\\hline 6934\end{array}$ When the multiplier, as in this case, is not higher than 12, we must first place it with the units-figure under that of the number to be multiplied, or *multiplicand*.

Then we begin to multiply, saying, *twice* 7 *is* 14—*put down* 4 *and carry* 1, that is, we put down the 4 *units* under the units-figure, and carry the 1, which means 1 *ten*, to be added to the *tens*.

We then go on, saying, *twice* 6 *is* 12, *and* 1 (that is, the 1 carried) *is* 13 (that is, 13 *tens*)—*put down* 3 *and carry* 1, that is, we put down the 3 *tens* under the tens-figure, and carry the 1, which means 1 *ten of tens* or 1 *hundred*, to be added to the hundreds; and so on.

Ex. 8.

(1.) 234321 × 2. (2.) 153263 × 2. (3.) 264372 × 2.
(4.) 257086 × 2. (5.) 275294 × 2. (6.) 357342 × 3.
(7.) 469875 × 2. (8.) 573684 × 3. (9.) 536874 × 3.

(10.) 542354 × 4.
(11.) 258796 × 3.
(12.) 671839 × 4.
(13.) 457936 × 4.
(14.) 623420 × 5.
(15.) 596787 × 4.
(16.) 465738 × 5.
(17.) 268795 × 5.
(18.) 430253 × 6.
(19.) 656770 × 5.
(20.) 671984 × 6.
(21.) 3785400 × 6.
(22.) 523143 × 7.
(23.) 485789 × 6.
(24.) 567134 × 7.
(25.) 479365 × 7.
(26.) 631042 × 8.
(27.) 287654 × 7.
(28.) 345672 × 8.
(29.) 867564 × 8.
(30.) 623147 × 9.
(31.) 377697 × 8.
(32.) 461587 × 9.
(33.) 457697 × 9.
(34.) 6200500 × 10.
(35.) 398795 × 9.
(36.) 747302 × 10.
(37.) 523456 × 11.
(38.) 634567 × 12.
(39.) 842567 × 11.
(40.) 563784 × 12.
(41.) 378245 × 11.
(42.) 560874 × 12.
(43.) 654738 × 11.
(44.) 728536 × 12.
(45.) 567348 × 11.
(46.) 766285 × 12.
(47.) 875694 × 11.
(48.) 965738 × 12.

Ex. 9.

(1.) In a school of forty-seven children, how many thumbs are there? how many fingers? how many toes?

(2.) Going along the road, I first met a flock of thirty-seven sheep, then twenty-nine geese, and then eighteen oxen. How many feet were there among them all?

(3.) There was a school-gallery with five benches. The elder children sat, nine in a row, upon the upper two benches, and the younger children, thirteen in a row, upon the other three benches, and there were seven little ones besides. How many children were there, altogether, in the school?

(4.) There are sixty-five houses in a parish, and each contains, on an average, six persons. How many people live in the parish?

(5.) I went a journey, and travelled, first, by coach for seven hours, at nine miles an hour, then by train for five hours, at twenty-eight miles an hour, and then on horseback for three hours, at eight miles an hour. How far did I travel?

(6.) The children of a school might have been all ranged in five three-sided classes, with five children in the middle, and six on each side, in each class. How many were they?

(7.) How many bottles are there in twenty-seven dozen of wine? How many *more* are there in a hundred and eleven dozen and eleven?

(8.) A man and his team walk nine miles in ploughing an acre of ground. How many miles will they walk in ploughing a field of thirteen acres?

(9.) There are twelve numbers published every year of the *Gospel Missionary*, and each number contains sixteen pages. How many pages do they make, when bound up in one volume?

(10.) How many pages would there be in seven volumes of the *Gospel Missionary?* And how many would there be, supposing one number were missing out of four of the seven volumes?

(11.) Take the number forty-seven nine times; add to the product the *difference* of nine score and eleven dozen; and take away from the result the *sum* of nine dozen and eleven score. What now remains?

(12.) A pair of sparrows, while feeding their young, will destroy four hundred and eighty-four caterpillars every day. How many will they destroy in seven days?

(13.) A fleet consisted of two ships of a hundred and twenty guns each, three of a hundred and ten guns, nine seventy-fours, and five frigates of forty-eight guns each. How many guns were there altogether?

(14.) In a village-school there were forty-seven children, and in the infant-school twenty-six. How many cakes were required to be made, so as to give four to each of the former, and two to each of the latter? and how many were left of this number on the day of distribution, when, the weather being wet and stormy, only thirty-eight of the school-children came, and only nine of the infants?

(15.) I want to send back seven dozen and seven empty bottles to a wine merchant, and I find that I have only seventy-seven in my cellar. How many more will be required to make up the full number?

(16.) A child has only twenty teeth in its first set; a full-grown man has thirty-two teeth. How many more teeth will there be among a class of twelve adults than among a class of twelve children, supposing none to have lost any?

(17.) At a school-feast a bag of nuts was distributed. Eleven children in the first class received each thirty-two nuts, twelve in the second received each twenty-four, seventeen in the third received a dozen each, and twenty-six others nine each. How many nuts were given away in all?

(18.) A farmer had a hundred and seventy-seven lambing ewes one season. Of these seventy-nine had twins, and the others single lambs. But he lost one of the former, and five of the latter, with their lambs in each case. How many had he in his flock at the end of the season?

(19.) A grocer receives three boxes of oranges. In one there were thirty-three dozen, in another, thirty-five dozen and ten, and in the third, thirty-seven dozen and five. How many oranges were there in all?

(20.) 'I have a dozen dozen nuts in my pockets,' said James. 'But I,' said John, 'have a dozen score in my bag.' How many more had John than James?

(21.) In four bags there are six hundred apples. In one there are six score and sixteen, in another, seven score and seventeen, and in a third, eight score and eighteen. How many were there in the fourth?

(22.) At a sale of a stock of wine, *A* bought seven dozen and six, *B* bought thirteen dozen and five, and *C* bought eleven dozen and two; and there were then fourteen dozen and four remaining. How many bottles of wine were there altogether at first?

(23.) One pipe will *fill* a cistern, at the rate of seven gallons a minute; another will *empty* it, at the rate of ten gallons a minute. The cistern was quite full, when a mischievous boy turned on both the cocks together, and in thirty-six minutes the cistern was emptied. How many gallons did it hold when full?

(24.) Farmer Brown bought of Farmer Johnson twenty-three bullocks, at eleven pounds each; and Farmer Johnson bought of Farmer Brown a hundred and twenty-seven ewes, at two pounds each. Which of them must pay money to the other, and how much?

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18. When the multiplier is found in the Multiplication Table, as the product of any two numbers, we may first multiply by one of those numbers, and then multiply the result by the other. It will generally be best to multiply by the *largest* factor first.

Ex. Multiply 476538 by 42.

$$\begin{array}{r} 476538 \\ 7 \\ \hline 3335766 \\ 6 \\ \hline 20014596 \end{array}\ \textit{Ans.}$$

Here the multiplier 42 is found in the Multiplication table, as the product of 6 *times* 7. We multiply, therefore, first by 7, and then by 6.

19. Numbers that are formed, like 42, by the multiplication together of two or more *factors*, are called *composite* numbers. Numbers which cannot be thus broken up into factors, as, 7, 11, 13, 17, 29, &c., are called *prime* numbers.

20. If the multiplier end with one or more *ciphers*, we may proceed as in the annexed example:

Ex. Multiply 320673 by 4400.

$$\begin{array}{r} 320673 \\ 11 \\ \hline 3527403 \\ 4 \\ \hline 1410961200 \end{array}$$

Here the multiplier 4400 = 44 × 100 = 4 × 11 × 100; and we multiply first by 11, then by 4, and then by 100, the last multiplication being effected by simply annexing two ciphers, by which the 14109612 *units* are changed into 14109612 *hundreds*.

Ex. 10.

(1.) 434746 × 14.
(2.) 360878 × 15.
(3.) 654273 × 16.
(4.) 406986 × 18.
(5.) 676827 × 21.
(6.) 528089 × 220.
(7.) 676384 × 2400.
(8.) 970632 × 25.
(9.) 486037 × 27.
(10.) 368736 × 28.
(11.) 406989 × 320.
(12.) 481896 × 33.
(13.) 590876 × 36.
(14.) 865437 × 480.
(15.) 748548 × 49.
(16.) 860379 × 5400.
(17.) 307685 × 63.
(18.) 586377 × 72.
(19.) 648892 × 77.
(20.) 470682 × 81.
(21.) 463574 × 108.
(22.) 674829 × 121.
(23.) 367684 × 132.
(24.) 674885 × 144.

Ex. 11.

(1.) There are fifteen companies in a regiment of soldiers, and forty-five men in each company. How many are there composing the whole regiment?

(2.) How many buttons will be wanted for a regiment of one thousand two hundred and seventy-nine men, allowing sixteen buttons for each coat?

(3.) How many quills can be plucked from six hundred and eighty-seven geese, if each wing supplies nine quills?

(4.) There are a hundred and seventy-six houses in a street in London, ninety-eight of which contain on an average nine persons each, and the rest fourteen persons. How many live in the street?

(5.) A dozen dozen is called a *gross*. How many are there in twenty-seven gross of steel pens?

(6.) The letter-carrier of a certain neighbourhood walks fourteen miles a day. How many miles does he walk in a year of three hundred and sixty-five days?

(7.) There are four-and-twenty sheets in a quire of paper. How many sheets are there in eighteen quires?

(8.) How many pounds of prize-money were divided among seventy-three men, so that each received for his share thirty-six pounds?

(9.) The sum of two numbers is three hundred and one, and the greater of the two is two hundred and seventy-seven. Find their product.

(10.) A railway-train travels at the rate of thirty-seven miles an hour. How far will it go in twenty-four hours?

(11.) A person, holding twenty-nine shares in the Great Western Railway, *sells out at eighty-eight* (that is, he sells each of his shares for eighty-eight pounds). How many pounds did the sale produce? And, if he bought them at seventy-two, how many pounds did he gain by the transaction?

(12.) A boy can point thirteen thousand five hundred and seventy-nine pins in an hour. How many can he point in six days of nine hours each?

(13.) An excursion-train contained forty-two carriages, nine of which were first-class, fourteen second-class, and the rest third-class carriages. Each of the first class carried twelve passengers, each of the second class, sixteen, and each of the third class, twenty-four. How many were there in the train altogether?

(14.) A mason's boy went up and down a ladder with mortar thirty-six times a day. The ladder had twenty-eight rounds. How many steps did he take upon it in the course of the day?

(15.) A body of soldiers can be arranged in a column with thirty-nine men in front and twenty-four in depth. How many are wanted to make up a *square*, with thirty-two men in front and thirty-two in depth?

(16.) In Lord Eliott's famous defence of Gibraltar (1779—1783), besides a vast number of shells and other missiles, there were fired by the English fifty-seven thousand one hundred and sixty-three shot, and a hundred and seventy-five thousand seven hundred and forty-one by the enemy (Spaniards and French). Reckoning the average weight of each ball as eighteen pounds, how many pounds' weight of iron was fired by each party in round shot only?

(17.) A cistern can be emptied by two pipes, one running fourteen gallons, and the other fifteen gallons, a minute. They were both opened together for fifteen minutes, and then the cistern, which had been quite full, was emptied. How many gallons did it hold when full?

(18.) A cattle-dealer went to a fair, with six hundred pounds to lay out. He bought twenty-eight oxen at seventeen pounds each, and seventeen hogs at six pounds each. How many pounds did he carry home of his money?

(19.) Ten thousand bricks are to be carted. When fourteen carts have carried off twenty-four dozen each, and thirteen others, twenty-one score each, how many remain to be carted?

(20.) In a large Sunday-school there were fourteen classes, and three forms for each class, each of which would hold seven children. How many could be seated altogether?

(21.) Each page of the *Gospel Missionary* contains twenty-nine lines, and each line on the average fifty-four letters. How many letters are used for a number of sixteen pages?

(22.) Find a number which is seventy times as great as seven times seventeen dozen.

(23.) In a large building there were sixty-seven windows. Each window had twelve rows, and each row had eight squares of glass. How many squares were wanted for the whole?

(24.) A railway company has seventy-five engines, five hundred and forty-seven carriages, and eight hundred and nineteen trucks. Each engine has eight wheels, each carriage, six, and each truck, four. On the average, there are fifteen spokes to each wheel. How many spokes are there altogether?

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21. When the multiplier is a *prime* number, we may proceed as in the following examples:—

Ex.

$$\begin{array}{r} 3964\times 59 \\ 6 \\ \hline 23784 \\ 10 \\ \hline 237840 \\ 3964 \\ \hline 233876 \end{array}$$

[Here the multiplier $59=60-1=6\times10-1$. We, therefore, multiply first by 6, then by 10, and then subtract once 3964. When practicable, let the product of the factors differ from the given number only by unity, for then the first line may be written down without any further multiplication. Thus $47=6\times8-1$, rather than $6\times7+5$. Similarly $37=12\times3+1$ or $4\times9+1$.—ED.]

Or thus:

$$\begin{array}{r} 3964 \\ 59 \\ \hline 35676 \\ 19820 \\ \hline 233876 \end{array}$$

Here $59=50+9$; we first multiply by 9, then by 50, that is, we multiply by 5, and carry each figure of the product thus obtained one place to the left, which is as good as to multiply by 5 and by 10.

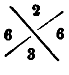
22. The latter method can be employed, whether the number be prime or composite.

$$\begin{array}{r}
1276\times1080\\
1080\\
\hline
102080\\
0000\\
1276\\
\hline
1378080
\end{array}
\qquad
\begin{array}{r}
1276\\
1080\\
\hline
102080\\
12760\\
\hline
1378080
\end{array}$$

Ex. In this example we multiply by 108, as above (of course, since 108 = 9 × 12, we *might* have multiplied first by 12, then by 9); and then, since 1080 = 108 × 10, we further multiply by 10, by annexing a cipher, which moves each figure of the former product one place to the left. Instead of having a row of ciphers, for the multiplier 0, such a sum is usually worked in the manner shown in the second process.

23. The *Answer* of any sum in Multiplication may be *proved* to be correct by the process of *casting out nines*, the reason of which is too complicated for this book, but is shown in Colenso's Algebra, Part ii., Art. 131. The method itself, however, is very easy of application, as follows:—Add up the figures in the upper number, or *multiplicand*: find (by the Mult. Table) the greatest number of *nines* which their sum contains, and set down the remainder; do the same with the figures of the lower number, or *multiplier*: then multiply together these two remainders, and do the same with the figures of this product: lastly, do the same with the product of the two numbers themselves. Then, if the sum be worked correctly, the two remainders last found will be identically the same. If they be *not* the same, it is certain that the sum is *wrong*; but if they *are* the same, we can only infer that the sum is *probably* right; because, if we had by mistake either left out, or put in, a figure 9 in the product of the two numbers, or if we had put down its figures in a wrong order, the above test would not enable us to detect the error.

The remainders are usually put down in the corners of a cross, as in the example here given:



$$\begin{array}{r}
8768\\
678\\
\hline
70144\\
61376\\
52608\\
\hline
5944704
\end{array}$$

Ex. Here, for the *multiplicand*, 8+7+6+8=29, and 29 = 27+2; we set down the remainder 2 in the upper angle of the cross: next, for the *multiplier*, 6 + 7 + 8 = 21, and 21 = 18+3; we set down 3 in the lower angle of the cross: then 2 × 3 = 6, which, being less than 9, must be itself set down in one of the other angles; lastly, for the product of the two numbers, 5+9+4+4+7+0+4 = 33, and 33 = 27+6; we set down the remainder, 6, in the fourth angle of the cross, and, since it is the same as that last set down, we may infer that the sum is probably correct.

Ex. 12.

(1.) 574284 × 13.
(2.) 672453 × 170.
(3.) 742889 × 109.
(4.) 867664 × 203.
(5.) 764074 × 29.
(6.) 439497 × 3100.
(7.) 774068 × 37.
(8.) 846675 × 308.
(9.) 749062 × 430.
(10.) 478567 × 47.
(11.) 360684 × 503.
(12.) 498658 × 59.
(13.) 784464 × 6100.
(14.) 946288 × 79.
(15.) 366849 × 805.
(16.) 765658 × 97.
(17.) 486870 × 5006.
(18.) 624774 × 4203.
(19.) 531828 × 2340.
(20.) 724465 × 3058.
(21.) 665838 × 4560.
(22.) 794683 × 5671.
(23.) 864069 × 6783.
(24.) 386847 × 7890.

---

24. The *square* of any number is obtained by multiplying that number by itself: thus, the square of 5 is 5 *times* 5, or 25.

For example, a square of children might be formed by making 25 children to stand in 5 rows with 5 in each row.

The *cube* of any number is obtained by multiplying the *square* of any number by the number itself: thus, the cube of 5 is 5 *times* 5 *times* 5, or 5 times 25, or 125.

Ex. 13.

(1.) Find the square and cube of thirteen.

(2.) Find the difference between the square of nine hundred and ninety-nine and the cube of ninety-nine.

(3.) A barrel of herrings contains two thousand three hundred and eighty-nine fish. How many are there in sixty-seven such barrels?

(4.) It takes thirteen buttons for a gentleman's shirt. When the buttons have been sewn to seventeen shirts, how many will remain out of two gross of buttons?

(5.) The boys of a school would form a square with six in each side; the girls would form one with seven in each side. How many children were there in the school? and how many more would be wanted, to make up a square with twelve in each side?

(6.) One hundred and forty-four hobnails are used for the sole of a *high-low* (man's half boot). How many nails will the shoemaker use to sole the high-lows of nineteen men?

(7.) A ship brought home three hundred and seventy-nine boxes of oranges, each containing two hundred and fifty-eight oranges. How many were there altogether?

(8.) A railway cost in making two thousand and seventy-eight pounds a mile. How many pounds did it cost for its whole length of fifty-seven miles?

(9.) A body of soldiers was formed into a square with thirty-eight men in each side. If they were arranged in a column, thirty-nine deep, with thirty-seven men in front, how many would be over?

(10.) There is a number which exceeds nine dozen and nine by eight score and eight; and there is another, which nine score and nine exceeds by eight dozen and eight. Find the product of these two numbers.

(11.) Great Britain and Ireland contain about ninety thousand square miles of land, and the population is reckoned at three hundred and eight souls to every square mile, nearly. Find the whole population of the two islands.

(12.) If the thirty-seventh part of a tub of herrings was sixty-five herrings, how many fish were there in the barrel?

(13.) There were two hundred and seventy-five shares in a mine, and, one year, the profit per share was a hundred and fifty-two pounds. What did a person receive, who held eleven shares? and what was the whole amount of profits, divided that year among the shareholders?

(14.) A grazier bought eighty-seven oxen at thirteen pounds each, and sold two-and-fifty of them at seventeen pounds each, and the rest at twenty pounds each. What did he gain by his bargain?

(15.) There are two numbers, thirty-five and fifty-three. By how much does the difference of their cubes exceed the cube of their difference?

(16.) It takes four hundred and seventy-five stitches to hem neatly one side of a pocket-handkerchief. How many stitches would it take for seventeen handkerchiefs?

(17.) There are fifty-nine labourers in a parish, and, on the average, each earns twenty-three pounds a year. How many pounds are paid away in wages among them all, in the course of the year?

(18.) A person had eighty-three shares in the London and North-Western Railway. He might have sold out at a hundred and twenty-seven: but he missed the opportunity, and at last was obliged to sell at a hundred and ten. How many pounds did he obtain for his shares? and how many did he lose in consequence of his mismanagement?

(19.) Find the difference between the square of a thousand and nine, and the cube of a hundred and nine.

(20.) A good cow yields a hundred and sixty-eight pounds of butter *per annum*. It takes two hundred and fifteen thousand cows to supply London with butter. How many pounds of butter are annually consumed in that city?

(21.) The earth is distant from the sun ninety-five millions of miles, and the planet Neptune is nineteen times farther off. At what distance from the sun does Neptune revolve?

(22.) Two men were gathering walnuts. One took up two hundred and forty-nine score and three, and the other four hundred and seventeen dozen and one. How many did the first take up fewer than the second, and how many did they pick up between them?

(23.) In Alison's History of Europe there are twenty volumes, in each volume three hundred and seventy-two pages, in each page forty-two lines, and in each line forty-five letters. How many letters does the whole work contain?

(24.) Suppose three volumes of the above History were lost, and thirty-two pages torn out of one of the others, how many letters would there be in the set, as it now stands?

---

25. DIVISION is the method of finding how often one number is contained in another, that is, how often one number must be taken to make up another. Hence, Division bears the same reference to *Subtraction* that Multiplication does to *Addition*: for we might go on subtracting the divisor from the dividend, and then again from the first remainder, then from the second remainder, then from the third, and so on, until the final remainder is either *zero* or *less* than the divisor itself; and if we counted the number of times we had subtracted it, this would be the result required, or, as it is called, the *quotient*.

But the Multiplication Table will enable us much more easily to divide one number by another. Thus, since 7 *times* 9 *is* 63, if we divide 63 by 7, the quotient will be 9, or if we divide by 9, the quotient will be 7.

26. When the divisor is not higher than 12, we may proceed as follows:

Ex. $4\overline{)372303}$
$93075\frac{3}{4}$

First, set a loop before the number to be divided, or *dividend*; and in that loop place the *divisor*. Then take the first figure of the dividend, 3 (which means 3 *hundred-thousands*), and consider if 4 will go into it. It will not: then take the first *two* figures of the dividend, 37 (which means 37 *ten-thousands*), and consider if 4 will go in this number, 37. It will go 9 times, with 1 over: set down the 9 (which means 9 *ten-thousands*) in the ten-thousands-column, under the 7 of the dividend, and carry the 1 (which means 1 *ten-thousand*), to be taken with the next figure of the dividend, 2 (which means 2 *thousands*). We have now 1 *ten-thousand* and 2 *thousand*, or 12 *thousand*; and 4 goes in 12 3 times, with none over: set down the 3 (which means 3 *thousands*) in the thousands-column, under the 2 of the dividend, and carry nothing. We have now 3 *hundreds*, and, as we cannot divide 3 by 4, we take the next figure also of the dividend, and so we have 30 *tens*; and 4 goes in 30 7 times, with 2 over; set down the 7 (which means 7 *tens*) in the tens-column, and carry the 2 tens to be taken with the 3 units of the dividend, making altogether 23 units. Then 4 goes in 23 5 times, with 3 over; set down the 5 (which means 5 *units*) in the units-column, and place, as is usual, the final remainder 3 over the divisor with a line between them, as $\frac{3}{4}$.

It appears, then, that 4 will be contained 93075 times in 372303, with 3 over: that is, we might subtract 4 from 372303 so many (93075) times, and we should then have 3 remaining.

27. The quantity $\frac{3}{4}$ in the last example is called a *fraction*, of which we shall speak more at length presently.

When we have no need to state the actual value of the remainder in any case, it will be sufficient to indicate that there *is* a remainder, and not a clear integral quotient, by appending the sign (+).

Thus the last answer may be written 93075 (+).

Ex. 14.

(1.) 468206 ÷ 2.
(2.) 563874 ÷ 2.
(3.) 384576 ÷ 2.
(4.) 765798 ÷ 2.
(5.) 753913 ÷ 2.
(6.) 670491 ÷ 3.
(7.) 297135 ÷ 2.
(8.) 476805 ÷ 3.
(9.) 3[illegible]9654 ÷ 3.
(10.) 642716 ÷ 4.
(11.) 156789 ÷ 3.
(12.) 763528 ÷ 4.
(13.) 250473 ÷ 4.
(14.) 676035 ÷ 5.
(15.) 177855 ÷ 4.
(16.) 885637 ÷ 5.
(17.) 317204 ÷ 5.
(18.) 7342842 ÷ 6.
(19.) 260707 ÷ 5.
(20.) 984474 ÷ 6.
(21.) 296531 ÷ 6.
(22.) 342650 ÷ 7.
(23.) 471215 ÷ 6.
(24.) 563472 ÷ 7.
(25.) 690548 ÷ 7.
(26.) 765432 ÷ 8.
(27.) 877654 ÷ 7.
(28.) 3462144 ÷ 8.
(29.) 812347 ÷ 8.
(30.) 7856424 ÷ 9.
(31.) 987648 ÷ 8.
(32.) 3700125 ÷ 9.
(33.) 776655 ÷ 9.
(34.) 23456789 ÷ 10.
(35.) 1234567 ÷ 9.
(36.) 567213 ÷ 10.
(37.) 43698567 ÷ 11.
(38.) 47620332 ÷ 12.
(39.) 57362459 ÷ 11.
(40.) 68742372 ÷ 12.
(41.) 37652473 ÷ 11.
(42.) 56773417 ÷ 12.
(43.) 65507329 ÷ 11.
(44). 55762080 ÷ 12.
(45.) 66138028 ÷ 11.
(46.) 73128468 ÷ 12.
(47.) 87312467 ÷ 11.
(48.) 1481481468 ÷ 12.

Ex. 15.

(1.) I looked under a school-desk, and saw fourteen feet. How many children were sitting at the desk?

(2.) I counted the fingers of all the children in the school, and found there were five hundred and thirty-six. How many children were there?

(3.) There were seventy-two children in a Sunday-school, and they walked two and two to church. How many rows did they make? How many rows would there have been, if they had walked three and three?

(4.) A boy had ninety-seven filberts. He kept thirty-four for himself, and divided the rest equally among his nine class-fellows. How many did he give to each?

(5.) There were a hundred and eleven children in a village-school, who were arranged so as to stand along three sides of the room, the same number on each side. How many stood on each side?

(6.) A crop of potatoes was laid up in eleven heaps. How many on the average did each heap contain, if the whole crop consisted of three thousand and three potatoes?

(7.) How often must we take the number *seven* to make eight hundred and nineteen? How often the number *nine*?

(8.) Divide a paper of two hundred and sixty-four pins equally into eight papers, and also into eleven papers.

(9.) The less of two numbers is eleven, and their difference is nine score and seven. What is the quotient of one by the other?

(10.) Two thousand six hundred and ninety-one poles were used in a certain hop-garden, and three are wanted for each plant. How many hop-plants were there?

(11.) A smith uses nine nails in putting on one shoe: and in one day he used up four hundred and thirty-two nails. How many hoofs did he shoe?

(12.) A bag of nuts was divided equally among eleven school-boys. There were two hundred and twelve nuts in the bag. How many did each boy get, and how many were over?

(13.) A tradesman made a profit of six hundred and seventy-nine pounds in seven years. How many pounds did he gain each year on the average?

(14.) In a house, with twelve windows, there were a hundred and ninety-two squares of glass. How many squares had each window on the average?

(15.) How many dozen apples are there in nineteen score and four? How many half-dozens in twenty-seven score and eighteen?

(16.) A wood of one thousand nine hundred and ninety-five trees is to be thinned, by cutting down one tree in seven. How many will be left after this clearing?

(17.) In a besieged town there were only two hundred and three hundred-weight of flour left in store, and seven hundred-weight were consumed daily. How many days would the flour last?

(18.) There are twelve equal piles of cannon-balls, and it is known that the whole number is forty-two thousand and twenty-four. How many are there in each pile?

(19.) One man could reap a field in fifty-four hours. In how many hours would *two* such men reap it? In how many would *three* such men? In how many, *nine*?

(20.) There is a number such that, when multiplied by eleven, the product is seven hundred and fifty-nine. Find it.

(21.) A gentleman left property to the amount of four thousand pounds. He ordered by his will that, after paying his debts, amounting to three hundred and ninety-eight pounds, and a legacy of a hundred and forty-four pounds to a charity, the rest should be equally divided among his seven children. How many pounds did each of them receive?

(22.) A farmer at a fair bought three hundred pounds' worth of cattle, buying sheep at two pounds, and oxen at nine pounds each. His sheep cost him a hundred and seventy-four pounds. How many of each kind did he buy?

(23.) There were a hundred and thirty-three boys in a school, who were ranged in seven classes. In the first there were nine boys, in each of the second, third, and fifth, twice as many, in the fourth, twenty-four, and seventeen in the sixth. How many were there in the seventh class, and how many *on the average* in each?

(24.) Five men pumped water from a brewer's vat. One pumped, for *five* minutes, *fifteen* gallons in a minute; another, for *six* minutes, *sixteen* gallons a minute; another, for *seven* minutes, *seventeen* gallons a minute; and so on. How many gallons did they pump altogether? and how many did the first pump *less*, and the last, *more*, than if they had each pumped an equal quantity of the whole?

28. When the divisor is found in the Multiplication Table, being a *composite* number, made up of factors each less than 12, we may divide first by one of them, and then by the other. It is best to divide by the *least* of them first.

Ex. 1.	Ex. 2.	Ex. 3.
3)763245÷15	4)346243÷28	2,0)28759,3÷20
5)254415	7)86560(+)	$14379\frac{13}{20}$.
50883	12365(+)	

In Ex. 3 is seen the method of dividing most easily, when the divisor ends with a cipher.

If it be desired to find the actual remainder in such a case as Ex. 2, it may be done as follows:

4)346243
7)86560+3
12395+23

Here the first remainder is three units of the *first* line; the second remainder is 5 units of the *second* line; but each of these units is equivalent to 4 of those in the first line; hence the five are equivalent to 20 of those in the first line, and, adding the first remainder, 3, we have the whole final remainder, 23, and the answer may be written $12365\frac{23}{28}$.

Ex. 16.

(1.) 27643532÷14.
(2.) 35762445÷15.
(3.) 34672352÷16.
(4.) 47623554÷18.
(5.) 35672360÷20.
(6.) 40673427÷20.
(7.) 56248733÷20.
(8.) 86245973÷30.
(9.) 37802616÷24.
(10.) 76438775÷25.
(11.) 78234657÷40.
(12.) 67123747÷50.
(13.) 67001234÷49.
(14.) 73123084÷45.
(15.) 81234733÷70.
(16.) 91236407÷77.
(17.) 10546279÷84.
(18.) 36427567÷80.
(19) 44567357÷96.
(20.) 56731247÷108.
(21.) 63472463÷120.
(22.) 70623435÷121.
(23.) 68437123÷132.
(24.) 35712729÷144.

---

29. When the divisor is any prime or composite number, greater than 12, we may proceed as follows:

Ex. 1.	Ex. 2.
28)346243($12365\frac{23}{28}$	29,00)13467,89($464\frac{1189}{2900}$
28	116
66	186
56	174
102	127
84	116
184	11,89
168	
163	
140	
23	

Ex. 2 shows the method of proceeding when the divisor ends with one or more ciphers.

Ex. 17.

Divide 987654321 by

(1.) 37. (2.) 430. (3.) 5900. (4.) 76.
(5.) 870. (6.) 9500. (7.) 1070. (8.) 253.
(9.) 507. (10.) 2005. (11.) 3509. (12.) 46800.

Ex. 18.

(1.) How many score are there in eighteen dozen and four?

(2.) How many dozen, and how many gross, are there in twenty-one score and twelve?

(3.) Out of a regiment of twelve hundred men, sixty were officers. How many men were there to each officer?

(4.) A common field, of two hundred and eighty-six acres, is to be divided equally among thirteen persons. How many acres will fall to the share of each?

(5.) The dividend of a sum in division is nine hundred and eighty-seven, the quotient is fifty-four, and the remainder is fifteen. What is the divisor?

(6.) A cart and a carriage were bought for ninety-five pounds, and the carriage cost eighteen times as much as the cart. How many pounds did each cost?

(7.) By how much does the thirty-seventh part of three thousand six hundred and sixty-three fall short of a hundred?

(8.) How many solid squares, with eight men on each side, can be formed out of a million of men?

(9.) Three hundred and thirty-six bees will weigh an ounce, and four thousand and thirty-two will just fill up a quart-pot. How many ounces will a swarm weigh, that just filled two quarts?

(10.) What number must be added to the square of eighty-three, that it may be exactly divisible by the square of thirty-eight?

(11.) The produce of a hundred and ten sugar-canes is six pounds of sugar. How many pounds were raised from a plantation, in which there were a million two hundred and thirty-three thousand two hundred and ten canes?

(12.) For one page of a book, containing thirty-six lines, two thousand and fifty-two letters were used. How many letters were used on the average for each line?

(13.) The length of a railway is eighty-one miles, and it cost, in making, a million of money, all but fifty-five pounds. How many pounds did it cost per mile?

(14.) How many shares must be made of the above railway to produce the above amount, at forty-five pounds each?

(15.) Two trains start at 10 A.M. from London and Edinburgh, four hundred and twenty-eight miles apart. The average speed of each is eight-and-twenty miles an hour. Between what hours will they pass each other?

(16.) There were three hundred and sixty men in a besieged garrison, with provisions for eighty days. At the end of sixty days, as no relief appeared, two hundred of the number were sent secretly away. How many days longer will the provisions last?

(17.) A body of soldiers is drawn up in a column, fourteen deep, with a hundred and seventy-seven men in front. If the depth had been increased by seven, what would have been the number of men in front?

(18.) A gentleman gave away, one Christmas, two hundred and sixteen shillings among six men and nine women, giving to each man thrice as much as he gave to each woman. How many shillings did he give to each?

(19.) If nineteen oxen cost two hundred and sixty-six pounds, and one ox is worth seven sheep, what is the price of a sheep in pounds?

(20.) Two hundred and seventy-six spiders will produce as much silk as twenty-three silkworms. How many spiders will be required to produce as much as one silkworm?

(21.) A draper began business, three-and-twenty years ago, with a capital of two hundred and seventy-five pounds. His stock is now valued at a thousand two hundred and eighty-six pounds; and he has also debts due to him, to the amount of four hundred and fifty-seven pounds, of which, however, eighty-nine pounds are *bad* debts. He himself owes sixty-eight pounds. How many pounds has he gained upon the *average* per annum since he has been in business?

(22.) A vessel can be filled in fifty-seven minutes by one pipe, which discharges six gallons a minute, and emptied in thirty-eight minutes by another. How many gallons a minute does the latter discharge? and in how many minutes would the vessel be emptied, if both pipes were running together?

(23.) The earth's diameter at the equator is seven thousand nine hundred and twenty-five miles in length. How many hours would it take a railway-engine, going at the rate of five-and-twenty miles an hour, to travel that distance?

(24.) A bag of a thousand nuts was brought into a school, where there were forty-three children, and they were told to take one each in turn, till all were gone. How many nuts would each get in this way? And how many would be over?

---

30. To find the *half* of any number, we have only to divide by 2, to find the *third*, by 3, to find the *fourth*, by 4, and so on.

Ex. Find the half of 237468, and the fifth of 523768.

$$\begin{array}{r} 2\underline{)237468} \\ 118734 \end{array} \qquad\qquad \begin{array}{r} 5\underline{)523768} \\ 104753\frac{3}{5} \end{array}$$

31. To find *two-thirds* of any number, we must take *one-third* of the number, and then double it, or, in other words, we must first divide by 3, and then multiply by 2. In practice, however, it will be found most convenient, first to multiply by 2, and then to divide by 3.

So, likewise, to find *three-fourths* of any number, we must first multiply by 3, and then divide by 4; to find *two-fifths*, we must first multiply by 2, and then divide by 5, &c.

Ex. Find two-thirds of 173065 and four-sevenths of 567266.

$$\begin{array}{r} 173065 \\ \underline{2} \\ 3\underline{)346130} \\ 115376\frac{2}{3} \end{array} \qquad\qquad \begin{array}{r} 567266 \\ \underline{4} \\ 7\underline{)2269064} \\ 324152 \end{array}$$

32. *One-half* is represented by $\frac{1}{2}$, *two-thirds* by $\frac{2}{3}$, &c.: and such expressions, as has been already mentioned in **27.**, are called *fractions*, that is, *broken pieces*, of a number.

The upper number in any fraction is called the *numerator*, and the lower, the *denominator*. The reason for these names will appear hereafter.

33. When such quantities stand by themselves, it is understood that they represent fractions of *unity*, whatever the unit may be, according to the nature of the question in which it occurs.

Thus the unit might be the number 1. or the number 100, or an apple, or a dozen of eggs, or a thousand pounds, or a bag of nuts, &c.; and $\frac{2}{3}$ would mean two-thirds of *one* such unit, while $\frac{2}{3}$ of 12 would mean two-thirds of *twelve* such units.

34. From what has just been said, it follows that $\frac{3}{4}$ represents *three-fourths of one*: whereas in **27.** we have used it to represent 3 divided by 4, or *one-fourth of three.* It is easily seen, however, that these two expressions are identical.

Thus, we shall get the same, whether we divide *one* orange into four equal parts, and then take *three* of those parts (which is to take *three-fourths of one*), or divide *three* oranges, each into four equal parts, and then take *one* part out of each orange (which is to take *one-fourth of three*). Of course, the number of parts *left* would be different in the two cases, in the former *one*, in the latter *nine*: but the number of parts *taken* will be the same in both cases, so that *three-fourths of one* is the same thing as *one-fourth of three*, and the same expression, $\frac{3}{4}$, may be used for both of these quantities. Hence it appears that to express $3 \div 4$, we may write $\frac{3}{4}$, as in **27.**; so one-fifth of twenty-seven, or $27 \div 5$, may be written $\frac{27}{5}$, &c.

35. To multiply a number by such a quantity as $3\frac{4}{5}$ means, to take, first, *three times* the number, and then *four-fifths* of it, and to add these together.

So, to multiply by $1\frac{1}{2}$, is to take *once* the number and *half* the number, and to add these together. And this is what is meant by taking *half as much again as* any number.

Ex. 1. Multiply 271346 by $7\frac{2}{3}$.

$$\begin{array}{r} 271346\times 7 \\ 7 \\ \hline 1899422 \\ 180897\frac{1}{3} \\ \hline 2080319\frac{1}{3} \end{array} \qquad \begin{array}{r} 271346\times\frac{2}{3} \\ 2 \\ \hline 3)\overline{542692} \\ \hline 180897\frac{1}{3} \end{array}$$

Ex. 2. Multiply 310425 by $1\frac{1}{5}$.

$$\begin{array}{r} 310425 \\ 1\frac{1}{5} \\ \hline 310425 \\ 62085 \\ \hline 372510 \end{array}$$

[*Obs.* Ex. 1. might also be done, rather more easily, by multiplying the number by 8, and subtracting $\frac{1}{3}$ of it; because $7\frac{2}{3}=8-\frac{1}{3}$.—ED.]

Ex. 19.

(1.) Find the *half* of 379124 and of 578948.
(2.) Find the *third* of 623472 and of 607821.
(3.) Find the *fourth* of 734656 and of 624032.
(4.) Find *two-thirds* of 473484 and of 604836.
(5.) Find *three-fourths* of 687348 and of 302424.
(6.) Find *two-fifths* of 708235 and of 682347.
(7.) Find *five-sixths* of 712345 and of 311274.
(8.) Find *four-sevenths* of 612346 and of 998817.
(9.) Multiply 246786 by $7\frac{1}{2}$, and 365073 by $5\frac{1}{3}$.
(10.) Multiply 461472 by $3\frac{1}{4}$, and 531475 by $9\frac{1}{6}$.
(11.) Multiply 612347 by $14\frac{2}{3}$, and 825696 by $16\frac{3}{4}$.
(12.) Multiply 734417 by $20\frac{3}{5}$, and 540029 by $24\frac{5}{6}$.

Ex. 20.

(1.) From London to Norwich, by the Eastern Union (Colchester) line, is a hundred and twelve miles. By the Eastern Counties (Cambridge) line, it is an eighth as far again. How many miles is it by this line?

(2.) John Brookes gathered four hundred and seventy-six apples from his orchard this year: but Thomas Smith gathered half as many again, and William Brown, as many as the other two together. How many had they between them?

(3.) *A* had twenty-eight marbles in his pocket, *B* had twenty-two. They played together, and, at first, *A* lost half his marbles to *B*, but, after that, *B* lost a third of his to *A*. How much has each now?

(4.) From a tank of water, containing thirteen hundred gallons, John pumped a hundred and ninety-six gallons, Thomas half as many again, and William half as many again as the other two together. How many gallons remained in the tank?

(5.) Out of a heap of seven hundred and eight potatoes, one-third were diseased, and one-fourth of the rest were kept for seed. How many were for use?

(6.) Says John to James, 'If you give me thirteen of your nuts, I shall then have only half as many as you.' Now James has fifty-seven nuts: how many has John?

(7.) Out of a bag of thirty-six shillings, I gave a third to *A*, and a fourth to *B*, two-thirds of the remainder to *C*, and the rest to *D*. How many shillings did each receive?

(8.) What is that number, whose seventh part is seven score, seven dozen, and seven? And what is the eleventh part of that number?

(9.) A tradesman began business with a capital of three hundred pounds, and increased it every year by a fifth of what it was at the beginning of it. How many pounds had he at the end of the first year? And how many at the end of the second?

(10.) Tom's father was two-and-forty years old, when Tom's schoolmaster told him, that, in twelve years' time, he would be just half the age of his father at that time? What was Tom's age?

(11.) By how many does the fifth part of seven dozen and one exceed the seventh part of five score and five?

(12.) A man, who had a legacy of four hundred pounds, bought with it eleven acres and a half of land, at thirty-four pounds an acre. How many pounds had he now left of his legacy?

(13.) A person leaves by will five hundred and twenty-five pounds to each of his five daughters, a third as much again to each of his three sons, and the rest of his estate to an hospital. He died worth five thousand pounds. How many pounds went to the hospital?

(14.) Find the difference of two-thirds of a hundred and eleven, and three-fourths of a hundred and twelve, and add the result to four-fifths of a hundred and ten.

(15.) Which is worth most, eleven-twelfths of a hundred and fifty-six pence, or nine-elevenths of a hundred and seventy-six pence?

(16.) A market-woman had in her basket twenty-six eggs. Now, if she had had half as many more, a dozen, a score, and three, how many eggs would she have had?

(17.) In a certain village there were a hundred and forty-four men, and a hundred and thirty-two women. The boys were a fourth as many again as the men, and the girls a third as many again as the women. How many were there in the parish altogether?

(18.) Six thousand bricks were carried off by twelve carts. How many would be carried by a third as many carts again, each a fourth as large again as the former?

(19.) Out of a flock of a hundred and twenty sheep and lambs, a sixth were ewes, with each two lambs; and there were a fourth as many again, with each one lamb. How many were the others, which had not yet lambed?

(20.) There were three hundred and sixty books belonging to a parochial lending library. Three-fifths of them were out, one-twelfth were at the rectory to be repaired, and one-fifth as many more were lost. How many were left upon the shelves?

(21.) There were sixty-five pairs of birds in a rookery at the beginning of the season, each of which had on the average four young ones. Three-fifths of the young ones were shot during the spring. How many birds were there after that?

(22.) A man went by train a hundred and ten miles on Monday, and, on Tuesday, half as far again; he went by coach forty-five miles on Wednesday, and, on Thursday, a third as far again; he walked sixteen miles on Friday, and, on Saturday, a fourth as far again. How far did he travel altogether?

(23.) I have two strings, one of which is sixty-four yards long. If I cut off a quarter of this and give it to the other, it will become sixty yards long. If, instead, I had cut off a quarter from the second, and had joined it to the first, what would have been its length in yards?

(24.) The book *A* has thirty-six lines in each page, and forty-two letters in each line. The number of lines in each page of *B* is five-sixths of the number in each page of *A*; but *B* has in each line a seventh as many letters again as *A*. Find the number of letters in a page of *A*, and of *B*.

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36. Examples of the following kind are usually given under the *Rule of Three*, but may be very easily solved by a more simple way of reasoning.

Ex. 1. If six bushels of wheat cost twelve pounds (£12), what will thirteen bushels cost?

Here 6 bushels cost £12;
therefore, 1 bushel costs $\frac{1}{6}$ of £12,
and 13 bushels cost $\frac{13}{6}$ of £12, or £26.

Ex. 2. If fifty-seven oxen are bought for £513, how many can be had for £387, at the same rate?

Here £513 will buy 57 oxen;
therefore, £1 will buy $\frac{1}{513}$ of 57 oxen,
and £387 will buy $\frac{387}{513}$ of £57, or 43 oxen.

Ex. 21.

(1.) If five chairs cost one pound (£1), what would be the cost of twenty-five such chairs? What of eighty?

(2.) If eighteen horses cost in keeping £6 a week, what will ninety horses cost per week? What will twelve horses cost?

(3.) If thirty ounces of gold be sold for £105, what will be the value of eighteen ounces? What of eighty-eight?

(4.) If I travel by railroad twenty-four miles for six shillings (6*s.*), what shall I have to pay for going seventy-six miles? and how many miles might I go for 17*s.*?

(5.) If I buy forty-five sheep for £50, what should I have to give for twenty-seven? and how many might I buy for £60?

(6.) If three hundred and seventy-five apples be sold for 12*s.*, what must be given for a thousand apples? and how many may be had for 20*s.*?

(7.) If eight pounds of coffee cost 10*s.*, what would be the cost of a hundred pounds? and how many pounds may be had for 25*s.*?

(8.) If fifteen days' wages for one man amount to 25*s.*, what would be the wages of ten men for nine days each?

(9.) If the rent of forty-two acres of land be £66, what will be the rent of ninety-one acres of the same land? and how many acres of it may be had for £121?

(10.) If thirty-five loads of hay can be had for £100, what must be given for sixty-three loads of the same? And how many loads can I buy for £180?

(11.) I gave £88 for forty dozen of wine, and a friend asks me to let him have fifteen dozen of it. What ought I to charge him?

(12.) A farmer took a flock of forty-five ewes to market, and sold twenty-seven of them for £51. What would he have made if he had sold the whole at the same rate?

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37. Examples of this kind are also given commonly under the *Rule of Three.*

Ex. If 5 men will plough a field in 36 hours, how many hours would it take 3 men to plough the same field?

Here 5 men can do the work in 36 hours;
therefore, 1 man would do it in 36 × 5, or 180 hours,
and 3 men would do it in 180 ÷ 3, or 60 hours.

Ex. 22.

(1.) If 50 workmen can do a piece of work in 12 days, how many days will it take 75 workmen to do the same work?

(2.) If 14 men can reap a field of corn in 15 hours, how many hours will it take 10 men to reap the same field?

(3.) If 112 sheep can be grazed in a field for 9 days, for how many days might 84 sheep have been grazed in the same field?

(4.) If 10 men will consume a store of provisions in 24 days, how many days would it last 15 men?

(5.) If 20 men can build a wall in 16 days, how many men would build it in 10 days?

(6.) If £100 will support a family of 9 persons for 8 months, for how many months would it support a family of 12 persons?

(7.) If a pipe, throwing off 10 gallons a minute, will empty a cistern in 15 minutes, how many minutes would it take a pipe, running 25 gallons a minute, to empty it?

(8.) There was food in a besieged garrison to last the 75 men, who were in it, for 39 days. If 10 men were sent away, for how many days would the provisions last?

(9.) If a person, travelling 9 hours a day, would finish a journey in 4 weeks, how many weeks would he take for it, if he travelled 12 hours a day?

(10.) A coach used to go from London to Liverpool, at the rate of 9 miles an hour, in 24 hours. How many hours will the train take, at the rate of 36 miles an hour?

(11.) A besieged town, containing 22,400 inhabitants, has provisions to last 3 weeks. How many must be sent away, that they may be able to hold out for 7 weeks?

(12.) If a ton of turnips will last 25 sheep a fortnight, how many days would it last for 35 sheep?

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38. Measures of Money.

4 farthings (*f.*) make 1 penny (*d.*).
12 pence — 1 shilling (*s.*).
20 shillings — 1 sovereign or pound (£).

One farthing, or *fourth* of a penny, is denoted by $\frac{1}{4}$*d.*; *two* farthings, or a *halfpenny*, by $\frac{1}{2}$*d.*; *three* farthings, by $\frac{3}{4}$*d.*

A *florin* = 2 shillings.
A *crown* = 5 shillings.
A *guinea* = 21 shillings.

d.		*s.*	*d.*	*d.*		*s.*	*d.*
12	make	1	0	80	make	6	8
20	—	1	8	84	—	7	0
24	—	2	0	90	—	7	6
30	—	2	6	96	—	8	0
36	—	3	0	100	—	8	4
40	—	3	4	108	—	9	0
48	—	4	0	110	—	9	2
50	—	4	2	120	—	10	0
60	—	5	0	130	—	10	10
70	—	5	10	132	—	11	0
72	—	6	0	140	—	11	8

144*d.* make 12*s.*

39. Addition of Money.

	£	s.	d.
Ex.	13	0	8¼
	2	5	6
	23	4	7½
	37	8	0¾
	0	13	4¼
	76	12	2¾

Here, adding up the farthings in the right-hand column, I have 7*f*., which is 1¾*d.*; I set down ¾*d.*, and carry 1*d.* to the pence column. Adding up the pence with the 1*d.* carried, I have 26*d.*, which is 2*s.* 2*d.*; I set down 2*d.*, and carry 2*s.* to the second, or shillings, column. Adding up the shillings column with the 2*s.* carried, I have 32*s.*, which is £1 12*s*; I set down the 12*s.*, and carry £1 to the first, or pounds, column: and so on.

Ex. 23.

	£	s.	d.
(1.)	37	13	6
	29	12	4
	6	3	9
	55	17	2
	7	11	10

	£	s.	d.
(2.)	6	14	7
	38	6	6
	73	0	11
	5	17	4
	99	18	9

	£	s.	d.
(3.)	77	11	4
	8	12	7
	59	10	0
	2	17	11
	14	0	5

	£	s.	d.
(4.)	3	7	6½
	69	11	10
	13	0	4¼
	37	13	2¾
	26	15	7

	£	s.	d.
(5.)	18	15	7¼
	76	14	2
	0	10	2¾
	13	13	3½
	66	4	7¼

	£	s.	d.
(6.)	43	17	5½
	9	19	8¾
	46	7	4½
	35	8	8
	19	9	7½

	£	s.	d.
(7.)	231	13	5¾
	13	0	0½
	57	15	7
	102	1	2¾
	0	7	4½

	£	s.	d.
(8.)	1	2	3¾
	130	17	1½
	37	15	4
	239	10	3¾
	101	11	10

	£	s.	d.
(9.)	70	17	6½
	102	12	10¼
	35	14	6
	311	11	11¾
	2	5	10

	£	s.	d.
(10.)	87	17	9½
	129	2	8¾
	47	6	5
	115	13	7½
	49	19	9¼

	£	s.	d.
(11.)	113	10	11¼
	76	15	7½
	82	9	4
	166	14	8¾
	29	7	11

	£	s.	d.
(12.)	17	6	5¼
	132	13	4
	98	7	8½
	266	0	7¾
	87	17	11

40. SUBTRACTION OF MONEY.

	£	*s.*	*d.*
Ex.	23	6	8½
	19	17	10¾
	3	8	9¾

Here, as I cannot take ¾*d.* from ½*d.*, I borrow 1*d.*, and take ¾*d.* from it, leaving ¼*d.*, which, with the ½*d.* in the upper line, makes ¾*d.* to be set down. As I have borrowed 1*d.*, I must either *take away* 1*d.* from the 8*d.* of the upper line, or *add* 1*d.* to the 10*d.* of the lower line. It will be best to do the latter. Then 11*d.* *from* 8*d.*, *I cannot*: I must, therefore, borrow 1*s.* or 12*d.*; and 11*d.* from 12*d.* leaves 1*d.*, to which I must add the 8*d.* of the upper line, making 9*d.* to be set down. Having borrowed 1*s.*, I must carry 1*s.*, to be added to the 17*s.* of the lower line: then 18*s.* *from* 6*s.* I *cannot*; I must, therefore, borrow £1 or 20*s.*; and 18*s.* from 20*s.* leaves 2*s.*, to which I must add the 6*s.* of the upper line, making 8*s.* to be set down. Then, having borrowed £1, I must carry £1, to be added to the £19 of the lower line, making £20; and £20 from £23 leaves £3.

Ex. 24.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(1.)	54	15	6¾	(2.)	61	17	10	(3.)	75	18	6
	37	12	2¼		23	14	8¼		6	13	8

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(4.)	82	11	9	(5.)	93	11	7	(6.)	101	13	0
	77	17	7		6	14	1½		99	2	7

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(7.)	112	0	5	(8.)	124	1	8½	(9.)	133	2	7½
	109	9	2¾		99	11	4¾		27	13	7¾

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(10.)	104	10	0	(11.)	152	0	0	(12.)	160	0	1¾
	77	10	5		39	0	3¾		158	19	2¾

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(13.)	225	0	1¼	(14.)	300	1	2½	(15.)	357	13	7
	105	0	10½		111	11	11¾		89	0	9½

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(16.)	444	4	4½	(17.)	500	0	0¼	(18.)	505	10	1¼
	77	7	7¾		234	15	6½		379	19	11¾

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(19.)	618	13	0	(20.)	666	16	6	(21.)	702	3	4¼
	508	17	10¼		88	18	8½		508	14	17½

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(22.)	800	1	0	(23.)	927	0	8	(24.)	1000	1	0¼
	608	17	11¾		77	11	9		959	17	10½

Ex. 25.

(1.) A parish overseer went out one day to collect the rates. He took at the first house 3*s.* 9*d.*, at the second, £2 0*s.* 1½*d.*, at the third, 11*s.* 7*d.*, at the fourth, £7 6*s.* 8*d.*, and at the fifth, 17*s.* 4½*d.* How much had he by this time in his pocket?

(2.) Going on to the next house, where he had to take £8 17*s.* 7½*d.*, he was paid with a £10 note. How much had he to give in change, and how much money had he now in his pocket?

(3.) An old woman died, and, on searching her premises, there were found, in an old stocking, seven sovereigns and a half sovereign; in a tea-pot, three five-pound notes and eleven sovereigns, with 15*s.* 16*d.* in silver; and, in a corner of a drawer, a bag, containing, in gold and silver, £7 17*s.* 8*d.* How much did the whole amount to?

(4.) Thomas Brown earns 9*s.* a week, his wife 3*s.* 6*d.*, one of his sons 5*s.* 6*d.*, and another 2*s.* 9*d.* They have 1*s.* 8*d.* a week to pay for rent. How much have they for food and clothing?

(5.) John Brookes made £6 10*s.* 6*d.* by his last harvest. But he owed £3 13*s.* 6*d.* for rent, £1 2*s.* 9*d.* for shoes, 12*s.* 7*d.* for flour, and 7*s.* 8*d.* for clothes. How much had he left of his harvest-earnings, after paying these debts?

(6.) A farmer gave a sheep, worth 34*s.* 6*d.*, in exchange for a pig, valued at 17*s.* 3*d.*, and a bushel of barley, worth 14*s.* 8*d.* How much money must be paid him, to make the bargain a fair one?

(7.) I went into the town with a £5 note in my pocket; and paid half-a-crown for a pair of gloves, 8*s.* 3*d.* for a hat, 9*s.* 9*d.* for a pair of shoes, and a bill at the grocer's amounting to £3 19*s.* 2½*d.* How much had I now in my pocket?

(8.) Take a penny-farthing from eighteen-pence, and take what remains from a crown, and take what remains from a sovereign, and tell me, what now remains?

(9.) *A* has in his pocket £1 5*s.* 7*d.*, and *B* has £1 3*s.* 10*d.* If *A* gives to *B* 19*s.* 4*d.*, and *B* gives to *A* 17*s.* 10½*d.*, who will have most, and by how much?

(10.) At a Missionary Meeting four collecting-boxes were opened. In the first there was 17*s.* 2½*d.*, in the second, 11*s.* 5*d.*, in the third, £1 0*s.* 7¾*d.*, and in the fourth, 13*s.* 4½*d.* How much did these sums amount to in all?

(11.) Said a man to his wife one day, 'Every week, my dear, I give you one pound one: now let me know how you spend it.' So the wife began to reckon: 'Half-a-crown for rent, three-and-sixpence for bread, two-and-twopence for butter, sugar, and tea, tenpence for coals, and sixpence for wood and coke, twopence halfpenny for needles, pins, and thread, and sixpence halfpenny for soap, three-and-sixpence for meat, three shillings for potatoes and greens, and threepence halfpenny for milk.—She stopped a minute, and said, 'Pray, how much is that?'

(12.) 'Then every morning for your breakfast I for you must something make,
And once or twice a week, you know, you'll have a pound of steak.
That will average one-and-sevenpence more, but that is not enough;
You have ninepence for tobacco, fourpence halfpenny for snuff.
And since you take me very close,—you must confess you do,—
There's three-pence halfpenny every week for soda, starch, and blue;
A pint of beer you have each day—there's tenpence halfpenny gone:
Now reckon up, and see what's left out of your one pound one.'

41. Multiplication of Money.

	£	*s.*	*d.*
Ex. 1.	37	11	3½
			3
	112	13	10½

	£	*s.*	*d.*
Ex. 2.	107	13	5¾
			11
	1184	9	3¼

Ex. 26.

	£	*s.*	*d.*			£	*s.*	*d.*	
(1.)	37	11	2	× 2.	(2.)	47	0	11¼	× 2.
(3.)	57	11	10½	× 2.	(4.	63	7	3½	× 3.
(5.)	73	8	8½	× 4.	(6.)	67	18	11¼	× 3.

	£	s.	d.		£	s.	d.
(7.)	76	13	$4\frac{1}{4} \times 4$.	(8.)	80	7	$6\frac{1}{2} \times 5$.
(9.)	92	0	$9\frac{1}{2} \times 6$.	(10.)	87	15	$7\frac{1}{2} \times 5$.
(11.)	97	10	$2\frac{3}{4} \times 6$.	(12.)	99	12	$0\frac{1}{2} \times 7$.
(13.)	101	3	$3\frac{3}{4} \times 8$.	(14.)	127	5	$2\frac{1}{4} \times 7$.
(15.)	139	17	$6\frac{1}{2} \times 8$.	(16.)	147	0	$9\frac{1}{2} \times 9$.
(17.)	152	11	$7\frac{1}{4} \times 10$.	(18.)	166	16	$6\frac{3}{4} \times 9$.
(19.)	171	13	$7\frac{1}{2} \times 10$.	(20.)	177	17	$0\frac{3}{4} \times 11$.
(21.)	180	0	$11\frac{1}{4} \times 12$.	(22.)	183	3	$3\frac{1}{4} \times 12$.
(23.)	192	12	$8\frac{1}{2} \times 12$.	(24.)	203	17	$9\frac{3}{4} \times 12$.

Ex. 27.

Find the value of

(1.) 7 yards of muslin, at £1 13*s*. $6\frac{1}{2}$*d*. per yard.
(2.) 9 ounces of silver, at 4*s*. 9*d*. per ounce.
(3.) 5 hogsheads of beer, at £4 7*s*. 5*d*. per hogshead.
(4.) 11 gallons of rum, at 13*s*. 8*d*. per gallon.
(5.) 6 ounces of gold, at £3 2*s*. $8\frac{1}{2}$*d*. per ounce.
(6.) 11 bushels of wheat, at 8*s*. $7\frac{1}{4}$*d*. per bushel.
(7.) 8 pounds of nutmegs, at 7*s*. $7\frac{1}{2}$*d*. per pound.
(8.) 10 bales of cotton, at £9 12*s*. 4*d*. per bale.
(9.) 12 loads of hay, at £4 6*s*. 9*d*. per load.
(10.) 7 chests of tea, at £25 7*s*. $9\frac{1}{2}$*d*. per chest.
(11.) 8 sacks of flour, at £2 11*s*. 5*d*. per sack.
(12.) 12 cows, at £9 9*s*. 9*d*. each.

Ex. 28.

(1.) What would a peal of eight bells cost, if each bell cost £52 6*s*. 8*d*., and the fixing of the whole set £31 6*s*. 8*d*. more?

(2.) In a certain printing-office there were seven men employed at 50*s*. a week, nine at 31*s*. 6*d*., eleven at 14*s*., and twelve boys at 4*s*. 3*d*. What was the whole amount of weekly wages?

(3.) How much is left out of £4, after paying for three pairs of shoes at 9*s*. 4*d*. each, six pairs of stockings at 1*s*. $8\frac{1}{2}$*d*. each, and shirts, &c. to the amount of £1 9*s*. $9\frac{1}{2}$*d*.?

(4.) A farmer took a sow and her nine pigs to market. The sow he sold for 37*s*. 6*d*., and the pigs for 8*s*. 9*d*. each, and he then bought five lambs at 9*s*. 10*d*. each. How much money did he carry home?

(5.) What is the poor-rate upon £11 at $8\frac{1}{2}$*d*. in the £? What is it upon £7 at 2*s*. $11\frac{3}{4}$*d*. in the £?

(6.) A person bought nine shares in a railway, for each of which he gave £13 7*s.* 6*d.* He afterwards sold five of them at £23 12*s.* 6*d.* each. How much more money did he want to make up his original outlay?

(7.) A family gleaned twelve bushels of wheat one season. What was it worth at 4*s.* 7½*d.* a bushel?

(8.) A hive of bees will yield on an average six pounds of honey, which may be sold at 11½*d.* a pound. How much may a cottager expect to gain in a year, one with another, by keeping nine hives of bees?

(9.) A bankrupt's estate can only pay 13*s.* 5½*d.* in the £. What shall I receive for a debt he owes me of £5, and how much shall I lose upon it?

(10.) A party of six travelled by the Great Western Railway from London to Plymouth, for which distance the fares are, first-class, 56*s.*; second-class, 38*s.* 6*d.*; third-class, 20*s.* 7*d.* How much did they pay for six second-class tickets? What did they save by not travelling first-class? What would they have saved more by travelling third-class?

(11.) A man bought a pig for thirteen-and-fourpence, and it cost him, for five weeks' fattening, two-and-tenpence halfpenny a week. It weighed nine stone when killed, and was sold at six-and-fivepence halfpenny a stone. What was the gain upon it?

(12.) A dealer bought nine oxen, at £11 17*s.* 8*d.* each. One of them died; but the others he sold again, with a profit of £2 12*s.* 2½*d.* upon each. What was his whole gain by the transaction?

	£	*s.*	*d.*			£	*s.*	*d.*
[**42**. Ex. 1.	57	11	6½ × 36		Ex. 2.	107	0	11¾ × 100
			4					10
	230	6	2			1070	9	9½
			9					10
	2072	15	6	Ed.]		10704	17	11

Ex. 29.

	£	*s.*	*d.*		£	*s.*	*d.*
(1.)	29	11	10½ × 14.	(2.)	43	0	5½ × 18.
(3.)	47	2	3¼ × 20.	(4.)	55	15	7½ × 33.
(5.)	64	14	7¼ × 45.	(6.)	67	2	8¼ × 56.
(7.)	72	0	10½ × 63.	(8.)	75	4	0¾ × 70.
(9.)	87	17	5¾ × 84.	(10.)	94	14	3¼ × 120.
(11.)	96	17	0½ × 132.	(12.)	99	19	11¾ × 144.

43. The following is the most convenient way of multiplying by a number greater than 12, if not found in the Multiplication Table :—

	£	s.	d.	
1.	3	13	7½	× 17
			10	
	36	16	3	
	25	15	4½	= 7 times 1st line
	62	11	7½	

	£	s.	d.	
[Ex. 2.	0	15	2¼	× 59
			10	
	7	11	10½	
			6	
	45	11	8	
	0	15	2¼	
	44	16	0¾	

See **21.**, p. 19. Ed.]

	£	s.	d.			£	s.	d.	
Ex. 3.	5	14	6½	× 347					
			10						
	57	5	5		=	5	14	6½	× 10
			10						
	572	14	2		=	5	14	6½	× 100
			3						
	1718	2	6		=	5	14	6½	× 300
	229	1	8	= 4 times 2rd line	=	5	14	6½	× 40
	40	1	9½	= 7 times 1st line	=	5	14	6½	× 7
	1987	5	11½		=	5	14	6½	× 347

Ex. 30.

Find the value of

(1.)	13 bullocks,	at £9 10*s.* 11*d.* each.
(2.)	19 sheep,	at £1 17*s.* 4*d.* each.
(3.)	22 ounces of gold,	at £3 17*s.* 7½*d.* per ounce.
(4.)	57 coom of wheat,	at £1 1*s.* 3¼*d.* per coom.
(5.)	73 chests of tea,	at £12 11*s.* 10½*d.* per chest.
(6.)	97 hogsheads of sugar,	at £6 15*s.* 4½*d.* per hogshead.
(7.)	135 acres of land,	at £1 11*s.* 10*d.* per acre.
(8.)	143 barrels of herrings,	at £1 9*s.* 6½*d.* each.
(9.)	207 tons of iron,	at £3 3*s.* 3*d.* per ton.
(10.)	308 pounds of spice,	at 19*s.* 8½*d.* per pound.
(11.)	379 gallons of brandy,	at £1 13*s.* 9*d.* per gallon.
(12.)	497 yards of velvet,	at £1 6*s.* 5½*d.* per yard.

Ex. 31.

(1.) If you had a sovereign and half-a-crown, and were to pay out of it for fifteen pounds of cheese at 7½*d.* a pound, and seventeen pounds of sugar at 8½*d.* a pound, how much change would you carry home ?

(2.) There are fifty-four labourers in a parish, and they earn, on an average, each 8*s*. 8*d*. a week. How much will be required to pay their whole weekly wages?

(3.) A waggon and team and nineteen bullocks were bought at a sale for £306 12*s*. Reckoning the four horses at £33 12*s*. 6*d*. each, and the bullocks at eight guineas each, what was the cost of the waggon?

(4.) A mercer bought a dozen dozen pair of gloves at 2*s*. 2½*d*. a pair. What did the whole parcel cost him? and what did he gain by selling them at 3*s*. 6*d*. a pair?

(5.) A tradesman became insolvent. His debts amounted to £87, and his goods, when sold, were sufficient to pay 7*s*. 8½*d*. in the £. What was their whole value?

(6.) An ox was killed weighing fifty-nine stone. Twenty-three stone being sold at 3*s*. 7*d*. a stone, and the rest at 3*s*. 2¾*d*., what did it sell for altogether?

(7.) A farmer's wife carried to market in one year fifty-seven fowls and twenty-nine ducks. The fowls she sold at 1*s*. 9*d*. each, and the ducks at 2*s*. 3*d*.; and she had besides 1237 eggs, which she sold, on the average, for three farthings each. What did she make by her poultry, allowing for their cost in fattening £6 11*s*. 6*d*.?

(8.) The Jewish shekel was worth 2*s*. 3½*d*. What was the value of a talent, which contained three thousand shekels? What was the value of ten thousand talents?

(9.) What sum must be divided among twenty-three men, so that each may receive £11 11*s*. 11¼*d*.?

(10.) A farmer sold at a fair one hundred and nineteen sheep, at £1 13*s*. 7*d*. each, and bought nineteen oxen at ten guineas each. How much money did he carry home?

(11.) What is the price of eight packs of cloth, each pack containing eight parcels, each parcel eight pieces, and each piece fifty yards, at 3¾*d*. a yard?

(12.) A gentleman's *gross* income is £457 *per annum*. But his outgoings in rates and taxes, &c. amount to 5*s*. 10½*d*. in the £. What is his *net* income?

---

44. DIVISION OF MONEY.

Ex. 1.
$$\begin{array}{rrrl} & \pounds & s. & d. \\ 3) & 7 & 13 & 5\frac{1}{4} \div 3 \\ \hline & 2 & 11 & 1\frac{3}{4} \\ \hline \end{array}$$

Ex. 2.
$$\begin{array}{rrrl} & \pounds & s. & d. \\ 8) & 11 & 17 & 9 \div 8 \\ \hline & 1 & 9 & 8\frac{1}{2}(+) \\ \hline \end{array}$$

Ex. 32.

	£	s.	d.			£	s.	d.	
(1.)	7	16	8	÷ 2.	(2.)	13	0	3½	÷ 2.
(3.)	15	2	7½	÷ 3.	(4.)	31	15	5	÷ 4.
(5.)	7	13	10¼	÷ 5.	(6.)	37	19	4½	÷ 6.
(7.)	37	2	10½	÷ 7.	(8.)	43	7	2	÷ 8.
(9.)	80	8	4½	÷ 9.	(10.)	113	13	6½	÷ 10.
(11.)	207	9	3½	÷ 11.	(12.)	814	13	9	÷ 12.

Ex. 33.

(1.) Five persons on a journey spend £7 9*s.* 5¾*d.* What was each person's share of the expenses?

(2.) A master paid in wages, at the week's end, £1 13*s.* 6*d.* There were two men, who had worked six days each, and three boys, who were paid each half-a-crown. What were the daily wages of each man?

(3.) Seven men bought a cheese between them, which cost £1 7*s.* 8½*d.* They divided it equally; what had each to pay?

(4.) A farmer buys fourteen sheep at 31*s.* 4*d.* each. Three of them having died, at what price did he sell the rest, so as to gain 38*s.* by the bargain?

(5.) I had in my purse £3 18*s.* 9*d.*, and the gold in it was worth eight times as much as the silver. What were the separate amounts of gold and silver?

(6.) A grocer mixed seven pounds of tea at 3*s.* 10*d.* with five pounds at 4*s.* 4*d.*, and sold the whole at 4*s.* 2*d.* a pound. How much did he gain by the transaction? and at what price should he have sold the mixture, so as to have dealt fairly?

(7.) Divide £10 among five persons, so that one may have 10*s.* more than each of the others.

(8.) I bought nine fowls for £1 0*s.* 7*d.* What was the price of one fowl?

(9.) A servant had saved in seven years £44 5*s.* 9½*d.* What had she saved in each year on the average?

(10.) A father left £500 at his death, to be divided among his seven children; but his eldest son was to receive twice as much as any of the rest. What did he receive? and what did the youngest?

(11.) A gentleman distributed £1 13*s.* 9*d.* among three poor labourers and their wives, giving to each man twice as much as to each woman. What did he give to each?

(12.) The expenses of a party of nine were 3*s.* 9½*d.* each; but two sneaked off without paying their shares. What must the others each pay in addition to discharge the bill?

45. Ex. 1.

£ s. d.
4) 356 17 4 ÷ 32
8) 89 4 4
11 3 0½

£ s. d.
or 32) 356 17 4 (£11
32
36
32
4
20
97 (3s.
96
1
12
16 (0d.
4
64 (2f.
64

Ex. 2.

£ s. d.
1,00) 2,35 7 6 ÷ 100
20
7,07
12
,90
4
3,60

Ans. £2 7s. 0¾d. (+)

Ex. 34.

	£	s.	d.			£	s.	d.	
(1.)	561	12	8	÷14.	(2.)	561	12	2¼	÷45.
(3.)	761	8	0	÷64.	(4.)	652	14	2	÷100.
(5.)	26	19	7	÷100.	(6.)	656	17	0	÷72.
(7.)	8765	14	6¾	÷99.	(8.)	2343	15	0	÷1000.
(9.)	723	19	2	÷1000.	(10.)	767	16	3	÷108.
(11.)	9793	8	9	÷1210.	(12.)	6229	3	4	÷10000.

Ex. 35.

	£	s.	d.			£	s.	d.	
(1.)	1234	15	4	÷17.	(2.)	2037	6	10¾	÷23.
(3.)	3125	0	6	÷34.	(4.)	4002	13	4¼	÷41.
(5.)	5678	11	3	÷52.	(6.)	6087	6	4¼	÷65.
(7.)	6999	19	11	÷76.	(8.)	8010	1	10¼	÷89.
(9.)	10425	15	9	÷123.	(10.)	23567	16	3	÷345.
(11.)	35672	19	6	÷456.	(12.)	45678	6	11¼	÷567.

Ex. 36.

(1.) If seventy-five oxen can be bought for £1647 8s. 5¼d., what is the price of one?

(2.) Divide £1901 2s. 2¼d. into ninety-nine equal portions.

(3.) How may a sum of £12 11s. 6¾d. be distributed among five men and six women, so as to give each man thrice the share of each woman?

(4.) A mixture is made of nine gallons of spirits at 12s. 6d. per gallon, six gallons at 18s 9d. and fifteen gallons at 22s. 8d. What is the proper price of a gallon of it?

(5.) If I buy forty-five sheep for £81 10s. 4d., and sell seven of them at 34s. 10d. each, at what price must I sell the rest so as to gain £10 by the bargain?

(6.) Forty-three persons embark between them £250 16*s.* 1*d.* in a speculation, which yields altogether £15006 5*s.* 11$\frac{3}{4}$*d.* What is the share of profit for each person, after allowing for his original outlay?

(7.) A gentleman left to twenty-seven poor people in a parish the sum of £1 1*s.* 1*d.* each. But four had died, and the whole was divided equally among the rest. What did each receive?

(8.) A farmer rents of one landlord fifty-seven acres at 27*s.* an acre, of another twenty-two acres at 33*s.* 6*d.* an acre, and of a third twenty-nine acres at 50*s.* an acre. What rent does he pay per acre *on the average?*

(9.) In a certain savings' bank, seventeen persons out of one village deposited in one year the sum of £53 16*s.* 3$\frac{3}{4}$*d.* What was the deposit of each on the average?

(10.) A man buys thirty-five bushels of potatoes for £4 7*s.* 1*d.* He sells ten bushels of them at 3*s.* 2*d.* each; but finds that six bushels of the remainder are diseased and good for nothing. At what price per bushel must he sell the rest, so as not to lose anything upon the whole by the bargain?

(11.) The yearly expense of a village school was £30. There was an endowment in land to the amount of £8 3*s.* 5*d.* The clergyman gave £10, and the landlords and farmers between them seven guineas. The rest was made up by the school-pence of the children, of whom there were forty-three attending the school. What did each of them pay yearly on the average?

(12.) Divide £10 10*s.* 10*d.* among three men, four women, and six children, so that each man may receive thrice as much as each child, and each woman twice as much.

46. Ex. 1.

$$
\begin{array}{rrl}
\pounds & s. & d. \\
15 & 6 & 7\times 1\frac{3}{4} \\
 & & 1 \\ \hline
15 & 6 & 7 \\
11 & 9 & 11\frac{1}{4} \\
26 & 16 & 6\frac{1}{4} \\ \hline
\end{array}
\qquad
\begin{array}{rrrl}
 & \pounds & s. & d. \\
 & 15 & 6 & 7\times\frac{3}{4} \\
 & & & 3 \\ \cline{2-4}
4) & 45 & 19 & 9 \\ \cline{2-4}
 & 11 & 9 & 11\frac{1}{4} \\ \cline{2-4}
\end{array}
\qquad \text{or} \qquad
\begin{array}{rrll}
\pounds & s. & d. & \\
15 & 6 & 7 & \\
 & & 1 & \\ \cline{1-3}
15 & 6 & 7 & \\
7 & 13 & 3\frac{1}{2} & =\frac{1}{2}\text{ the 1st line} \\
3 & 16 & 7\frac{3}{4} & =\frac{1}{2}\text{ the above} \\ \cline{1-3}
26 & 16 & 6\frac{1}{4} & \\ \cline{1-3}
\end{array}
$$

[Or

$$
\begin{array}{rrll}
\pounds & s. & d. & \\
15 & 6 & 7\times(2-\frac{1}{4}) & \\
 & & 2 & \\ \cline{1-3}
30 & 13 & 2 & \\
3 & 16 & 7\frac{3}{4} & =\frac{1}{4}\text{ the 1st line} \\ \cline{1-3}
26 & 16 & 6\frac{1}{4} & \\ \cline{1-3}
\end{array}
$$

See *Obs.* p. 31. Ed.]

Ex. 2.

£	s.	d.	
25	8	10	$\times 407\frac{3}{5}$
		10	
254	8	4	
		10	
2544	3	4	
		4	
10176	13	4	
178	1	10	= 7 times the 1st line
15	5	$3\frac{1}{2}$	(+) = $\frac{3}{5}$ of the 1st line
10370	0	$5\frac{1}{2}$	(+)

	£	s.	d.	
	25	8	10	$\times \frac{3}{5}$
			3	
5)	76	6	6	
	15	5	$3\frac{1}{2}$	(+)

Ex. 37.

(1.)	1	0	$9\frac{1}{2}$	$\times 3\frac{1}{2}$.	(2.)	8	0	8	$\times 5\frac{2}{3}$.
(3.)	7	16	9	$\times 9\frac{3}{4}$.	(4.)	8	13	$6\frac{1}{2}$	$\times 14\frac{1}{2}$.
(5.)	9	2	7	$\times 32\frac{2}{3}$.	(6.)	1	17	3	$\times 77\frac{3}{4}$.
(7.)	29	13	6	$\times 124\frac{3}{5}$.	(8.)	6	2	6	$\times 235\frac{3}{4}$.
(9.)	7	1	$1\frac{3}{4}$	$\times 357\frac{4}{5}$.	(10.)	8	2	4	$\times 409\frac{3}{4}$.
(11.)	9	11	4	$\times 1347\frac{5}{7}$.	(12.)	6	2	$8\frac{1}{2}$	$\times 3456\frac{9}{10}$.

Ex. 38.

(1.) *A* and *B* went to the gold-diggings. *A* brought back gold to the value of £166 13*s.* 4*d.* and *B* brought back half as much again. What value of gold did they bring back between them?

(2.) A man died without a will, leaving property to the amount of £299 19*s.* $4\frac{1}{2}$*d.* His widow became entitled to one third of this, and the rest was equally divided among his five children. What did the widow and each child receive?

(3.) Out of a sum of £4 7*s.* 6*d.* one-third is paid to *A*, and one-seventh to *B*. After this, three-elevenths of the remainder are paid to *A*, and the rest to *B*. What sums are now in the possession of *A* and *B*?

(4.) A person dies worth £1000, and leaves a third of his property to his wife, half to his son, and the rest to his daughter. The wife at her death leaves two-fifths of the legacy to her son, and the rest to her daughter. How much do they each now possess?

(5.) What must I give for three-quarters of a cheese, if the whole was worth £1 2*s.* 7*d.*? and what for three and a half such cheeses?

(6.) A farmer sold eleven coom and a half of barley at 15*s.* 2*d.* a coom, and bought seven sheep with the money. What did he give for each sheep?

(7.) A grocer bought twelve dozen pounds of tea at 3*s.* 6*d.*

a pound, and made by the sale a fourth as much again as he gave for it. What did he give for the whole? and at what price per pound did he sell it?

(8.) A farm of two hundred and fifty acres was rented at 33*s*. 6*d*. an acre; but, in consequence of low prices, one-tenth of the rent was taken off by the landlord. What was the reduced rent?

(9.) An Australian emigrant sent home ten ounces of gold, to be divided equally among his three sisters. The gold being sold at £3 17*s*. 10$\frac{1}{2}$*d*. per ounce, what was the value of the present to each?

(10.) On leaving home, *A* had a £5 note in his pocket. But he pays to *B* two-thirds of it, and to *C* three-fourths of what remained. How much had *A* now remaining?

(11.) A tailor bought 23$\frac{3}{4}$ yards of silk at 3*s*. 8*d*. per yard. What did the whole piece cost him?

(12.) How much would the tailor gain in (11.), if, after cutting off four yards for his own use, he had sold the rest of the silk at 4*s*. 7*d*. per yard?

---

47. Questions of the following kind are usually given under the Rule of Three. But it is useful here to observe, that a fraction (such as that which arises in the method by which such sums as these are here worked) may always be *simplified*, by dividing both the numerator and denominator by any number that will divide them both without remainder.

Thus $\frac{4}{8}$ is the same as $\frac{2}{4}$ or $\frac{1}{2}$; for it is the same thing, whether we divide the unit into *eight* equal parts, and take *four* of them, or into *four* equal parts, and take *two* of them, or into two equal parts, and take *one* of them.

So $\frac{10}{14}=\frac{5}{7}$, $\frac{12}{18}=\frac{2}{3}$, &c.

Ex. If thirty-two bottles of wine cost £5 1*s*. 4*d*., what will a dozen of the same wine cost?

Here 32 bottles cost £5 1*s*. 4*d*.;
therefore, 1 bottle costs $\frac{1}{32}$ of £5 1*s*. 4*d*.,
and 12 bottles cost $\frac{12}{32}$ of £5 1*s*. 4*d*. $=\frac{3}{8}$ of £5 1*s*. 4*d*.
= £1 18*s*. 0*d*., as before.

Ex. 39.

(1.) If five chairs cost 17*s*. 11*d*., what will be the cost of fifteen such chairs?

(2.) If twenty-seven horses cost in keeping £1 11*s*. 6*d*. a day, what will nine horses cost at the same rate?

(3.) If thirty ounces of gold be sold for £102 7*s.* 6*d.*, what is the value of twelve ounces?

(4.) If I travel by railroad forty-two miles for 10*s.* 6*d.*, what ought I to pay for travelling thirty-five miles on the same line?

(5.) If I buy forty-five sheep for £57 7*s.* 6*d.*, what should I give for fifty at the same rate?

(6.) If twenty-five apples be sold for 3*s.* 1½*d.*, what will a hundred and ninety-five cost?

(7.) If eight pounds of coffee cost 12*s.* 6*d.*, what should I have to give for twenty-nine pounds?

(8.) A man's wages for seventeen days amounted to £1 9*s.* 9*d.* What will they amount to for fifty days' work?

(9.) If nineteen men earn between them £1 11*s.* 8*d.* for a day's work, what would ninety-one men earn at the same rate?

(10.) If sixteen oranges are bought for a shilling, what is that a-piece? And what must I give for a hundred and six oranges?

(11.) If seventeen hundredweight of sugar cost £53 11*s.*, what will be the cost of a hundred and one hundredweight?

(12.) Fifty-three yards of cloth were purchased for £66 5*s.* What will be the cost of seventy-five yards of the same material?

(13.) A farmer rents forty-two acres of land for sixty-one guineas. What would be the rent of sixty-one acres, at the same rate?

(14.) I bought thirty-five loads of hay for £99 3*s.* 4*d.* What should I have to give for fifty-three loads of the same?

(15.) If twenty dozen of wine cost forty-five guineas, what would be the price of twenty-eight dozen of the same?

(16.) Farmer Brookes took three-and-twenty bullocks to market, and sold fourteen of them for £249 18*s.* What would he make by the others, if he sold them at the same rate?

(17.) There was a flock of a hundred and seventy-five sheep, which were reckoned to be worth, one with another, £186 5*s.* But the disease got among them, and forty-two died. What may the flock be supposed to be worth now, valuing the living sheep at the same rate as before?

(18.) If the income-tax on £73 is £2 2*s.* 7*d.*, what will it be on £28?

(19.) If fifteen pounds of salt cost 7*s.* 9¾*d.*, what must I pay for forty-one pounds?

(20.) What is the value of sheep per score, if a hundred and eleven are sold for £209 12*s.* 6$\frac{3}{4}$*d.*?

(21.) What do the poor-rates on twenty-five acres of land amount to, if upon ninety-seven acres they are £16 13*s.* 5$\frac{1}{4}$*d.*?

(22.) If thirty-nine children at a school-feast ate between them cake, &c., to the value of 14*s.* 7$\frac{1}{2}$*d.*, how much will it cost to provide for sixty-seven children at the same rate?

(23.) I gave 17*s.* 6*d.* for three dozen and a half of porter. What should I have to give for eleven dozen and nine of the same?

(24.) Fourteen copies of a book cost £1 6*s.* 3*d.* What would four-and-forty copies of the same book come to?

---

48. By *ten per cent.* is meant *ten* in every *hundred* units, the word *cent.* being abridged from the Latin *centum*, a hundred. Hence *ten per cent.* means 10 in 100, or 1 in 10, or $\frac{1}{10}$ in *every unit*; and, consequently, *ten per cent.* of any quantity is equivalent to $\frac{1}{10}$ of the *whole quantity.*

Ex. Find ten per cent. upon £77 17*s.* 3*d.*

$$\begin{array}{r|rrl} & \pounds & s. & d. \\ 10) & 77 & 17 & 3 \\ \hline & 7 & 15 & 8\frac{1}{2}\,(+) \end{array}$$

49. So *five per cent.* is 5 in every 100, or 1 in 20, or $\frac{1}{20}$ of any quantity. Hence five per cent. upon £1 is 1*s.*, and five per cent. upon £23 is 23*s.*

50. So, likewise, *three per cent.* is 3 in every 100, or $\frac{3}{100}$ of any quantity; which we may easily calculate by first multiplying it by 3, and then dividing the product by 100. And, similarly, with other rates of per-centage.

Ex. Find 3$\frac{1}{2}$ per cent. on £113 15*s.* 7*d.*

$$\begin{array}{rrrll} & \pounds & s. & d. & \\ & 113 & 15 & 7 & \\ & & & 3\frac{1}{2} & \\ \hline & 341 & 6 & 9 & \\ & 56 & 17 & 9\frac{1}{2} & =\frac{1}{2}\text{ the 1st line} \\ \hline 1{,}00) & 3{,}98 & 4 & 6\frac{1}{2} & \\ & 20 & & & \\ \hline & 19{,}64 & & & \textit{Ans.}\ \pounds3\ 19s.\ 7\frac{1}{2}d.\ (+) \\ & 12 & & & \\ \hline & 7{,}74 & & & \\ & 4 & & & \\ \hline & 2{,}98 & & & \end{array}$$

51. There are various kinds of business transactions in which per-centages are charged. Thus, *Interest* is a per-centage paid for the use of money, which is lent by one person to another. The money lent is called the ***Principal,*** and the interest is charged at so much per cent. upon the principal for each year.

Ex. Find the interest for three years on £113 15*s.* 7*d.* at $3\frac{1}{2}$ per cent. per annum.

We may proceed, as in the last example, to find the interest on £113 15*s.* 7*d.* for one year, namely, £3 19*s.* $7\frac{1}{2}$*d.*, and then multiply this by 3, for the three years ; but it is generally most convenient to multiply by the number of years *before* dividing by 100, as below.

	£	*s.*	*d.*	
	113	15	7	
			$3\frac{1}{2}$	= rate per cent.
	341	6	9	
	56	17	$9\frac{1}{2}$	
	398	4	$6\frac{1}{2}$	
			3	= number of years
1,00)	11,94	13	$7\frac{1}{2}$	
	20			
	18,93			*Ans.* £11 18*s.* 11*d.*
	12			
	11,23			
	4			
	,94			

N.B. If we had followed the other method, by taking 3 × £3 19*s.* $7\frac{1}{2}$*d.*, we should have had the result £11 18*s.* $10\frac{1}{2}$*d.*: the difference arises from our leaving out the remainder (+) in the latter process. [It would be easier in this and similar instances, instead of multiplying separately by $3\frac{1}{2}$, and then by 3, to multiply by ($3\frac{1}{2} \times 3$), or $10\frac{1}{2}$, and then divide by 100. Or $10\frac{1}{2}$ per cent. might be found readily, as in **48.**, by taking $\frac{1}{10}$ of the principal, and $\frac{1}{20}$ of that, and adding.

Thus	£113	15	7
$\frac{1}{10}$	11	7	$6\frac{1}{2}$+
$\frac{1}{20}$	0	11	$4\frac{1}{2}$+
	£11	18	11+

Hence, to find the interest on any sum for any number of years, we have the following

Rule. Multiply the principal by the rate per cent., and then by the number of years, and divide the result by 100.

Ex. 40.

(1.) Find 10 per cent. on £27 13*s.* 4*d.*
(2.) Find 5 per cent. on £175 17*s.* 4*d.*
(3.) Find 4 per cent. on £131 15*s.* 7*d.*
(4.) Find 3 per cent. on £221 14*s.* 8*d.*

Find the interest on

(5.) £172 15*s.* 6*d.* for one year, at $3\frac{1}{2}$ per cent.
(6.) £213 10*s.* for one year, at $4\frac{1}{2}$ per cent.
(7.) £371 14*s.* 8*d.* for two years, at 6 per cent.

(8.) £283 14*s*. 6*d*. for two years, at 3¾ per cent.
(9.) £223 13*s*. 4*d*. for three years, at 5 per cent.
(10.) £315 4*s*. 9*d*. for four years, at 4¼ per cent.
(11.) £405 for two years and a half, at 3½ per cent.
(12.) £513 13*s*. 6*d*. for 3 years and a half, at 4¾ per cent.

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52. The last example is one of *Simple* Interest, in which the principal remains the same for every year.

In *Compound* Interest, the interest of any year is added at the end of it to the principal, and the increased amount is taken for the principal of the following year: and so on.

Ex. Find the compound interest on £173 17*s*. 6*d*. at 4 per cent., for two years.

```
   £    s.  d.       £    s.  d.
  173  17   6       173  17   6=Princ. of 1st year
            4         6  19   1=Int. of 1st year
 -------------      ------------
 6,95  10   0       180  16   7=Princ. of 2nd year
   20                         4
 -----              ------------
 19,10              7,23   6  4     £    s.  d.
    12                20           180  16   7  =Princ. of 2nd year
 -----              -----            7   4   7¾=Int. of 2nd year
  1,20              4,66           -------------
     4                12           188   1   2¾=Amt. at end of 2nd year
 -----              -----          173  17   6  =Original Principal
   ,80              7,96           -------------
 -----                 4           £14   3   8¾=Total Interest
                    -----          -------------
                    3,84
                    -----
```

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53. *Fire Insurance* is a per-centage, paid yearly to an Insurance Company for securing property from fire. This is reckoned at a certain fixed rate, depending upon the nature of the property and the danger of fire to which it is exposed, per cent. upon the whole amount of property to be insured. The whole annual payment is called the *Premium*, and the paper or parchment by which the company binds itself to secure the insurer from loss, so long as he pays the premium regularly, is called the *Policy* of Insurance.

Of the same kind is *Hail-and-Storm Insurance*, by which a farmer may secure compensation for any injury that may be done to his crops by hail or rain.

Life Insurance is a per-centage, paid yearly to a company for securing a sum of money upon the death of a person. The premium of a Life Policy is charged per cent. upon the sum to be assured, at a rate depending upon the age and healthiness of the person, whose life is insured, at the time the Policy was first taken out.

In each of the above cases the premium must be paid every year while the policy is in force; but the following charges are, from their nature, paid only once:—

Insurance from Sea-risk is a per-centage charged upon the value of a cargo, to secure it against wreck or damage by sea;

Commission is a per-centage paid to an agent for buying or selling goods;

Brokerage is a smaller per-centage of the same nature as the last, paid usually for transacting money concerns;

Banker's Discount is a per-centage charged, at some fixed rate, generally 5 per cent., for paying money before it is due. It is usually reckoned as the *interest* upon the sum in question, at the given rate, for the given time. Thus, if I took to a banker a *Bill* or *Note of Hand* of a person whose credit was good, by which he engaged to pay the sum of £100 at the end of a year, the banker would discount it at 5 per cent. by paying me £95, keeping for himself £5, which is the interest of £100 for a year.

Strictly speaking, the banker ought to pay me rather more than £95; for he ought to pay me such a sum as, put out to interest during the year, would at the end of it produce with its interest the same amount as the value of the Bill, which would then become due, namely, £100. Now, as £100 would produce £5 interest, of course, £95 would produce less than £5; and, therefore, my £95, put out to interest, would *not* produce £100 at the end of the year, and thus I shall be somewhat a loser, and the banker a gainer, by this transaction. The *true* discount might be found, but the method is too complicated for this book. (See Colenso's Arithmetic, p. 105.)

Tradesmen's Discount is an allowance of 5 per cent. for ready-money payment of an account; and it is generally calculated, with sufficient accuracy, by reckoning a shilling for every pound in the account, and 3*d*. besides for every 5*s*., if there are any shillings besides the pounds.

Thus the discount upon an amount of £10 11*s*. 3*d*. would be 10*s*. 6*d*., and the ready-money payment, consequently, would be £10 0*s*. 9*d*.

Ex. 41.

(1.) What discount would a tradesman allow upon an account of £13 6*s*. 4*d*.?

(2.) I went to pay a bill of £13 16*s*. 8*d*., with three five-pound notes in my pocket. What change shall I get, allowing tradesmen's discount of 5 per cent.?

(3.) What should I have to pay for insuring a vessel and cargo, worth £3225, at $3\frac{1}{4}$ per cent.?

(4.) What is the premium on a life-policy of £3287 13*s*. 4*d*. at $3\frac{1}{2}$ per cent.?

(5.) A merchant insures to the value of £765 upon goods of the value of £734 8*s*., the insurance being charged at 4 per cent. Show that, in case of loss, he will recover by means of this insurance both the value of his goods and the premium.

(6.) Find the compound interest on £99 19*s*. 9*d*. for two years, at 3 per cent.

(7.) What is the premium for insuring goods to the value of £6417, at $2\frac{3}{5}$ per cent.?

(8.) I went to a banker with a good bill for £230, due at the end of half a year. What will he give me in cash, discount at 5 per cent.?

(9.) What is the brokerage on a money transaction of £273 15*s*. 6*d*., at $\frac{1}{6}$ per cent.?

(10.) Find the commission for the purchase of goods to the value of £1426 13*s*. 4*d*., at $2\frac{3}{4}$ per cent.

(11.) At $4\frac{3}{8}$ per cent. show that, by insuring to the value of £447 6*s*. 8*d*., I may cover both my goods, worth £427 15*s*. 3*d*., and my premium.

(12.) Find the compound interest on £333 3*s*. 3*d*., at $3\frac{1}{3}$ per cent., for three years.

Ex. 42.

(1.) Out of a crop of potatoes 35 per cent. were diseased. How many bushels of potatoes were lost out of the whole crop of 175 bushels?

(2.) On opening the tin canisters, which contained Goldner's preserved meats for the British Navy, it was found that 87 per cent. were made up of all kinds of filthy garbage. How many pounds were good out of 3146 pounds of this material?

(3.) In a box of oranges 23 per cent. were decayed. How many were sound out of four hundred and seventy-five?

(4.) When a book is published, the publisher and retail-bookseller take between them 35 per cent. of the selling price. How much is left for printing, binding and the author's remuneration, upon an edition of 2000 copies of a book, which is sold at 4*s*. 6*d*.?

(5.) Common air is composed of two gases, oxygen and nitrogen, 23 per cent. by weight being oxygen. What weight of each is there in 500 grains of air?

(6.) A savings' bank gives interest at the rate of $2\frac{5}{8}$ per cent. per annum. What will £3·6*s.* 8*d.* amount to in one year?

(7.) A workman is paid 35 per cent. upon the selling price of the articles which he is employed to make. What will he get upon an article which sells for 17*s.* 6*d.*?

(8.) The births annually in a parish were 4 per cent. upon the population, and the deaths $2\frac{1}{2}$ per cent. What was the population of the parish at the end of the year, when it was 4668 at the beginning?

(9.) In order to pay off a debt of £1550, which I owed at 5 per cent., I borrowed £800 at 4 per cent., and £750 at $4\frac{1}{2}$ per cent. What shall I save yearly by this arrangement?

(10.) The prime cost of a fifty-gallon cask of wine was £25, and ten gallons have been lost by leakage. At what price per gallon must the remainder be sold, so as to gain 10 per cent. upon the prime cost?

(11.) I bought goods for £48, and sold half of them at a gain of 15 per cent. For how much must I sell the rest, so as to gain 20 per cent. upon the whole?

(12.) A grocer mixes 26 pounds of tea at 5*s.* 3*d.*, 32 pounds at 5*s.* 7*d.*, and 36 pounds at 6*s.* 1*d.* At what rate per pound must he sell the mixture, so as to gain upon the whole 40 per cent.?

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54. REDUCTION OF MONEY.

Ex. 1. Reduce £15 7*s.* $0\frac{3}{4}$*d.* to farthings.

$$\begin{array}{rrr}
\pounds & s. & d. \\
15 & 7 & 0\frac{3}{4} \\
20 & & \\
\hline
307 & & \\
12 & & \\
\hline
3684 & & \\
4 & & \\
\hline
14739 & & \\
\hline
\end{array}$$

Ex. 2. Reduce 25977 farthings to pounds.

$$\begin{array}{l}
4\underline{)\ 25977f.} \\
12\underline{)\ 6494} \ \ldots\ \frac{3}{4}d. \\
2{,}0\underline{)\ 54{,}1} \ \ldots\ 2d. \\
\quad \underline{\pounds 27\ 1s.\ 2\frac{3}{4}d.}
\end{array}$$

Ex. 43.

Reduce

(1.) £22 to pence.

(2.) 351 shillings to farthings.

(3.) 4321 crowns to halfpence.

(4.) £1234 to pence.

(5.) £15 13*s.* 6*d.* to pence.

(6.) £3 14*s*. 10$\frac{1}{4}$*d*. to farthings.
(7.) £42 11*s*. 6*d*. to sixpences.
(8.) 117 half-crowns to halfpence.
(9.) £50 17*s*. 8*d*. to pence. (10.) £73 0*s*. 2$\frac{1}{2}$*d*. to farthings.
(11.) £86 13*s*. 4$\frac{1}{2}$*d*. to halfpence.
(12.) £99 19*s*. 11$\frac{3}{4}$*d*. to farthings.

Ex. 44.

Reduce

(1.) 123456*s*. to pounds. (2.) 234567*d*. to pounds.
(3.) 345678*f*. to pounds. (4.) 456789*d*. to crowns.
(5.) 500801*fl*. to pounds. (6.) 765432*f*. to florins.
(7.) 345627 halfpence to pounds.
(8.) 672378*f*. to pounds.
(9.) 216475 fourpenny bits to pounds.
(10.) 891007 threepenny bits to pounds.
(11.) 331133*f*. to sovereigns.
(12.) 767574 halfpence to pounds.

Ex. 45.

(1.) Find the value of a million of farthings.

(2.) On Oct. 7, 1851, a hundred and nine thousand nine hundred and fifteen persons visited the Crystal Palace. Reckoning them all at a shilling a head, how many pounds were realised on that one day?

(3.) A miser died, and was found to have buried in his garden a pot of money, containing six hundred and thirteen half-crowns. What were these worth?

(4.) *A* has £3 3*s*. 3$\frac{3}{4}$*d*. in his pockets, and *B* has £2 0*s*. 10$\frac{1}{4}$*d*. How many farthings have they between them, and how much money must *A* give to *B*, that the latter may have two thousand farthings?

(5.) If three thousand people pass over Waterloo Bridge, London, in a day, each paying a toll of one halfpenny, what is the amount thus paid daily?

(6.) There are about twenty thousand five hundred and seventy-five copies of the Times sold daily. What does the daily sale of these amount to, at 3*d*. each?

(7.) A French *franc* is worth 10*d*. What is the value in English money of 500 francs?

(8.) What amount of income-tax at 7*d*. in the pound will be paid by a gentleman whose income is £1475?

(9.) If each person in the parish, on the average, were to contribute one penny a month (or a shilling a year) to the Gospel-Propagation Society, what sum would thus be raised in a parish of four hundred and twenty-five people? And what in one of four thousand five hundred?

(10.) On the same supposition, what sum would be raised for Church Missions throughout all England, reckoning its population at sixteen millions?

(11.) The property of a parish is assessed at £1120, and a poor-rate is levied of 8*d.* in the pound. What is the amount of the whole rate?

(12.) Take a shilling, a penny, and a halfpenny, and add to them half-a-crown and sixteen pence halfpenny; and see how often the whole sum is contained in £5.

(13.) Find the value of a *lac* (a hundred thousand) of Calcutta rupees, reckoning the rupee as worth 2*s.* Find also the value of the above sum, reckoning the rupee at its true value, which is a farthing less than 2*s.*

(14.) How many pounds are there in nine hundred and ninety-nine guineas?

(15.) How many guineas are there in nine hundred and ninety-nine pounds?

(16.) How many yards of silk, at 6*s.* 8*d.* a yard, can be bought for £7?

(17.) To how many persons might a sum of £20 5*s.* 2*d.* be distributed, giving £1 11*s.* 2*d.* to each?

(18.) How many half-crowns are there in a hundred and five half-guineas?

(19.) How many half-guineas are there in eighty-four crowns?

(20.) A nugget of gold was sold for £54 9*s.* 9*d.*, at £4 10*s.* $9\frac{3}{4}$*d.* an ounce. How many ounces did it weigh?

(21.) An equal number of men and women earned together £4 5*s.* in five days. The men earned 1*s.* 10*d.*, the women 1*s.*, a day. How many were there of each?

(22.) How much water must be added to a cask containing ten gallons of spirits, at 13*s.* a gallon, to allow of the price being reduced to 8*s.* 8*d.* a gallon?

(23.) If the property of the country be valued at a hundred and eighty-six millions and a half, what will be the produce of the income-tax, at 7*d.* in the pound?

(24.) The United States' dollar is worth 4*s.* 1*d.* What is the value of the President's salary, which is 25,000 dollars?

55. Practice is a short method of finding the value of any quantity of merchandise, when the value of a unit of any kind is given. The way of proceeding in different cases will appear from the following examples.

Ex. 1.

		36 at £3 10*s.*
		3
		108
10*s.*	$\frac{1}{2}$	18
		£126

Here we first find the value of £3 × 36; and then, observing that 10*s.* is £$\frac{1}{2}$, we find the value of 10*s.* × 36 by taking $\frac{1}{2}$ of £36 or £18, and thus we get £126 for the whole product of £3 10*s.* × 36.

Ex. 2.

		25 at £2 16*s.* 8*d.*		
		2		
		50		
10*s.*	$\frac{1}{2}$	12	10	
6*s.* 8*d.*	$\frac{1}{3}$	8	6	8
		£70	16	8

Here, having found, as before, the value of £2 10*s.* × 25, we observe that the remainder of the price, 6*s.* 8*d.*, is £$\frac{1}{3}$, and we find the value of 6*s.* 8*d.* × 25 by taking $\frac{1}{3}$ of £25 or £8 6*s.* 8*d.*

For this purpose it should be observed that

10*s.* = £$\frac{1}{2}$ 6*s.* 8*d.* = £$\frac{1}{3}$ 6*d.* = $\frac{1}{2}$*s.* 4*d.* = $\frac{1}{3}$*s.*
5*s.* = £$\frac{1}{4}$ 3*s.* 4*d.* = £$\frac{1}{6}$ 3*d.* = $\frac{1}{4}$*s.* 2*d.* = $\frac{1}{6}$*s.*
2*s.* 6*d.* = £$\frac{1}{8}$ 1*s.* 8*d.* = £$\frac{1}{12}$ 1$\frac{1}{2}$*d.* = $\frac{1}{8}$*s.* 1*d.* = $\frac{1}{12}$*s.*

Ex. 46.

		£	*s.*	*d.*			£	*s.*	*d.*
(1.)	23 at	3	10	0.	(2.)	29 at	5	6	8.
(3.)	37 at		5	0.	(4.)	39 at	4	4	0.
(5.)	41 at	7	3	4.	(6.)	43 at		2	6.
(7.)	53 at	2	1	8.	(8.)	62 at	6	11	0.
(9.)	77 at		16	8.	(10.)	83 at	8	13	4.
(11.)	95 at	3	11	8.	(12.)	127 at		12	6.

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56. **Ex. 1.**

		37 at £5 17*s.* 6*d.*		
		5		
		185		
10*s.*	$\frac{1}{2}$	18	10	0
5*s.*	$\frac{1}{2}$	9	5	0
2*s.* 6*d.*	$\frac{1}{2}$	4	12	6
		£217	7	6

Having found, as before, the value of 37 times £5 10*s.*, we observe that, out of the remaining 7*s.* 6*d.* of the price, 5*s.* is $\frac{1}{2}$ of 10*s.*, and, therefore, we may obtain the value of 37 times 5*s.* by taking $\frac{1}{2}$ of 37 times 10*s.*, that is, by taking $\frac{1}{2}$ of £18 10*s.* = £9 5*s.*; so 2*s.* 6*d.* is $\frac{1}{2}$ of 5*s.*, and we may obtain the value of 37 times 2*s.* 6*d.* by taking $\frac{1}{2}$ of 37 times 5*s.*, that is, by taking $\frac{1}{2}$ of £9 5*s.* = £4 12*s.* 6*d.* [See **58.** Ed.]

Ex. 2. 71 at £4 8*s.* 11½*d.*

		4		
		284		
5*s.*	¼	17	15	
3*s.* 4*d.*	$\frac{1}{6}$	11	16	8
7½*d.*	⅛	2	4	4½
		£315	16	0½

Here we find, as in Ex. 1, the value of 71 times £4 5*s.*: then we observe that of the remaining 3*s.* 11½*d.*, 3*s.* 4*d.* is £$\frac{1}{6}$; and we may find the value of 71 times 3*s.* 4*d.*, by taking $\frac{1}{6}$ of £71 =£11 16*s.* 8*d.* Again, 7½*d.* being ⅛ of 5*s.*, 71 times 7½*d.*=⅛ of 71 times 5*s.* =⅛ of £17 15*s.*=£2 4*s.* 4½*d.* ED.]

Ex. 47.

	£	*s.*	*d.*		£	*s.*	*d.*
(1.) 142 at	2	15	0.	(2.) 267 at	3	7	6.
(3.) 317 at	3	12	0.	(4.) 482 at		17	6.
(5.) 528 at	8	17	0.	(6.) 677 at	2	18	6.
(7.) 761 at		14	6.	(8.) 874 at	4	15	6.
(9.) 973 at	2	6	6.	(10.) 1062 at	7	13	6.
(11.) 1189 at	5	18	9.	(12.) 1299 at		16	9.

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57. Ex. 89 at 3*s.* 11¾*d.*

		89		
3*s.* 4*d.*	$\frac{1}{6}$	14	16	8
5*d.*	⅛	1	17	1
2½*d.*	½		18	6½
¼*d.*	$\frac{1}{10}$		1	10¼
		£17	14	1¾

Here 3*s.* 4*d.* is $\frac{1}{6}$ of £1;
5*d.* is ⅛ of 3*s.* 4*d.*;
2½*d.* is ½ of 5*d.*;
¼*d.* is $\frac{1}{10}$ of 2½*d.*;
by observing which facts, we may obtain by Division the corresponding lines of the sum.

58. When, as in the last examples, the price is very nearly a *complete number* of pounds or shillings, it is often convenient to suppose it increased to that number, for which we may calculate by Practice or common Multiplication, and then calculate separately for the part which has been added to the true price, and subtract it from the former result.

Ex. 89 at 3*s.* 11¾*d.*

4*s.*	⅕	£17	16	
			1	10¼
		£17	14	1¾

Here we add ¼*d.* to the price, and take it as 4*s.*, and find the value of 89 times 4*s.*=£37 16*s.*, from which we subtract 89*f.*=22¼*d.*=1*s.* 10¼*d.*, and get the same result as before.

[*Obs.* All *these* examples might be done more readily by compound multiplication.—ED.]

Ex. 48.

	£	*s.*	*d.*		£	*s.*	*d.*
(1.) 217 at	1	19	3½.	(2.) 235 at	2	17	7¾.
(3.) 309 at		18	4¾.	(4.) 412 at	5	14	5¼.

		£	s.	d.			£	s.	d.
(5.)	473 at	2	8	8½.	(6.)	513 at	1	9	7¾.
(7.)	624 at		17	11½.	(8.)	712 at		15	10.
(9.)	777 at	3	7	10¾.	(10.)	814 at	4	11	9¼.
(11.)	923 at	2	15	10¼.	(12.)	1111 at	11	11	11¾.

Ex. 49.

(1.) A bankrupt's debts amount to £324, and he can just pay 11*s.* 6½*d.* in the £. What is his estate worth?

(2.) What do the wages of 135 railway labourers amount to for a week, at 16*s.* 6*d.* each?

(3.) A dealer bought 207 sheep at £1 3*s.* 6*d.* each, and sold 100 at £1 4*s.* 10*d.*, and the rest at £1 7*s.* 9*d.* each. What was his gain upon the bargain?

(4.) What is the rent of 179 acres of land, at £1 11*s.* 3*d.* per acre?

(5.) A bankrupt owes me a debt of £143, and his estate can only pay 12*s.* 3½*d.* in the £. What shall I get for my debt?

(6.) Find the cost of 537 pounds of sugar, at 5½*d.* each.

(7.) Farmer Jones took to market 137 quarters of wheat, and was offered 47*s.* 9*d.* a quarter. But he would not take it, and the next market-day he was obliged to sell it at 43*s.* 6*d.* a quarter. How much did he lose in consequence?

(8.) I sold 29 oxen at £13 12*s.* 8*d.* each, and bought 153 sheep at £1 16*s.* 4*d.* each. How much money did I carry home?

(9.) A fruiterer bought 97 boxes of oranges, and paid for each box 16*s.* 10½*d.* What did he pay for them all?

(10.) Each person in a clothing-club put in 13*s.* of his own, and had 17*s.* 9*d.* to lay out at the end of the year. There were 47 persons in the club. How much money was *given away* between them? And how much had they to lay out altogether?

(11.) A tailor bought 59 yards of ribbon at 2*s.* 8½*d.*, and 61 yards at 2*s.* 7¾*d.* What did the whole cost him?

(12.) What are the rates upon a farm, which is assessed at £235, at 2*s.* 4½*d.* in the £?

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59. Bills of Parcels. These are examples of the way in which tradesmen's bills are made out. In the Exercise which follows, the pupil should be required to allow discount at 5 per cent.

Ex. J. Brookes, Esq., Bought of Robert Moore,

1853.			£	s.	d.
Feb. 7.	17 pounds of	loaf sugar, at 5½d.		7	9½
	28 „	soft sugar, at 4¼d.		9	11
March 9.	11 „	tea, at 4s. 4d.	2	7	8
	15 „	rice, at 3¾d.		4	8¼
	7 „	currants, at 6½d.		3	9½
			£3	13	10¼
		Discount, 5 per cent.		3	6
			£3	10	4¼

Ex. 50.

(1.) James Harvey, Esq., Bought of Thomas Falgate,

1853.			£	s.	d.
May 2.	11 pounds	of beef, at 7d.			
17.	13 „	„ mutton, at 6½d.			
30.	15 „	„ veal, at 5¾d.			
June 11.	26 „	„ pork, at 5½d.			
18.	7 „	„ sausages, at 9½d.			

(2.) Mrs. Mason, Bought of James Brown,

1853.		£	s.	d.
Jan. 3.	29 yards of calico, at 7¼d.			
	13 pairs of stockings, at 1/6½			
	17 yards of linen, at 2/2¾			
	8 yards of flannel, at 1/4			
	3 pairs of gloves, at 1/5			

(3.) Mr. Johnson, Bought of E. Marr,

1853.		£	s.	d.
Aug. 3.	To malt, 9 quarters, at 24/4			
	hops, 19 pounds, at 1/3½			
29.	To beans, 17 bushels, at 4/7			
	pease, 23 „ at 3/11½			
	tares, 13 „ at 1/9			

(4.) Mrs. Gardiner, Bought of S. Green & Co.,

1853.			£	s.	d.
Jan. 22.	23	yards of satin, at 5/7			
	31 „	sarsenet, at 3/4			
	11 „	brocade, at 18/5			
	9 „	velvet, at 9/8			
	29 „	muslin, at 1/10½			

(5.) Mr. Williams, To M. Wilde & Sons,

1853.		£	s.	d.
March 3.	To 44 feet of stout pipe, at 5½*d.*			
	8 joints, at 1/3			
	18 pipe-hooks, at 1½*d.*			
July 13.	To 15 squares of glass, at 1/3			
	Time, 2 men, each 2 days, at 3/6			

(6.) H. Barton, Esq., To W. H. Barkham,

1853.		£	s.	d.
Oct. 14.	44 yards of tapestry carpet, at 4/9			
Nov. 17.	10 „ diaper, at 2/4			
	20 „ binding, at 1½*d.*			
Dec. 1.	Restringing 23 window blinds, at 4*d.*			
	78 yards of line, at 1*d.*			

(7.) The Rev. J. Morant, Bought of R. Webster,

1853.		£	s.	d.
Feb. 11.	13 roan Bibles, at 1/1½			
	123 Bristol tracts, at ½*d.*			
	22 prayer-books, at 4*d.*			
	7 ditto at 1/8			
	33 Testaments, at 4½*d.*			

(8.) Mr. James, Bought of W. Jones,

1853.		£	s.	d.
Sept. 13.	5 dozen and four eggs, at 6*d.* a doz.			
	7 pounds of bacon, at 8¾*d.*			
	10 „ soap, at 5½*d.*			
	9 „ butter, at 1/3			
	8 „ cheese, at 8½*d.*			

(9.) Mrs. Collins, Bought of R. Powell,

1853.		£	s.	d.
June 1.	To 5 pairs of blankets, at 13/6			
	18 tea-cloths, at 7½*d.*			
	12 glass-cloths, at 10½*d.*			
	7 yards of ribbon, at 9½*d.*			
	13 „ French merino, at 4/6			

(10.) J. Parkyn, Esq., Bought of R. Seaman,

1853.		£	s.	d.
April 3.	5 dozen of pale sherry, at 43/6 .			
	7 ,, rich port, at 39/6 . .			
May 15.	3 ,, Dublin stout, at 6/6 .			
	6 bottles of French brandy, at 4/3 .			
	68 ,, at 3s. per doz. .			

(11.) Mr. G. Thomson, Bought of T. Priest,

1853.		£	s.	d.
Dec. 3.	250 envelopes, at 9d. per 100 . .			
	3 dozen copy-books, at 4½d. each .			
	A gross of penholders, at 3½d. per doz. .			
	8 penknives, at 10½d. . . .			
	6 quires of foolscap, at 7½d. . .			

(12.) John Cross, Esq., To William Clark,

1853.		£	s.	d.
May 27.	To 55 feet run of deal, at 3d. . .			
	19 yards partitioning, at 5/6 .			
June 5.	To 23 feet of moulding, at 2½d. . .			
	13 ,, elm plank, at 1/6 . .			
10.	To 37 ,, beech plank, at 9d. . .			

PART II.

MEASURES OF WEIGHT.

60. *Avoirdupois Weight.*

Used for all common goods.

16 drams (*dr.*) make	1 ounce (*oz.*)
16 ounces	1 pound (*lb.*)
14 pounds	1 stone (*st.*)
28 pounds (2 st.)	1 quarter (*qr.*)
4 quarters (112 lbs.)	1 hundredweight (*cwt.*)
20 hundredweight	1 ton.

Ex. 51.

Reduce

(1.) 37 tons to pounds. (2.) 125 cwt. to ounces.
(3.) 117 pounds to drams. (4.) 73 tons to ounces.
(5.) 25 stone to ounces. (6.) 13 quarters to drams.
(7.) 3 tons 5 cwt. to pounds. (8.) 9 st. 11 lbs. to ounces.
(9.) 11 cwt. 1 qr. 11 lbs. to ounces.
(10.) 2 tons 23 lbs. 11 oz. to drams.
(11.) 15 st. 13 lbs. 11 oz. 9 drs. to drams.
(12.) 1 ton 2 qrs. 3 lbs. 4 oz. to ounces.

Ex. 52.

Reduce

(1.) 31213 lbs. to tons. (2.) 56003 oz. to tons.
(3.) 10000 drs. to pounds. (4.) 12021 oz. to cwt.
(5.) 56201 drs. to stones. (6.) 72654 drs. to quarters.
(7.) 87654 st. to tons. (8.) 91235 lbs. to tons.
(9.) 61004 drs. to quarters. (10.) 51002 oz. to stones.
(11.) 33033 oz. to cwt. (12.) 867891 drs. to tons.

Ex. 53.

	lb.	oz.	dr.
(1.)	5	6	7
	13	2	11
	23	13	4
	7	11	8

	qr.	lb.	cz.
(2.)	3	17	13
	2	21	7
	1	19	15
	2	23	10

	cwt.	qr.	lb.
(3.)	12	2	21
	9	0	17
	14	1	25
	13	3	11

	st.	lb.	oz.
(4.)	2	11	13
	1	7	15
	3	4	2
	2	6	14

	cwt.	qr.	lb.
(5.)	2	2	22
	3	3	13
	4	0	7
	7	1	18

	lb.	oz.	dr.
(6.)	17	15	14
	10	9	11
	8	12	7
	13	5	11

	st.	lb.	oz.	dr.
(7.)	2	5	14	13
	1	8	10	8
		4	11	14
	2	0	12	7

	cwt.	qr.	lb.	oz.
(8.)	17	1	21	10
		2	15	14
	31	2	18	9
	43	3	7	15

	tons	cwt.	qr.	lb.
(9.)	20	19	1	18
	17	11	0	23
	5	14	3	7
	10	12	2	25

	cwt.	qr.	lb.	oz.
(10.)	37	2	25	14
	9	1	1	11
	41	0	13	9
	20	1	26	15

	tons	cwt.	qr.	lb.
(11.)	25	18	2	25
	3	15	1	6
	17	10	0	22
	6	17	1	27

	st.	lb.	oz.	dr.
(12.)	2	9	4	13
	1	4	15	7
	3	11	0	8
		7	10	11

Ex. 54.

	lb.	oz.	dr.
(1.)	37	1	13
	23	12	14

	st.	lb.	oz.
(2.)	3	1	8
	1	5	9

	cwt.	qr.	lb.
(3.)	23	0	11
	18	2	17

	st.	lb.	oz.
(4.)	2	8	11
	1	12	15

	cwt.	qr.	lb.
(5.)	13	2	3
	2	3	22

	lb.	oz.	dr.
(6.)	42	3	11
	17	14	13

	st.	lb.	oz.	dr.
(7.)	2	4	11	8
		6	15	11

	cwt.	qr.	lb.	oz.
(8.)	47	2	0	5
	19	3	23	12

	tons	cwt.	qr.	lb.
(9.)	20	0	0	0
	11	17	2	25

	cwt.	qr.	lb.	oz.
(10.)	13	0	0	7
	6	2	17	9

	tons	cwt.	qr.	lb.
(11.)	25	13	2	11
	9	17	2	15

	st.	lb.	oz.	dr.
(12.)	3	6	4	2
	1	8	13	11

Ex. 55.

(1.) 13 lbs. 11 oz. 10 drs. × 2. (2.) 3 st. 8 lbs. 13 oz. × 5.
(3.) 15 cwt. 2 qrs. 17 lbs. 11 oz. × 7.
(4.) 32 tons 17 cwt. 1 qr. 19 lbs. × 10.
(5.) 5 st. 13 lbs. 13 oz. × 14. (6.) 12 cwt. 3 qrs. 22 lbs. × 24.
(7.) 2 st. 4 lbs. 11 oz. 8 drs. × 36. (8.) 13 cwt. 11 oz. × 63.
(9.) 14 cwt. 1 qr. 25 lbs. × 13. (10.) 9 lbs. 10 oz. 11 drs. × 31.
(11.) 17 tons 11 cwt. 3 qrs. 7 lbs. × 53.
(12.) 3 st. 7 lbs. 11 oz. 13 drs. × 123.

Ex. 56.

(1.) 27 lbs. 13 oz. 14 drs. ÷ 2. (2.) 13 st. 9 lbs. 14 oz. × $4\frac{3}{5}$.
(3.) 63 cwt. 0 qr. 11 lbs. 14 oz. ÷ 8.
(4.) 35 tons 18 cwt. 2 qrs. 15 lbs. × $2\frac{4}{11}$.
(5.) 72 st. 13 lbs. 13 oz. ÷ 15. (6.) 132 cwt. 3 qrs. 21 lbs. ÷ 42.
(7.) 13 st. 4 lbs. 11 oz. 8 dr. ÷ 100.
(8.) 47 cwt. 2 qrs. 1 lb. 4 oz. ÷ 132.
(9.) 41 cwt. 3 qrs. 16 lbs. ÷ 23. (10.) 53 lbs. 7 oz. 11 drs. ÷ 38.
(11.) 19 tons 7 cwt. 5 lbs. ÷ 67.
(12.) 27 st. 13 lbs. 8 oz. 6 drs. ÷ 130.

Ex. 57.

(1.) How many pounds are there in 101 tons? How many tons in 10001 pounds?

(2.) Reduce a million of drams to tons.

(3.) The great bell of St. Paul's, London, weighs 5 tons 2 cwt. 1 qr. 22 lbs., and the Great Tom of Christ Church, Oxford, weighs 7 tons 11 cwt. 3 qrs. 4 lbs. Find the sum and difference of their weights, and express each result in pounds.

(4.) The great bell at Moscow weighs 443772 pounds. Express this in tons.

(5.) The iron railings round St. Paul's weighed 200 tons 81 lbs. What did they cost, at 7*d.* per lb.?

(6.) If 1 cwt. 1 lb. cost £119 4*s.* $9\frac{1}{4}$*d.*, what is the cost of 1 lb.?

(7.) What is the weight of copper in a million of pence, each weighing an ounce?

(8.) London is supplied with butter by 225000 cows, which produce 17176 tons 6 cwt. 3 qrs. 4 lbs. yearly. What will a single cow produce?

(9.) At a school-feast there were 53 children, who consumed between them 43 lbs. 1 oz. of cake. How much was that for each on the average?

(10.) Eleven sacks of potatoes weighed together 27 st. 7 lbs. What was the weight of each?

(11.) A grocer bought 3 casks of sugar, each containing 3 cwt. 2 qrs. 13 lbs. The first month he sold 1 cwt. 2 qrs. 25 lbs., the second, 2 cwt. 0 qr. 19 lbs., and the third, 3 cwt. 1 qr. 26 lbs. How much had he taken out of the third cask?

(12.) The bulk of the sun is 1367631 times that of the earth. Supposing I took a marble, weighing one ounce, to represent the size of the earth, what must be the weight of a block of the same marble which shall represent the size of the sun?

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61. *Troy Weight.*

Used for Gold, Silver, and Jewelling, and in Philosophical Experiments.

24 grains (*gr.*) make .	1 pennyweight (*dwt.*)
20 pennyweights . .	1 ounce (*oz.*)
12 ounces	1 pound (*lb.*)

N. B. The troy pound and ounce are not the same as the avoirdupois pound and ounce; but—

1 lb. troy $= 12 \times 20 \times 24 = 5760$ grs., 1 oz. tr. $= \frac{5760}{12} = 480$ grs.,
1 lb. avoirdupois $= 7000$ grs., 1 oz. av. $= \frac{7000}{16} = 437\frac{1}{2}$ grs.

62. Apothecaries, in making up their medicines, employ the troy *grain*, *ounce*, and *pound*, but not the pennyweight; and they arrange their table of weights as follows:—

20 grains make	1 scruple,
3 scruples	1 dram,
8 drams	1 ounce,
12 ounces	1 pound.

Ex. 58.

Reduce

(1.) 214 lbs. to pennyweights. (2.) 307 oz. to grains.
(3.) 35 lbs. 7 oz. to pennyweights.
(4.) 12 oz. 3 dwt. to grains. (5.) 13 lbs. 13 dwt. to grains.
(6.) 24 lbs. 9 oz. 17 grs. to grains.
(7.) 10000 dwt. to pounds. (8.) 10000 grs. to ounces.
(9.) 12345 grs. to pounds. (10.) 33333 dwt. to pounds.
11.) 45678 dwt. to pounds. (12.) 56789 gr. to pounds.

Ex. 59.

Add

	lb.	oz.	dwt.		oz.	dwt.	gr.		lb.	oz.	dwt.	gr.
(1.)	13	7	19	(2.)	7	15	21	(3.)	11	11	19	23
	6	11	17		3	18	9		7	8	9	10
	8	3	5			16	18		13	6	17	22
	14	9	13		5	14	23		10	0	14	17

From

	lb.	oz.	dwt.		oz.	dwt.	gr.		lb.	oz.	dwt.	gr.
(4.)	11	2	13	(5.)	7	15	2	(6.)	19	9	9	1
Take	8	9	15		6	18	23		15	11	17	19

(7.) 17 lb. 5 oz. 13 dwt. × 7. (8.) 13 oz. 5 dwt. 21 gr. ÷ 9.

(9.) 10 lb. 11 oz. 12 dwt. 13 gr. × 11. (10.) 11 lb. 8 oz. 17 dwt. × $3\frac{5}{8}$.

(11.) 9 oz. 19 dwt. 19 gr. ÷ 35. (12.) 85 lb. 7 oz. 15 dwt. 1 gr. ÷ 67.

Ex. 60.

(1.) Find the value in grains of a dram avoirdupois, and a dram of apothecaries' weight.

(2.) Taking the weight of a sovereign as 123 grains, what is the weight of £1000 in gold?

(3.) How many sovereigns can be made out of a pound weight of gold?

(4.) Sixty-six shillings are made out of a pound weight of silver. Find the weight of a single shilling.

(5.) A Jewish *shekel* weighed 219 grains, and was worth 2*s.* $3\frac{1}{2}$*d.* Find the weight of a *talent*, which contained 3000 shekels; and the value of ten thousand talents.

(6.) What is the value of a pound weight of Australian gold, at £3 3*s.* $3\frac{1}{2}$*d.* per ounce?

(7.) Ten wedges of gold, weighing in all 12 lbs. 9 oz. 15 dwt., are to be coined into sovereigns. Find the weight of each wedge, and the number of sovereigns that will be coined, supposing them to weigh each 123 grains.

(8.) Twenty-four pence are coined from a pound (av.) of copper. What is the difference between the weights of a penny and a sovereign?

(9.) If an ounce of wrought silver cost 5*s.* 6*d.*, what is the value of a silver cup weighing 1 lb. $11\frac{1}{2}$ oz.?

(10.) The Jewish *mina* or *pound* (St. Luke xix.) weighed

50 shekels of silver. Find its weight and value from the *data* above, in (5).

(11.) A silversmith made three dozen silver spoons, each weighing 2 oz. 5 dwt. 6 grs., and a dozen silver forks, each weighing 1 oz. 10 dwt. 22 grs. How much silver did he use for the whole?

(12.) One-eleventh of every sovereign is copper alloy. How much of pure gold is there in a bag of 100 sovereigns?

MEASURES OF LENGTH.

63. *Long Measure.*

12 inches (*in.*) make .	1 foot (*ft.*)
3 feet	1 yard (*yd.*)
$5\frac{1}{2}$ yards	1 rod, pole, or perch (*po.*)
40 poles (220 yds.) .	1 furlong (*fur.*)
8 furlongs (1760 yds.)	1 mile (*m.*)
3 miles	1 league (*lea.*)

Besides the above, the following measures of length are used:

A hand = 4 inches, a fathom = 6 feet, a degree = $69\frac{1}{2}$ miles.

64. A difficulty occurs under this rule in passing from yards to poles, by reason of the fractional number of yards in one pole. It is best to bring the yards into *half*-yards, by multiplying by 2; and then, since 1 pole = $5\frac{1}{2}$ yards = 11 *half*-yards, if we divide by 11, we shall get the number of poles required, and the remainder (if any) will be in *half*-yards, which may be brought into yards by dividing by 2.

Ex. 1. Reduce 1001 feet to poles.

3) 1001 ft.
333 . . . 2 ft.
2
11) 666
60 . . . 6 half-yards = 3 yards. *Ans.* 60 po. 3 yds. 2 ft.

Ex. 2. Reduce 54955 inches to poles.

12) 54955 in.
3) 4579 . . . 7 in.
1526 . . . 1 ft.
2
11) 3052
277 . . . 5 half-yards = $2\frac{1}{2}$ yds. = 2 yds. 1 ft. 6 in.

Here we have a remainder 2 yds. 1 ft. 6 in., to be added to the former remainders, 1 ft., 7 in., making altogether 2 yds. 2 ft. 13 in. = 2 yds. 3 ft. 1 in. = 3 yds. 1 in.

Ans. 277 po. 3 yds. 1 in.

Ex. 61.

Reduce

(1.) 75 miles to feet.
(2.) 3 furlongs to inches.
(3.) 33 m. 7 fur. to yards.
(4.) 1 lea. 2 m. 3 fur. to feet.
(5.) 5 fur. 115 yds. 5 in. to inches.
(6.) 3 m. 100 yds. 7 in. to inches.
(7.) 134507 ft. to miles.
(8.) 225000 yds. to leagues.
(9.) 57008 in. to furlongs.
(10.) 378125 ft. to miles.
(11.) 400000 in. to miles.
(12.) 175423 ft. to leagues.

Ex. 62.

	lea.	m.	fur.
(1.)	15	1	7
	3	0	6
	12	2	5
	7	1	3

	m.	fur.	po.
(2.)	41	5	37
	16	4	25
	39	7	9
	8	0	32

	fur.	po.	yds.
(3.)	17	23	4
	9	17	2
	23	0	3
	11	35	1

	po.	yds.	ft.
(4.)	23	3	1
	18	4	2
	27	0	2
	6	4	0

	yds.	ft.	in.
(5.)	15	1	7
	23	2	9
	35	0	6
	7	2	11

	m.	fur.	yds.
(6.)	14	7	119
	9	3	96
	27	6	213
	8	5	198

	m.	yds.	ft.
(7.)	7	1324	2
	19	876	1
	27	990	2
	6	1543	0

	fur.	yds.	ft.
(8.)	3	178	2
	1	156	1
	4	98	2
	5	136	1

	lea.	m.	fur.	yds.
(9.)	14	2	7	142
	7	1	5	219
	6	0	6	75
	4	2	3	187

	m.	fur.	yds.	ft.
(10.)	23	5	119	2
	19	0	173	1
	8	7	65	2
	32	6	147	2

	m.	yds.	ft.	in.
(11.)	13	550	1	7
	9	1230	2	11
	4	658	0	9
	23	742	2	10

	fur.	yds.	ft.	in.
(12.)	7	135	2	7
	8	96	1	11
	3	125	0	4
	6	48	1	9

Ex. 63.

	lea.	m.	fur.
(1.)	17	2	5
	11	2	7

	m.	fur.	po.
(2.)	39	5	22
	12	7	29

	fur.	po.	yds.
(3.)	13	33	2
	9	37	4

	po.	yds.	ft.
(4.)	22	2	0
	19	3	2

	yds.	ft.	in.
(5.)	14	1	4
	10	2	11

	m.	fur.	yds.
(6.)	13	5	183
	9	7	214

	m.	yds.	ft.
(7.)	23	76	1
	6	157	2

	fur.	yds.	ft.
(8.)	7	135	1
	3	206	2

	lea.	m.	fur.	yds.
(9.)	5	2	7	156
	2	2	7	200

	m.	fur.	yds.	ft.
(10.)	17	0	0	0
	13	1	135	2

	m.	yds.	ft.	in.
(11.)	33	100	1	5
	26	1000	2	9

	fur.	yds.	ft.	in.
(12.)	17	125	1	10
	8	187	2	11

Ex. 64.

(1.) 37 yds. 2 ft. 11 in. × 4.
(2.) 25 m. 7 fur. 200 yds. × 7.
(3.) 5 lea. 1 m. 1000 yds. × 10.
(4.) 6 fur. 20 po. 5 yds. × 11.
(5.) 3 lea. 2 m. 5 fur. × 14.
(6.) 5 yds. 10 in. × 72.
(7.) 13 m. 7 fur. 180 yds. × 88.
(8.) 23 po. 3 yds. 2 ft. × 100.
(9.) 33 yds. 2 ft. 11 in. × 17.
(10.) 17 lea. 1 m. 1021 yds. × 47.
(11.) 2 lea. 1 m. 7 fur. 37 po. × 113.
(12.) 3 fur. 30 po. 2 yds. 1 ft. × 235.

Ex. 65.

(1.) 15 yds. 1 ft. 9 in. ÷ 3.
(2.) 17 m. 5 fur. 144 yds. ÷ 7.
(3.) 22 yds. 1 ft. 11 in × $2\frac{4}{5}$.
(4.) 47 m. 7 fur. 200 yds. × $3\frac{7}{10}$.
(5.) 37 fur. 179 yds. ÷ 18.
(6.) 14 lea. 1 m. 7 fur. ÷ 24.
(7.) 141 yds. 2 ft. 3 in. ÷ 81.
(8.) 200 m. 311 yds. 2 ft. ÷ 121.
(9.) 3 yds. 1 ft. 6 in. × $41\frac{7}{18}$.
(10.) 33 m. 1 fur. ÷ 200.
(11.) 5 lea. 1 m. 986 yds. × $38\frac{5}{13}$.
(12.) 289 yds. 2 ft. 9 in ÷ 213.

Ex. 66.

(1.) The earth is about 132 millions of feet round at the equator. Find its circumference in miles.

(2.) Sound travels at the rate of 1120 feet in a second. How far off is a thunder-cloud when the clap follows the flash of lightning after an interval of $10\frac{1}{2}$ seconds?

(3.) The diameter of the earth at the equator is $7925\frac{1}{2}$

miles, and at the poles 7899 miles. By how many yards is the earth broader at the equator than it is at the poles?

(4.) A pint pot will hold 9000 barleycorns, and three of these, placed one after another, would reach an inch. How far would they all reach?

(5.) In a certain house there are two staircases. Each staircase consists of 16 stairs, and each stair is 10 inches high. What is the height of the top floor above the ground?

(6.) A cannon-ball travels at the rate of 1000 feet in a second. How far will it travel in 12 seconds?

(7.) A man plants a row of cabbages in his allotment, setting the plants, fifty in number, 15 inches apart. What is the length of the row?

(8.) Soldiers, in ordinary marching, take 75 steps a minute; in quick marching, 108. How far would a regiment advance in twenty minutes, both in ordinary and quick marching, reckoning each step as 2 ft. 8 in.?

(9.) Sixty-three children met for a school-feast. What length of forms will be required to seat them, allowing 16 inches for each child?

(10.) *The circumference of any circle is (nearly)* $3\frac{1}{7}$ *times its diameter.* What is the size round of a tree which is 3 ft. 6 in. across through the middle?

(11.) A mill-sail is seven yards long, and goes round ten times in a minute. How far does its end travel in an hour or sixty minutes?

(12.) A cocoon of silk was unwound by 270 turns of a wheel of 14 inches' diameter. What was its length?

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65. *Cloth Measure.*

$2\frac{1}{4}$ inches make . . .	1 nail (*na.*)
4 nails	1 quarter (*qr.*)
4 quarters . . .	1 yard (*yd.*)
5 quarters . . .	1 ell.

Ex. Reduce 1025 inches to nails.

$$\begin{array}{l} 1025 \text{ in.} \\ \underline{4} \\ 9)\,\underline{4100} \text{ quarter-inches.} \\ \underline{455} \ldots 5 \text{ quarter-inches} = 1\tfrac{1}{4} \text{ in.} \end{array}$$

Here, since 1 nail $= 2\frac{1}{4}$ inches $= 9$ quarter-inches, we multiply by 4 to bring the inches into quarter-inches, and then divide by 9.

Ex. 67.

	yds.	qr.	na.
(1.)	17	3	1
	23	2	3
	45	0	3
	7	1	2

	ells	qr.	na.
(2.)	11	3	3
	6	4	1
	27	2	3
	14	0	2

	yds.	qr.	na.	in.
(3.)	52	2	3	$1\frac{1}{4}$
	37	0	3	2
	9	1	0	$1\frac{1}{3}$
	19	3	2	$0\frac{3}{4}$

	ells	qr.	na.	in.
(4.)	16	4	3	$0\frac{1}{2}$
	18	3	2	2
	29	0	1	$1\frac{1}{4}$
	7	4	2	$1\frac{1}{2}$

	yds.	qr.	na.
(5.)	33	1	0
	17	3	3

	ells	qr.	na.
(6.)	47	0	0
	16	3	3

	yds.	qr.	na.	in.
(7.)	19	0	2	1
	18	2	3	$1\frac{1}{2}$

	ells	qr.	na.	in.
(8.)	7	1	1	$0\frac{3}{4}$
	4	3	2	$1\frac{1}{2}$

(9.) 15 yds. 2 qrs. 3 na. $1\frac{1}{3}$ in. × 7.
(10.) 17 yds. 3 qrs. 3 na. $0\frac{1}{2}$ in. ÷ 11.
(11.) 22 yds. 3 qrs. 1 na. × $12\frac{2}{5}$.
(12.) 34 yds. 3 qrs. 3 na. $0\frac{1}{4}$ in. × $13\frac{3}{4}$.

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66. Land is usually measured by a *chain*, called Gunter's chain, which is 22 yards (or 4 poles) long; consequently—

22 yards (4 poles) make .	1 chain (*ch.*)
10 chains	1 furlong.

The chain is divided into 100 equal parts, called *links*.

67. Hence the number of chains in any given number of links may be obtained at once by dividing it by 100, or marking off the last two figures by a dot.

Thus 3781 links = 37.81 chains = 37 ch. 81 links.

68. So, too, any number of chains and links may be written down as so many chains, by marking off the links as hundredths of a chain by means of a dot, observing to insert a cipher after the dot, when the number of links happens to be less than 10.

Thus 37 ch. 81 links = 3781 links = 37.81 chains;
but 37 ch. 7 links = 3707 links = 37.07 chains.

69. These expressions may now be treated in addition, &c. just like whole numbers, the last two figures in any result being always marked off as so many *hundredths of a chain*, or links.

Ex. 1. *Add*
Chains.
37.81
12.78
44.05
94.64 = 94 ch. 64 lks.

Ex. 2. *Subtract*
Chains.
37.81
12.78
25.03 = 25 ch. 3 lks.

Ex. 3.
Chains.
37.81 × 5
5
189.05 = 189 ch. 5 lks.

Ex. 4.
Chains.
9) 12.79 ÷ 9
1.42 (+) = 1 ch. 42 lks. (+)

The truth of the above may be seen in any case by turning the given number of chains and links into links, as follows:—

Ex. 3. 37.81 ch. = 3781 links.
5
18905 links = 189.05 chains.

70. To reduce any number of chains and links to *yards*, we have only to mark off the links as above, and then multiply by 22, marking off now the last two figures as *hundredths of a yard*.

And these yards may be reduced to *feet* by multiplying by 3, and marking off the last two figures as *hundredths of a foot*.

Ex. 37 ch. 81 l. = 37.81 chains.
22
7562
7562
831.82 yards = 831$\frac{82}{100}$ yds.
3
2495·46 feet = 2495$\frac{46}{100}$ feet.

For 37.81 chains = $\frac{1}{100}$ of 3781 chains = $\frac{1}{100}$ of 3781 × 22 yards
= $\frac{1}{100}$ of 83182 yards = 831.82 yards;
and 831.82 yards = $\frac{1}{100}$ of 83182 yards = $\frac{1}{100}$ of 83182 × 3 feet
= $\frac{1}{100}$ of 249546 feet = 2495.46 feet.

Or we may wish to reduce to feet, not only the whole number of yards, but only the fractional or *decimal* part, as it is called.

Thus 831.82 yards = 831 yds. 2$\frac{46}{100}$ feet
3
2.46 feet

For .82 yards = $\frac{1}{100}$ of 82 yards = $\frac{1}{100}$ of 82 × 3 feet
= $\frac{1}{100}$ of 246 feet = 2.46 feet.

Ex. 68.

(1.) The four sides of a field are found to be 23 chains 19 links, 17 chains 34 links, 6 chains 85 links, and 24 chains 62 links. How many yards round was the field?

(2.) The sides of a triangular field measured 21½ chains, 14 chains 11 links, and 8 chains 10 links, respectively. *A* runs along the longest side, and *B* along the other two. What distance in yards will *B* run more than *A*?

(3.) From one end of a village to the other it is just 250 chains. How many miles is this?

(4.) A four-sided church-green is 6 chains 18 links across one way, and 3 chains 82 links the other. How many steps would a man take to walk quite round it, reckoning each step as a yard?

(5.) Twelve persons at a clothing club bought each 3 yds. 1 qr. 2 na. of flannel, and five others bought each 2 yds. 3 qrs. How much remained out of a piece which contained 60 yards?

(6.) A tailor bought three pieces of cloth. In the first piece there were 23 yds. 2 qrs. 2 na., in the second, 25 yds. 1 qr. 1 na., and in the third, 27 yds. 3 qrs. How much did he buy altogether?

(7.) From the first of the above pieces, the tailor cut off one-half, from the second, one-third, and from the other, one-fourth. How much do these three pieces make between them?

(8.) How many yards are there in 25 ells of cloth? How many ells in 25 yards?

(9.) A draper bought 80 ells of cloth at 4*s.* 6*d.* per ell, and sold it at 4*s.* 6*d.* per yard. What did he gain by this transaction?

(10.) Find how many chains there will be in a distance of 12½ miles.

(11.) One side of a field was 9 chains 72 links long, another was half as long again, another was a third as long again as the first, and the fourth was half as long as the second and third together. What was the distance round the field in yards?

(12.) A piece of tape measures 24 yards and a nail. What length would 32 such pieces reach?

71. MEASURES OF SURFACE, OR SQUARE MEASURE.

(12 × 12 or)	144	square inches make	1 square foot
(3 × 3 or)	9	square feet . . .	1 square yard
(5½ × 5½ or)	30¼	square yards . .	1 square rod, pole, or perch (P.)
	40	poles	1 rood (R.)
	4	roods (4840 sq. yds.)	1 acre (A.)
	640	acres	1 square mile.
(4 × 4 or)	16	square poles = (22 × 22 or 484 sq. yds.) . . .	1 square chain
	10	square chains . .	1 acre.

Ex. 69.

Reduce

(1.) 13A. 3R. 35P. to poles.
(2.) 18 sq. yds. 2 sq. ft. 63 sq. in. to sq. inches.
(3.) 54A. 2R. 27P. to poles.
(4.) 11 sq. yds. 8 sq. ft. 100 sq. in. to sq. inches.
(5.) 29A. 0R. 16P. to sq. yards.
(6.) 29 sq. yds. 6 sq. ft. 95 sq. in. to sq. inches.
(7.) 3R. 16P. 13 sq. yds. to sq. feet.
(8.) 28P. 21 sq. yds. 3 sq. ft. to sq. inches.
(9.) 10000 poles to acres.
(10.) 100000 sq. inches to sq. yards.
(11.) 10000 sq. yards to acres.
(12.) 10000000 sq. inches to acres.

Ex. 70.

Add

	s. yds.	s. ft.	s. in.
(1.)	23	7	120
	35	6	47
	6	2	135
	15	8	98

	A.	R.	P.
(2.)	13	2	27
	42	3	13
	27	0	7
	3	1	32

	s. yds.	s. ft.	s. in.
(3.)	97	2	85
	12	7	123
	8	6	74
	45	4	19

	A.	R.	P.	s. yds.
(4.)	37	3	29	17
	4	2	11	28½
	16	3	17	30
	3	0	33	15¼

	R.	P.	s. yds.	s. ft.
(5.)	3	37	29	8
	2	29	18	5
	3	16	23	7
	1	21	25	6

	P.	s. yds.	s. ft.	s. in.
(6.)	33	23	7	130
	17	18	6	98
	25	7	3	76
	32	16	8	129

Subtract

	s. yds.	s. ft.	s. in.
(7.)	19	7	100
	17	8	121

	A.	R.	P.
(8.)	37	3	27
	19	0	35

	s. yds.	s. ft.	s. in.
(9.)	123	1	100
	99	8	132

	A.	R.	P.		s. yds.	s. ft.	s. in.		A.	R.	P.
(10.)	123	2	13	(11.)	47	0	59	(12.)	225	1	11
	87	3	35		38	7	117		88	2	18

Ex. 71.

(1.) 17A. 3R. 35P. × 7.
(2.) 36 sq. yds. 4 sq. ft. 121 sq. in. × 11.
(3.) 27A. 3R. 37P. × 23.
(4.) 3 sq. yds. 1 sq. ft. 100 sq. in. × 127.
(5.) 63A. 0R. 15P. ÷ 5.
(6.) 223 sq. yds. 7 sq. ft. 120 sq. in. ÷ 7.
(7.) 345A. 1R. 5P. ÷ 29.
(8.) 436 sq. yds. 8 sq. ft. 76 sq. in. + 134.
(9.) 17A. 3R. 34P. × $14\frac{1}{2}$.
(10.) 76 sq. yds. 6 sq. ft. 76 sq. in. × $36\frac{1}{2}$.
(11.) 25A. 1R. 35P. × $43\frac{3}{5}$.
(12.) 9 sq. yds. 7 sq. ft. 104 sq. in. × $51\frac{5}{8}$.

Ex. 72.

(1.) A father left a small farm of 35A. 3R. 34P. among his three sons, giving half of it to one, a third to another, and the rest to the third. What were their respective portions?

(2.) England contains about 50000 square miles. How many acres are there in its surface?

(3.) A farmer rented four fields. One measured 11A. 2R. 13P., another 8A. 15P., a third 23A. 1R. 30P., and the fourth was half as large again as the third. How many acres did he rent altogether?

(4.) Six persons bought between them a piece of land, containing 5A. 2R. 26P. What will be the share of each, if it be equally divided?

(5.) A man has an allotment of 2R. 34P. He sows one-third of it with wheat, and plants one-quarter of the rest with potatoes. How much more ground will he have?

(6.) A hay-field, containing 16A. 3R. 22P., was mowed by fourteen men. How much did each of them mow on the average?

(7.) Nineteen men reaped in one day 9A. 1R. 4P. of corn. Seven of them reaped, on an average, half an acre each. What did each of the others reap?

(8.) From a glebe-field of 1A. 1R., a site of 1R. 28P. is given for a school. How much of the field now remains?

(9.) If the land remaining in the last question be divided into four allotments, what will be the size of each?

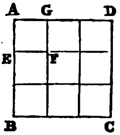
(10.) A farmer has a field of 7A. 2R. 13P., and another of 3A. 1R. 25P. He throws down the hedge between them, thereby gaining 17 poles of ground. What is now the size of the field?

(11.) A farm consisted of 105A. 2R. 23P. arable, and 54A. 1R. 37P. pasture, 2A. 2R. 15P. woodland, and 3A. 0R. 35P. orchard. What was the whole extent of it?

(12.) A common field of 19 acres is to be divided into 11 equal parts, after allowing 1R. 19P. for a roadway. What will be the size of each part?

---

72. Suppose *ABCD* to represent the surface of a table which is a *yard square*, that is, a yard long and a yard broad, so that $AB =$ a yard or 3 feet, and $AD = 3$ feet. Divide then *AB* into 3 equal parts, and *AD* into 3, as in the figure, and through the points of division draw lines parallel to *AB* and *AD*. By this means we shall have divided the whole surface into small figures, such as *AEFG*, all equal to one another; and since $AE = 1$ foot, and $AG = 1$ foot, it is plain that the surface *AEFG* measures a foot everywhere, a foot long and a foot broad, that is, *AEFG* is a *square foot*, and so are all the other small figures.



Now the number of these figures is $3 \times 3 = 9$, each horizontal row of three square feet (where 3 is the number of feet in the length of *AD*) being repeated three times (where 3 is the number of feet in the length of *AB*); and so we see the reason why a *square yard* contains

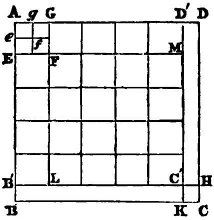
$3 \times 3 = 9$ *square feet.*

73. In like manner, a *square foot* contains $12 \times 12 = 144$ *square inches*; a *square chain* contains $4 \times 4 = 16$ *square poles*, or $22 \times 22 = 484$ *square yards*.

So, too, a *square pole* contains $5\frac{1}{2} \times 5\frac{1}{2} = 30\frac{1}{4}$ *square yards*; for we have to multiply $5\frac{1}{2}$, that is, 5 and $\frac{1}{2}$, by $5\frac{1}{2}$, that is, by 5 and by $\frac{1}{2}$, and to add all these results together:

now $5 \times 5 = 25$, $5 \times \frac{1}{2} = \frac{5}{2} = 2$, $\frac{1}{2} \times 5\frac{1}{2} = 2\frac{1}{2}$, and $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$;
adding these together, we get $25 + 2\frac{1}{2} + 2\frac{1}{2} + \frac{1}{4} = 30\frac{1}{4}$.

74. It may be useful to illustrate the last result by a geometrical figure.



Let the figure represent a *square pole*, so that $AB = 5\frac{1}{2}$ yards, and $AD = 5\frac{1}{2}$ yards; let $AB' = 5$ yards $= AD'$, and, dividing AB', AD', each into five equal parts, draw the parallel lines, as in the figure. Then the figure $AB'C'D'$ contains $5 \times 5 = 25$ surfaces, each a yard square: and since DD' represents half a yard, the figure $DHC'D'$ will be half of $AB'LG$, and, for a like reason, the figure $BKC'B'$ will be half of $AD'ME$, that is, half of $AB'LG$; hence the figures $DHC'D'$, $BKC'B'$, will be together equal to $AB'LG$, that is, to 5 square yards. And there remains still the corner figure $CKC'H$, which has each of its sides half a yard, and is, therefore, equal to the figure $Aefg$, which is plainly $\frac{1}{4}$ of $AEFG$, that is, $\frac{1}{4}$ of a square yard. Hence the whole figure $ABCD$ contains 25 square yards + 5 square yards + $\frac{1}{4}$ square yard $= 30\frac{1}{4}$ square yards.

Or the same might be shown thus:—

Since AD, in the above figure, $= 5\frac{1}{2}$ yards $= 11$ half-yards, suppose it to be divided into 11 equal parts, each equal to Ag, and suppose AB to be similarly divided. Then drawing parallel lines, as before, through the points of division, we should form 121 little squares, each equal to $Aefg$, and each being, consequently, a quarter of a square yard. Hence the whole figure $ABCD = 121$ quarters of a square yard $= 30\frac{1}{4}$ square yards.

N.B. Let the pupil observe that the square upon $\frac{1}{2}$ a yard, or a half-yard square, $=$ ($Aefg$ in the figure $= \frac{1}{4}$ of $AEFG =$) $\frac{1}{4}$ of a square yard.

Observe, also, the difference between *five square yards* and *five-yards square*, which latter expression means a square figure, measuring five yards every way, and which, therefore, contains twenty-five square yards.

---

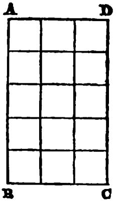
75. Since 1 sq. pole $= 30\frac{1}{4}$ square yards $= 121$ quarter sq. yds., in order to bring square yards into poles, we must multiply by 4, in order to bring them into quarters, and then divide by 121.

```
Ex.     1000 sq. yds.
           4
        ----
   121) 4000 (33
        363
        ---
         370
         363
         ---
           7 quarter sq. yds. = 1¾ sq. yds.
```

N.B. Observe that,

$\frac{1}{4}$ sq. yd. $= \frac{1}{4}$ of 9 sq. ft. $= 2\frac{1}{4}$ sq. ft. $=$ 2 sq. ft. 36 sq. in.
$\frac{1}{2}$ sq. yd. $= 4\frac{1}{2}$ sq. ft. $=$ 4 sq. ft. 72 sq. in.
$\frac{3}{4}$ sq. yd. $= 6\frac{3}{4}$ sq. ft. $=$ 6 sq. ft. 108 sq. in.

Hence the above Answer may be written 1 sq. yd. 6 sq. ft. 108 sq. in.



76. Let *ABCD* represent the surface of a table, whose length *AB* is 5 feet, and breadth *AD* is 3 feet. Then, dividing these lines as before, and drawing parallel lines through the points of division, we see that the surface is divided into $3 \times 5 = 15$ little squares, each of which is a square foot.

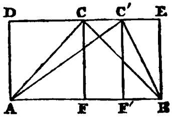
In like manner, the number of *square* feet, yards, &c., in any rectangular surface is found by multiplying together the number of *linear* feet in its length and breadth.

Ex. Find the surface of a floor, that is 17 ft. 8 in. long by 3 yards wide.

Here 17 ft. 8 in. = 212 in.
3 yds. = 108 in.
1696
2120
22896 sq. in. = 17 sq. yds. 6 ft. *Ans.*

N.B. Observe that, in the last Example, we do not multiply 212 *inches* by 108 *inches*, which would be as absurd as if we were to multiply 212 *shillings* by 108 *shillings*. What we have really done is merely to multiply the *numbers* 212 and 108 together, by which means we shall obtain a result which, we know from the above reasoning, will be the *number* of square inches in the surface.

77. In the annexed figure, let *ABED* be a rectangular figure; and let *C* and *F* be the middle points of the sides *DE* and *AB*. Join *AC*, *BC*, *CF*: then it is plain that the triangle *ACF* is half of the figure *ADCF*, and the triangle *BCF* is half of the figure *BECF*, and, therefore, the whole triangle *ACB* is half of the whole figure *ADEB*. And the area of *ADEB* is found by multiplying the number of feet. &c. in the *length AB* by the number of feet, &c. in the *breadth BE*, or *CF*; hence the area of



the triangle ACB is $\frac{1}{2}$ this product, or, if we call AB the *base*, and CF the *perpendicular*, it is

$$\frac{1}{2} \times \text{base} \times \text{perpendicular}.$$

This will help us, presently, to find the quantity of brickwork in the gable end of a house, of which ACB represents the roof.

N.B. If C' be *any* point whatever in DE, and $C'F'$ be drawn perpendicular to AB, then, joining AC', BC', we have triangle $AC'F'$ = half of $ADC'F'$, and triangle $BC'F'$ = half of $BEC'F'$, and, therefore, triangle $AC'B$ = half of $ADEB$ = triangle $ACB = \frac{1}{2}$ base × perpendicular, as before.

Ex. 73.

(1.) 2 yds. 2 ft. × 3 yds. 1 ft. (2.) 13 ft. 7 in. × 6 ft. 9 in.
(3.) 7 yds. 1 ft. × 9 yds. 2 ft. (4.) 16 ft. 3 in. × 10 ft. 7 in.
(5.) 25 yds. × 2 yds. 1 ft. (6.) 17 ft. × 6 ft. 8 in.
(7.) 17 yds. × 5 yds. 2 ft. (8.) 23 ft. × 11 ft. 11 in.
(9.) 13 yds. 2 ft. × 6 yds. (10.) 22 yds. 7 in. × 8 yds.
(11.) 10 yds. 3 in. × 2 ft. 5 in. (12.) 13 yds. 5 in. × 7 ft. 7 in.

Ex. 74.

(1.) A school-room measures 34 ft. 6 in. in one direction, and 22 ft. 3 in. in the other. What is the area of its floor?

(2.) A garden wall is 120 yards long and $9\frac{1}{2}$ feet high. How many square yards of brickwork are in it?

(3.) What is the size of a doorway, which is 7 ft. high and 2 ft. 4 in. broad?

(4.) Find the area of a barn-floor, which measures $52\frac{1}{2}$ ft. every way.

(5.) What is the difference between 10 square feet, and 10 feet square? between $10\frac{1}{2}$ square yards and $10\frac{1}{2}$ yards square?

(6.) A side of Russell Square in London is 660 feet. How many acres does it contain?

(7.) Find the area of a square building, whose side is 45 ft. 9 in.

(8.) A square piece of ground is enclosed for sheep by 56 hurdles, each 6 ft. long. What is the size of the enclosure?

(9.) How many yards of flooring will there be in a room which measures 14 ft. 3 in. by 13 ft. 4 in.?

(10.) How many yards of plastering will there be in the walls of the above room, supposing them 7 ft. high, and making no allowance for doors and windows?

(11.) A glebe-field was divided into six allotments, each 28 yards by $23\frac{1}{2}$ yards. What was the size of the field?

(12.) How many square feet of glass are there in a window which contains twelve panes, each 18 in. by 16 in.?

---

78. Since 1 chain contains 100 links, therefore, 1 *square* chain contains $100 \times 100 = 10{,}000$ *square* links: and, consequently, any number of square links may be expressed in square chains by dividing by 10,000, or marking off the last four figures with a dot.

Ex. How many square chains are there in a field, which measures 3 ch. 35 l. in one direction and 16 ch. 89 l. in the other?

Here 3 ch. 35 l. = 335 links
16 ch. 89 l. = 1689 links
3015
2680
2010
335
565815 square links = 56.5815 square chains.
$= 36\frac{5815}{10000}$ sq. ch. or 36 sq. ch. 5815 sq. l.

79. In like manner, since an acre = 10 square chains = 100,000 square links, any number of square chains may be converted into acres by dividing by 10, or moving the dot one place further to the left; and any number of square links may be converted into acres by dividing by 100,000, or marking off the last five figures with a dot.

Thus the field in the last example = 56.5815 square chains, or = 565815 square links = 5.65815 acres.

The remainder in such a case may be expressed in roods, poles, &c., by multiplying, successively, by 4, 40, &c., marking off always the last five figures with a dot.

5.65815 acres.
4
2.63260
40
25.30400

Ans. 5A. 2R. 25P. (+)

For .65815 acres $= \frac{65815}{100000}$ acres
$= \frac{1}{100000} \times 65815$ acres
$= \frac{1}{100000} \times 65815 \times 4$ roods
$= \frac{1}{100000} \times 263260$ roods $= 2.63260$ roods;
and so on.

Of course, the last remainder might be expressed, if it were desired, in square yards, square feet, and square inches, by multiplying, successively, by $30\frac{1}{4}$, 9, and 144.

80. It is useful to know the facts, (though they cannot here be proved), that

(i.) the *circumference* of any circle = $3\frac{1}{7}$ × its diameter,
and (ii.) the *area* of any circle = $3\frac{1}{7}$ × the square of its radius.

Ex. 1. Find the size round of a wheel, which is 4 ft. 8 in. wide across the centre.

Here the diameter = 4 ft. 8 in. = 56 in.;
and the circumference = $3\frac{1}{7} \times 56$ in. = 176 in. = 14 ft. 8 in.

Ex. 2. Find the area of the section of a tree, which measures 2 ft. 6 in. across the middle.

Here the radius = 1 ft. 3 in. = 15 in.

$$\begin{array}{l} \underline{15 \text{ in.}} \\ 75 \\ \underline{15} \\ 225 \text{ sq. in.} = \text{square on radius} \\ \underline{3\frac{1}{7}} \\ 675 \\ \underline{32\frac{1}{7}} \\ \underline{707\frac{1}{7}} \text{ sq. in.} = \text{area of section} \\ \qquad\qquad = 4 \text{ sq. ft. } 131\frac{1}{7} \text{ sq. in.} \end{array}$$

Ex. 75.

(1.) A church-green measures 3 chains 50 links by 3 chains 37 links. What is its size?

(2.) What is the size of the green in Ex. **68**, 4?

(3.) What is the area of a square field, whose side measured 7 chains 7 links?

(4.) There are seven miles of road in a parish. Reckoning their width as half a chain, how many acres of land consist of roads?

(5.) One field consisted of 3 square chains, and another was 3 chains square. How much land did they contain together?

(6.) A field is crossed by a roadway, 15 links wide, and 13 chains 43 links long. How much ground was lost in consequence?

(7.) What is the size of a clock-face, if the large hand be 4 inches long, and the face itself extending an inch beyond it?

(8.) How much ground will an ass have to graze over that is tethered by a string 40 yards long?

(9.) A railway is 120 miles long, and its average breadth is two-thirds of a chain. Show that it occupies exactly one square mile of land.

(10.) Out of a field of four acres, I cut out a rectangular piece, 3 chains 25 links in one direction, and 2 chains 75 links in the other. How much of the field is left?

(11.) What is the size of a round table, which measures 4 ft. 2 in. across at its widest part?

(12.) A string, 112 yards long, measures exactly round a square plot of ground. How large is the enclosure? If one end of the string were fixed, what would be the area of the circle described by the other?

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81. MEASURES OF SOLIDITY, OR CUBIC MEASURE.

1728 Cubic Inches make 1 Cubic Foot.
27 Cubic Feet „ 1 Cubic Yard.

Ex. 76.

Reduce

(1.) 26 cub. yds. to feet.
(2.) 15 cub. yds. 13 ft. to inches.
(3.) 25 cub. yds. 1000 in. to inches.
(4.) 150000 cub. inches to yards.
(5.) 276543 cub. inches to yards.
(6.) 100001 cub. inches to yards.

Add

	c. yds.	c. ft.	c. in.		c. yds.	c. ft.	c. in.		c. yds.	c. ft.	c. in.
(7.)	8	13	1234	(8.)	47	25	728	(9.)	29	16	107
	5	14	976		32	24	613		3	0	99
	2	5	854		51	23	427		14	23	765
	7	10	1012		61	22	352		37	19	1034

Subtract

	c. yds.	c. ft.	c. in.		c. yds.	c. ft.	c. in.		c. yds.	c. ft.	c. in.
(10.)	47	13	1236	(11.)	32	16	367	(12.)	22	7	857
	28	25	1500		18	25	982		19	24	1000

Ex. 77.

(1.) 3 c. yds. 5 c. ft. 627 c. in. × 7.
(2.) 17 c. yds. 17 c. ft. 187 c. in. × 11.
(3.) 25 c. yds. 125 c. in. × 15. (4.) 9 c. yds. 113 c. in. × 56.
(5.) 100 c. yds. ÷ 6. (6.) 87 c. yds. ÷ 12.
(7.) 6 c. yds. 11 c. ft. × $5\frac{3}{4}$.
(8.) 9 c. yds. 18 c. ft. 27 c. in. × $14\frac{3}{8}$.
(9.) 23 c. ft. 347 c. in. × 31.
(10.) 187 c. yds. 11 c. ft. 137 c. in. ÷ 28.
(11.) 1 c. yd. 12 c. ft. 1504 c. in. × $13\frac{3}{8}$.
(12.) 2 c. yds. 15 c. ft. 768 c. in. × $23\frac{5}{8}$.

82. Suppose we place, upon each of the little squares in the figure of (**72**), a *solid* (as, for instance, a brick), in the form of a *cubic foot*, that is measuring a foot every way — a foot long, a foot broad, and a foot high. We shall thus have a layer, or course, of such bricks *one* foot high, and containing as many cubic feet as there are square feet in the base. If upon this we pile another similar layer, we shall have the whole solid *two* feet high, and containing *twice* as many cubic feet as there are square feet in the base ; and so on.

Hence the whole number of cubic feet in any such solid will be found by multiplying together the number of *feet* in the height, and the number of *square feet* in the base; and this last, as in (**76**), we obtain by multiplying together the number of *feet* in the length and the number of *feet* in the breadth.

It appears, then, that the number of *cubic feet* (yards or inches) in any rectangular solid or space is found by multiplying together the number of *linear feet* (yards or inches) in its length, breadth, and height, (or thickness, as the height would be called, when small—as, for instance, in the case of a beam of timber).

Hence we see the reason why a cubic yard, which measures a yard, or 3 feet, every way, contains 3 × 3 × 3 = 27 cubic feet; and a cubic foot, which measures a foot, or 12 inches, every way, contains 12 × 12 × 12 = 1728 cubic inches.

Ex. Find the solid content of a beam of timber, 18 ft. long, 2 ft. 6 in. wide, and 1 ft. 6 in. thick.

$$\begin{array}{rr l}
\text{Here } 18\text{ ft.} = & 216 & \text{in.} \\
2\text{ ft. } 6\text{ in.} = & \underline{30} & \text{in.} \\
& 6480 & \text{sq. in.} \\
1\text{ ft. } 6\text{ in.} = & \underline{18} & \\
& 51840 & \\
& \underline{6480} & \\
& \underline{116640} & \text{cub. in.} = 67 \text{ cub. ft. } 864 \text{ cub. in.}
\end{array}$$

Ex. 78.

(1.) 6 ft. × 7 ft. × $10\frac{1}{2}$ ft. (2.) 3 ft. × $2\frac{1}{3}$ ft. × 5 ft.
(3.) $9\frac{1}{4}$ ft. × 8 ft. × 7 ft. (4.) 6 yds. × 3 ft. 6 in. × 7 ft.
(5.) 3 yds. × 1 yd. 1 ft. × 1 yd. 2 ft.
(6.) 4 yds. 2 ft. × 2 yds. × 8 yds. 1 ft.
(7.) 2 yds. × 2 ft. × 2 in.
(8.) 2 yds. 2 ft. × 2 ft. 2 in. × 2 yds. 2 in.
(9.) $3\frac{1}{3}$ yds. × $2\frac{1}{7}$ ft. × 1 in. (10.) $1\frac{1}{2}$ ft. × $2\frac{2}{3}$ ft. × $3\frac{3}{4}$ ft.
(11.) 1 yd. 2 in. × 2 yds. 3 in. × 3 yds. 4 in.
(12.) 1 yd. 2 ft. 3 in. × 2 yds. 2 ft. 3 in. × 3 yds. 2 ft. 3 in.

Ex. 79.

(1.) What is the solid content of a brick, which is 9 inches long, 4 inches wide, and 3 inches thick?

(2.) How many cubic feet are there in a block of marble, which is 6 feet long, 3 feet wide, and 2 feet thick?

(3.) How many inch cubes can be cut out of a foot cube? How many out of a yard cube?

(4.) The school-room door is 6 feet high, 2½ feet wide, and an inch thick. How many cubic feet of timber are in it?

(5.) How many solid yards of earth will be taken out in making a drain, 100 yards long, 2 feet broad, and 3 feet deep?

(6.) How many cubic feet of water can be contained in a cistern which is 2 feet deep, and has a square base, measuring a yard each way? and what will be its weight, allowing a cubic foot of water to weigh 1000 ounces?

(7.) How many cubic feet of timber are there in a spar which is 30 feet long, 2 feet 3 inches wide, and 2 feet 1 inch thick?

(8.) Find the solid content of a cube whose side is 5 feet 5 inches.

(9.) How much air is there in a room which is 10 feet long, 8 feet 6 inches broad, and 6 feet 6 inches high?

(10.) The length of the cubit was 22 inches. What was the size of the Ark, which was 300 cubits long, 50 broad, and 30 high?

(11.) What quantity of water is contained in a canal whose depth is 9 feet, width 21 feet, and length 12 miles?

(12.) Find the quantity of water in a round washing-tub, whose base is 2 feet 4 inches across, and depth 2 feet.

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83. MEASURES OF CAPACITY.

4 Gills (*g.*) or Noggins make	1 Pint (*pt.*)
2 Pints . . .	1 Quart (*qt.*)
2 Quarts . .	1 Pottle (*pot.*)
4 Quarts . . .	1 Gallon (*gal.*)
2 Gallons . . .	1 Peck (*pk.*)
4 Pecks . . .	1 Bushel (*bus.*)
4 Bushels . . .	1 Coom (*co.*)
8 Bushels . . .	1 Quarter (*qr.*)
5 Quarters . . .	1 Load (*loa.*)

Coal Measure.

3 Bushels . . .	1 Sack
12 Sacks . . .	1 Chaldron.

But coals are most commonly now sold by weight.

A Barrel of *Beer* contains	36 Gallons
A Hogshead . . .	54 Gallons
A Hogshead of *Wine* .	63 Gallons
A Pipe . . .	2 Hogsheads.

Ex. 80.

Reduce

(1.) 37 gallons to pints.
(2.) 56 pecks to quarts.
(3.) 115 bushels to gallons.
(4.) 72 cooms to pecks.
(5.) 237 quarters to bushels.
(6.) 48 cooms to gallons.
(7.) 22 pks. 3 qts. 1 pt. to pints.
(8.) 47 bus. 3 pks. 1 gal. 3 qts. to quarts.
(9.) 13 co. 3 bus. 2 pks. 1 gal. to gallons.
(10.) 47 qrs. 1 co. 2 bus. 3 pks. to pecks.
(11.) 7 co. 3 bus. 2 pks. 1 gal. to gallons.
(12.) 42 qrs. 1 co. 2 bus. 3 pks. to pecks.

Ex. 81.

Reduce

(1.) 109 pints to gallons.
(2.) 47 quarts to pecks.
(3.) 151 gallons to bushels.
(4.) 382 pecks to cooms.
(5.) 190 bushels to quarters.
(6.) 686 pecks to cooms.
(7.) 440 pints to pecks.
(8.) 1012 quarts to bushels.
(9.) 935 gallons to cooms.
(10.) 1983 pecks to quarters.
(11.) 1447 gallons to cooms.
(12.) 891 pecks to quarters.

Ex. 82.

Add

	gals.	qts.	pts.
(1.)	13	2	1
	2	3	0
	15	0	0
	7	1	1

	pks.	gals.	qts.
(2.)	5	1	3
	6	1	1
	2	0	0
	3	0	2

	bus.	pks.	gals.
(3.)	17	2	1
	11	3	1
	3	0	0
	18	3	1

	co.	bus.	pks.
(4.)	23	3	2
	3	2	3
	21	2	3
	4	3	2

	qrs.	co.	bus.
(5.)	17	1	3
	16	0	3
	23	1	2
	31	1	2

	co.	bus.	pks.
(6.)	42	3	2
	7	2	3
	16	1	3
	5	2	1

Subtract

	pks.	gals.	qts.		bus.	pks.	gals.		gals.	qts.	pts.
(7.)	27	1	1	(8.)	56	1	0	(9.)	4	1	0
	18	1	3		27	3	1		3	2	1

	co.	bus.	pks.		qrs.	co.	bus.		co.	bus.	pks.
(10.)	43	2	2	(11.)	56	0	2	(12.)	54	1	2
	33	3	3		29	1	3		48	2	3

Ex. 83.

(1.) 7 gals. 1 qt. 1 pt. × 3. (2.) 9 co. 2 bus. 3 pks. × 7.
(3.) 34 bus. 3 pks. 1 gal. ÷ 9. (4.) 13 qrs. 1 co. 2 bus. ÷ 11.
(5.) 5 bus. 2 pks. ÷ 16.
(6.) 24 co. 1 pk. 2 qts. 1 pt. ÷ 45.
(7.) 12 pks. 1 gal. 2 qts. × $56\frac{1}{2}$.
(8.) 43 qrs. 3 bus. × $108\frac{3}{4}$. (9.) 11 pks. 1 gal. 2 qts. × 29.
(10.) 27 co. 2 bus. 3 pks. 1 gal. 3 qts. ÷ 67.
(11.) 15 gals. 2 qts. 1 pt. × $130\frac{3}{8}$.
(12.) 47 qrs. 3 bus. 1 qt. × $273\frac{5}{8}$.

Ex. 84.

(1.) A nine-gallon cask of beer had leaked; and, upon being tapped, it was found to yield only 7 gallons and a pint. How many pints had leaked away?

(2.) Part of an eighteen-gallon cask of beer was drawn, and the rest was bottled. There were four dozen and five bottles altogether, each holding a quart. How much had been *drawn?*

(3.) Out of a coom of barley, I brewed $54\frac{1}{2}$ gallons of beer, and filled with it four casks. What did they each hold?

(4.) A farmer's crop one year was 225 coom of wheat, 174 coom of barley, and 132 coom of oats. How many quarters of grain had he altogether?

(5.) A barn was broken open by thieves, and out of 20 quarters of wheat, which had been threshed, they left only nine coom and a gallon and a half remaining. How much was stolen?

(6.) If eight pecks and a half of seed are wanted for an acre, how many bushels will be wanted to sow a field of 7 acres?

(7.) A maltster bought a quantity of barley, which he brought home in three waggons. The first carried 43 coom

3 bushels, the second, 37 coom and half a bushel, and the third, 35 coom and a bushel and a half. How much did it amount to altogether?

(8.) Out of 15 coom of wheat, seven bushels were damaged. How many quarters of good corn remained of the lot?

(9.) A farmer reckoned his crop of wheat one year, from a field of 8 acres, as being 67 coom and a half. How much was this per acre?

(10.) A corn-dealer buys 3 qrs. of corn at 6*s.* 6*d.* a bushel, and 5 qrs. 3 bus. at 6*s.* a bushel. At what price per bushel must he sell the whole, so as to gain £2 15*s.* by the bargain?

(11.) A dishonest milkman mixes a pint of water with every two quarts of milk. How many gallons will he make in this way out of 20 gallons of pure milk?

(12.) A ship's crew, consisting of 36 men, found their water running so short, that each man could only be allowed a pint and a half a day, in order that it might hold out for ten days longer, in which time they expected to make the land. How much water was at this time on board?

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84. Measures of Time.

60 Seconds (*sec.*) make 1 Minute (*min.*)
60 Minutes . . 1 Hour (*hr.*)
24 Hours . . . 1 Day (*d.*)
7 Days . . . 1 Week (*w.*)
4 Weeks . . . 1 Lunar Month (*mo.*)
365 Days . . . 1 Year (*yr.*)

Minutes and seconds are denoted by the marks $'$ and $''$; thus $13^h\ 14'\ 56''$ means 13 hours 14 minutes 56 seconds.

Since 52 weeks, or 13 lunar months, contain 364 days, these are often reckoned as a year.

85. The year is also divided into 12 months, called *Calendar* Months, which contain unequal numbers of days:—

January	31.	May	31.	September	30.
February	28.	June	30.	October	31.
March	31.	July	31.	November	30.
April	30.	August	31.	December	31.

It will be seen that all of these contain 31 days, except February, which has 28, and those mentioned in the following rhymes, which have 30:—

'Thirty days have September,
April, June, and November.'

Every fourth year contains 366 days, and is called Leap-Year: and in such a year February has 29 days.

Ex. 85.

Reduce

(1.) 13 years to days. (2.) 5 weeks to hours.
(3.) 27 days to hours. (4.) 12 hours to seconds.
(5.) 13 weeks to minutes. (6.) 33 years to seconds.
(7.) 13 d. 7 h. 15 min. to minutes.
(8.) 11 w. 3 d. 14 h. to seconds.
(9.) 5 y. 123 d. to minutes.
(10.) 4 mo. 1 w. 13 h. to minutes.
(11.) 11 y. 225 d. 17 h. to minutes.
(12.) 3 y. 113 d. 35 min. to seconds.

Ex. 86.

(1.) 123456 days to years. (2.) 300125 hours to weeks.
(3.) 356123 hours to months.
(4.) 268137 seconds to hours.
(5.) 500000 minutes to weeks.
(6.) 305503 seconds to days. (7.) 471654 hours to months.
(8.) 376423 minutes to weeks.
(9.) 146357 seconds to days.
(10.) 365230 hours to years.
(11.) 653725 minutes to months.
(12.) 673856 seconds to days.

Ex. 87.

Add

	h.	min.	sec.
(1.)	6	27	49
	2	11	21
	4	8	17
	11	19	53

	d.	h.	min.
(2.)	16	8	27
	41	14	33
	3	7	14
	27	13	9

	h.	min.	sec.
(3.)	5	25	17
	6	32	43
	17	47	37
	8	18	29

	w.	d.	h.
(4.)	13	2	15
	11	6	11
	9	0	9
	17	4	18

	mo.	w.	d.
(5.)	18	2	4
	7	0	2
	16	1	6
	11	3	5

	w.	d.	h.
(6.)	17	4	17
	2	6	15
	11	1	9
	18	5	11

Subtract

	w.	d.	h.
(7.)	18	4	15
	12	5	22

	mo.	w.	d.
(8.)	13	0	0
	3	3	3

	w.	d.	h.
(9.)	11	1	11
	8	1	23

	yr.	d.	h.		d.	h.	min.		h.	min.	sec.
(10.)	5	0	17	(11.)	15	19	47	(12.)	12	1	12
	2	138	22		8	20	52		7	4	44

Ex. 88.

(1.) 4 h. 8 min. 47 sec. × 4. (2.) 17 d. 15 h. 23 min. × 8.
(3.) 11 w. 6 d. 18 h. ÷ 9. (4.) 13 mo. 3 w. 4 d. ÷ 10.
(5.) 3 d. 15 h. 35 m. × 35. (6.) 2 y. 135 d. 17 h. ÷ 72.
(7.) 17 h. 19 min. 21 sec. × $110\frac{2}{3}$.
(8.) 11 w. 4 d. 13 h. × $144\frac{3}{4}$. (9.) 4 mo. 3 w. 5 d. × 31.
(10.) 5 y. 200 d. 16 h. 59 min. 42 sec. ÷ 79.
(11.) 19 h. 18 min. × $123\frac{5}{8}$.
(12.) 6 d. 11 h. 35 min. 12 sec. × $202\frac{4}{7}$.

Ex. 89.

(1.) A watch loses 7 min. 13 sec. a day. How much time will it lose in a fortnight? If it agree with the true time, $\frac{1}{4}$ to 12, now, what time will it show at $\frac{1}{4}$ to 12, exactly a fortnight hence?

(2.) The sun's light takes 8 min. 13 sec. to reach the earth; and light travels at the rate of 192000 miles a second. How far is the sun from the earth?

(3.) Find the exact length of the lunar month, which contains 2551443 seconds.

(4.) If five minutes are wasted every day, how much time is lost in 20 years? (Mind the leap-year.)

(5.) How long would it take to count a million of shillings, at the rate of 100 a minute?

(6.) How many seconds has a boy lived, who is just twelve years old? (Mind the leap-year.)

(7.) How many times will a clock strike in twenty years?

(8.) Supposing a railway engine to travel at the rate of 25 miles an hour, how long would it take to go round the moon, the earth, and the sun respectively, reckoning their diameters as 2160, 7912, and 887076 miles. (See Ex. **66**, 10.)

(9.) If a railway engine were to travel, night and day, without stopping, at the rate of 100 miles in three hours, how long would it take to reach the moon, a distance of 240000 miles?

(10.) How long would the same engine take to reach the sun, a distance of 96 millions of miles?

(11.) If the distance to the sun could be traversed in one day, how long would it take to reach the nearest fixed star, a distance of $19\frac{1}{2}$ billions of miles?

(12.) How long ago was the light, by which it is seen at any moment, sent off from the star in (11)? See Ex. 2, above.

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86. When numbers are used with reference to the things numbered, as when we say 3 *apples*, 4 *pens*, 5 *shillings*, they are called *concrete* numbers. When they are used without any such reference, merely to indicate a certain number of units of the same kind, as when we say simply 3, 4, 5, they are called *abstract* numbers.

The concrete quantities, required in ordinary calculations, are those which are necessary to express *money*, *weight*, *space*, and *time*.

87. Division is required in Arithmetic, when the dividend is a concrete quantity, for two distinct purposes.

(i.) We may have to divide a concrete quantity by an *abstract* number; and this amounts to finding what is the value of a certain given part of the dividend.

Thus, to divide £3 7*s.* 6*d.* by 8, is to find the value of the eighth part of £3 7*s.* 6*d.*—namely, 8*s.* $5\frac{1}{4}$*d.*

(ii.) We may have to divide a concrete quantity by another *concrete* quantity of the same kind; and this amounts to finding how many times the divisor is contained in the dividend.

Thus, to divide £3 7*s.* 6*d.* by 16*s.* $10\frac{1}{2}$*d.*, is to find how many times 16*s.* $10\frac{1}{2}$*d.* is contained in £3 7*s.* 6*d.*—namely, 4 times.

The difference in the two operations will be plain by observing, that in the latter case we can find how many times the divisor is contained in the dividend, by *subtracting* it again and again; but we cannot do this with an example of the former case.

In instances of the former kind, the quotient may be obtained, as in the Examples hitherto given, and will always be a *concrete* quantity, of the same kind as the dividend.

In cases of the latter kind, the quotient may be obtained by reducing the two quantities to the same denomination, and then performing the division.

Ex. 1. Divide £5 1*s.* 3*d.* by 8*s.* $5\frac{1}{4}$*d.*

Here £5 1*s.* 3*d.* = 4860 *farthings.*
8*s.* $5\frac{1}{4}$*d.* = 405 *farthings.*

Hence we have

$$\begin{array}{r} 405)\,4860\,(12 \\ \underline{405} \\ 810 \\ \underline{810} \end{array}$$

Ans. 12.

Ex. 2. Divide 1 ton 11 cwt. 24 lbs. 8 oz. by 1 qr. 3 lbs. 8 oz.

Here 1 ton 11 cwt. 24 lbs. 8 oz. = 55944 oz.
1 qr. 3 lbs. 8 oz. = 504 oz.

$$\begin{array}{r} 504)\,55944\,(111 \\ \underline{504} \\ 554 \\ \underline{504} \\ 504 \\ \underline{504} \end{array}$$

Ans. 111.

Ex. 90.

£ *s.* *d.* £ *s.* *d.* £ *s.* *d.* £ *s.* *d.*

(1.) 22 15 $7\frac{1}{2}$ ÷ 2 10 $7\frac{1}{2}$. (2.) 13 18 $5\frac{1}{4}$ ÷ 1 5 $3\frac{3}{4}$.
(3.) 34 3 $5\frac{1}{4}$ ÷ 3 15 $11\frac{1}{4}$. (4.) 296 3 $1\frac{1}{2}$ ÷ 22 15 $7\frac{1}{2}$.
(5.) 227 16 3 ÷ 15 3 9. (6.) 85 10 $11\frac{1}{4}$ ÷ 28 10 $3\frac{3}{4}$.
(7.) 44 cwt. 2 qrs. 11 lbs. ÷ 1 cwt. 2 qrs. 17 lbs.
(8.) 40 cwt. 2 qrs. 22 lbs. ÷ 3 qrs. 22 lbs.
(9.) 29 miles 1 fur. 100 yds. ÷ 2 m. 3 fur. 100 yds.
(10.) 417 yds. 1 ft. 5 in. ÷ 3 yds. 2 ft. 1 in.
(11.) 122 qrs. 1 bus. 2 pks. 1 gal. ÷ 1 qr. 4 bus. 1 pk. 1 gal.
(12.) 1183A. 1R. 38P. ÷ 45A. 2R. 3P.

Ex. 91.

(1.) How often will a cart-wheel turn, in going a distance of 10 miles, if its circumference be 3 ft. 9 in.?

(2.) How many lengths of rope, each $4\frac{1}{2}$ yds. long, can be made out of a coil of 18 fathoms?

(3.) The length of a chain is 50 ft. 8 in. How many times will it wrap round a wheel which is 2 ft. 8 in. in circumference?

(4.) A well is 18 yds. 2 in. deep, and the wheel is 4 ft. 2 in. round. How many turns of the wheel will raise the bucket?

(5.) The upper floor of a house is 18 ft. 4 in. above the ground, and is to be reached by two equal staircases, separated by the landing of the first floor. How many stairs will there be in each staircase, each stair being 11 inches high?

(6.) A school-room desk is 11 ft. 4 in. long. How many boys can be seated at it, when each requires 1 ft. 5 in. in length of the desk?

(7.) Alison's History of Europe consists of a set of volumes, each 2 in. thick, and filling closely up a shelf, 3 ft. 6 in. long.

How many volumes are there? How many would there have been, if they had been 1½ in. thick?

(8.) A row of cabbages is planted, 33 ft. 10 in. long, and with the plants 14 inches apart. How many are there in the row?

(9.) A field of 2A. 1R. 18P. is to be divided into allotments each of 42P. How many such allotments can be made?

(10.) A father left a small farm of 37 acres to be divided equally among his children, each of whom received for his portion 7A. 1R. 24P. of land. How many were they?

(11.) A farmer wants to carry 28 quarters of corn to market. How many sacks must he provide for this purpose, each holding 3 bus. 2 pks.?

(12.) How many sacks of potatoes will make up the weight of 17 cwt., if each sack weighs 2 st. 6 lbs.?

(13.) A box of tea, containing 36 lbs., was given away among the poor women of a parish, each of them receiving 9 ounces. How many were they?

(14.) At a school-feast, 50 lbs. of cake were eaten. Allowing 12½ ounces for each child on the average, how many children were they?

(15.) Each person, on the average, breathes 28 cubic feet of air in an hour. If twelve tailors were working in a room, 14 feet long, 12 feet wide, and 6 feet high, and without any openings for fresh air, how many hours would the air in the room supply fresh air for each inspiration?

(16.) A French *franc* is worth 10*d.* How many are there in £24 6*s.* 8*d.*?

(17.) There are 9 oz. of iron in the blood of one man. How many men will furnish iron enough in their veins to make a ploughshare of 22½ lbs. weight?

(18.) How many parcels of sugar, each weighing 4½ lbs., can be made out of 3 cwt. 2 qrs. 22 lbs.?

(19.) It takes 4 yds. 1 qr. 3 na. of cloth to make a suit of clothes. How many suits can be made out of a piece of cloth containing 39 yds. 3 qrs. 3 na.?

(20.) A silversmith had orders to melt down a silver tankard, weighing 5 lbs. 2 oz. 11 dwt., and make spoons of it. How many did he make, each spoon weighing 2 oz. 12⅛ dwt.?

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88. Questions of the following kind are usually given under the Rule of Three, but may be solved, like those in

(**36**) and (**47**), after reducing the pair of like quantities, which occurs in each question, to the same common denomination.

Ex. 1. If 1 qr. 11 lbs. cost £59 6*s.* 3*d.*, what will 2 qrs cost, at the same rate?

Here 1 qr. 11 lbs. = 39 lbs.
2 qrs. = 56 lbs.
hence 39 lbs. cost £59 6*s.* 3*d.*;
therefore, 1 lb. costs $\frac{1}{39}$ of £59 6*s.* 3*d.*,
and 56 lbs. cost $\frac{56}{39}$ of £59 6*s.* 3*d.* = £85 3*s.* 4*d.*

Ex. 2. If I can buy 2 cwt. 3 qrs. 7 lbs. of goods for £1 8*s.* $1\frac{1}{2}$*d.*, what weight of goods can I buy for £20?

Here £1 8*s.* $1\frac{1}{2}$*d.* = 675 halfpence,
£20 = 9600 halfpence;
therefore, 675 halfpence will buy 2 cwt. 3 qrs. 7 lbs. or 315 lbs.,
and 1 halfpenny . . $\frac{1}{675}$ of 315 lbs.
and 9600 halfpence . . $\frac{9600}{675}$ of 315 lbs. = 4480 lbs. = 40 cwt.

Ex. 92.

(1.) How many yards of cloth can be had for £10 15*s.* when 37 yards cost £6 12*s.* 7*d.*?

(2.) If $1\frac{1}{2}$ cwt. of cheese cost £2 12*s.*, what must I give for 3 qrs. 7 lbs. of the same?

(3.) If 1 lb. 2 oz. of honey cost 1*s.* $10\frac{1}{2}$*d.*, what is the value of 1 cwt.?

(4.) If the coach fare for 57 miles be £1 4*s.*, how far ought I to go for £2 16*s.*?

(5.) If 87 yards of holland cost £6 0*s.* 10*d.*, how many can be had for £30?

(6.) If 18A. 1R. 20P. of land are let for £21, what would be the rent of 43A. 3R. of the same land?

(7.) If a lump of gold, weighing 1 lb. 2 oz., 5 dwts. 16 grs., be sold for £42 17*s.*, what is that per ounce?

(8.) If I pay 3*s.* 9*d.* per hundred for pens, how many shall I buy for £103 2*s.* 6*d.*?

(9.) What weight of sugar may be bought for £187 4*s.* when the cost of 3 cwt. 1 qr. is £13 17*s.* 4*d.*?

(10.) The carriage of a parcel of goods, weighing 1 ton 3 cwt. 2 qrs., being £1 7*s.*, what will be the charge for four other parcels, weighing each 1 ton 2 qrs. 7 lbs.?

(11.) How many gals. of wine, at the rate of £15 18*s.* 2*d.* per 23 gallons, can be bought for £58 15*s.* 10*d.*?

(12.) If the poor-rate on £23 17*s.* 6*d.* be £3 19*s.* 7*d.*, what is that on the £?

89. The following are examples in Practice, where the price of a unit is given, as in (**55**), &c., but the quantity, whose value is to be found, is not expressed in the same denomination as the given unit.

Ex. Find the value of 7 cwt. 3 qrs. 11 lbs., at £2 13*s.* 1*d.* per qr.

[Here 7 cwt. 3 qrs. = 31 qrs., and the value of 31 qrs., at £2 13*s.* 1*d.* per qr., can be found by multiplying. We have next to find the value of 11 lbs., at £2 13*s.* 1*d.* per quarter; and for this we observe that 7 lbs. are $\frac{1}{4}$ of a quarter, and 4 lbs. are $\frac{1}{7}$ of a quarter: so that, dividing £2 13*s.* 1*d.* by 4 and by 7, and adding up, we shall have the value of 11 lbs., which we are then to add to the other.

		£	*s.*	*d.*		
		2	13	1 × 31		
				10		
		26	10	10		
				3		
		79	12	6	value of 30 qrs.	
		2	13	1	,, 1 qr.	
7 lbs.	$\frac{1}{4}$		18	$3\frac{1}{4}$	,, 7 lbs.	
4 lbs.	$\frac{1}{7}$		7	7	,, 4 lbs.	
		£83	6	$5\frac{1}{4}$	,, 31 qrs. 11 lbs.	Ed.]

Find the value of

Ex. 93.

(1.) 5 cwt. 15 lbs., at £2 2*s.* 6*d.* per lb.
(2.) 3 cwt. 1 qr. 8 lbs., at £2 4*s.* 4*d.* per quarter.
(3.) 7 cwt. 3 qrs. 11 lbs., at £1 8*s.* per cwt.
(4.) 3 qrs. 2 lbs. 10 oz., at £4 14*s.* 8*d.* per lb.
(5.) 2 qrs. 7 oz. 7 dr., at £1 6*s.* 8*d.* per lb.
(6.) 2 cwt. 2 qrs. 10 lbs., at £5 5*s.* per cwt.
(7.) 15 yds. 2 ft. 8 in., at £3 18*s.* per yard.
(8.) 32 yds. 2 ft. 5 in., at 4*s.* 6*d.* per foot.
(9.) 34A. 1R. 16P., at £2 3*s.* 4*d.* per acre.
(10.) 27A. 3R. 20P., at £1 8*s.* 8*d.* per acre.
(11.) 6 mo. 2 w. 4 d., at £1 11*s.* 6*d.* per month.
(12.) 7 mo. 3 w. 6 d., at £3 3*s.* per week.

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90. The method of Practice may be applied, not only when the given value is the *price* of a given unit, but whenever any given amount is charged for any reason whatever upon the unit.

Ex. A bankrupt pays 7*s.* $8\frac{1}{2}$*d.* in the £, and his debts are £2560. What was his estate worth?

[				
		2560		
5*s.*	$\frac{1}{4}$	640		
2*s.* 6*d.*	$\frac{1}{2}$	320		
$2\frac{1}{2}$*d.*	$\frac{1}{12}$	26	13	4
		£986	13	4

This means that, for every £ he owes, he can pay only 7*s.* $8\frac{1}{2}$*d.* Here, then, we have to find the value of 7*s.* $8\frac{1}{2}$*d.* × 2560, which must have been the value of his whole estate.—Ed.]

Ex. 94.

(1.) What is the pay of 147 factory-men for a week, at 14*s*. 3*d*. each?

(2.) A bankrupt's debts are £4976, and he is able to pay 11*s*. 8$\frac{3}{4}$*d*. in the £. What are his effects worth?

(3.) What must be given for a gold ornament, weighing 4 oz. 4 dwts. 22 grs., at the rate of £4 10*s*. per ounce?

(4.) How many acres will supply 47 horses, if each horse consumes annually the produce of 2A. 3R. 33P.?

(5.) If a gentleman's estate be worth £3185 per annum, and the land-tax be assessed at 2*s*. 9$\frac{1}{2}$*d*. in the £, what is his net income?

(6.) A piece of land, containing 37A. 2R. 35P., was sold at £53 16*s*. 8*d*. per acre. What did it cost altogether?

(7.) I have a claim upon the estate of the bankrupt in (2), to the amount of £776. What shall I receive for it?

(8.) Find the rent of a farm, of 357A. 2R. 25P., at £1 13*s*. 4*d*. per acre.

(9.) If a workman lay by 3*s*. 10$\frac{1}{2}$*d*. a week, what will he save in a year?

(10.) Reckoning a man's step as 2 ft. 10 in., how far will a regiment of soldiers, making 75 steps a minute, advance in an hour?

(11.) A grocer mixes 2 cwt. 15 lbs. of sugar, at £5 2*s*. 8*d*. per cwt., with 4 cwt. 30 lbs, at £2 6*s*. 8*d*. per cwt. Show that, if he sells the mixture at £3 5*s*. 4*d*. per cwt., he will neither gain nor lose by the transaction.

(12.) What will be the cost of replacing a cistern, to weigh 4 cwt. 1 qr. 7 lbs., at the rate of £2 6*s*. 4*d*. per cwt., if the plumber allows £1 10*s*. 4*d*. per cwt. for the lead of the old one, which weighs 3 cwt. 2 qrs. 19 lbs.?

TARE AND TRET.

91. By the *Gross Weight* of any quantity of goods is meant the whole weight both of goods and package.

Tare is an allowance made for the weight of the package, either at so much per barrel, box, sack, &c., or at so much per cwt. In the former case, the Tare may be found by Multiplication; in the latter, by Practice, or as in the Example below.

Tret is an allowance for waste of 1 lb. in 26 lbs., or $\frac{1}{26}$ of the whole weight of the *goods;* that is, after the Tare has been deducted from the Gross Weight.

Cloff is an allowance, after Tare and Tret have been deducted, generally of 2 lbs. on every 3 cwt., which is *sometimes* made to retail tradesmen, for the turning of the scales.

Suttle is the name given to the remainder, when any of the above allowances has been made.

Net (or Neat) *Weight* is the final result, after all the allowances have been made.

Ex. Find the net weight of 17 chests of tea, each weighing 1 cwt. 2 qrs. 13 lbs., tare 10 lbs. per cwt., tret as usual.

	cwt.	qrs.	lbs.	
	1	2	13	
			17	
	27	1	25	gross
	2	1	$22\frac{1}{2}$	tare
26)	25	0	$2\frac{1}{2}$	suttle
		3	23 (+)	tret
	24	0	$7\frac{1}{2}$	net

Tare		cwt.	qrs.	lbs.
For 27 cwt.	=	2	1	18
2 qrs.	=			$2\frac{1}{2}$
25 lbs. or $\frac{25}{112}$ cwt.	=			2 (+)
		2	1	$22\frac{1}{2}$

Ex. 95.

Find the net weight

(1.) Of 6 hhds. of sugar, each weighing 10 cwt. 2 qrs. 10 lbs., *tare* 1 qr. 14 lbs. per hhd.

(2.) Of 10 chests of tea, each weighing 1 cwt. 2 qrs. 3 lbs., *tare* 16 lbs. per chest.

(3.) Of 20 casks of butter, each weighing 4 cwt. 2 qrs. 10 lbs., *tare* 14 lbs. per cwt.

(4.) Of 7 hhds. of tobacco, each weighing 3 cwt. 1 qr. 8 lbs., *tare* 20 lbs. per cwt.

(5.) Of 16 barrels of rice, each weighing 3 cwt. 15 lbs., *tare* 1 qr. 10 lbs. per barrel, *tret* as usual.

(6.) Of 23 barrels of figs, each weighing 2 cwt. 1 qr. 8 lbs., *tare* 16 lbs. per cwt., *tret* as usual.

(7.) Of 13 butts of currants, each weighing 7 cwt. 1 qr. 10 lbs., *tare* 14 lbs. per cwt., *tret* as usual.

(8.) Of 10 casks of butter, each weighing 4 cwt. 0 qr. 6 lbs., *tare* 18 lbs. per cwt., *tret* as usual.

(9.) Of 16 bags of pepper, each weighing 75 lbs. 8 oz., *tare* $2\frac{1}{4}$ lbs. per bag, *tret* as usual.

(10.) Of 7 frails of raisins, each weighing 5 cwt. 2 qrs. 5 lbs., *tare* 23 lbs. per frail, *tret* as usual.

(11.) Of 15 barrels of nutmegs, each weighing 2 cwt. 2 qrs. 11 lbs., *tare* 16 lbs. per cwt., *tret* as usual.

(12.) Of 17 barrels of soap, each weighing 4 cwt. 3 qrs. 27 lbs., *tare* 20 lbs. per cwt., tret as usual.

---

92. Since by multiplying together the numbers of yards, feet, &c. in the *length* and *breadth* of any rectangular surface, we obtain the number of square yards, square feet, &c. in its area, it follows that, if we divide the square area by the *length*, we shall get the *breadth*, or, if we divide by the *breadth*, we shall get the *length* of it.

N.B. Remember to express the quantities concerned, before division, as quantities of the same denomination, so as not, for instance, to divide a number of square *feet* by a number of *inches*, &c.

Ex. What length of carpet, 2 ft. wide, will be wanted for a room, 14 ft. 6 in. square?

```
                                           in.  sq.in.  in.
     Here 14 ft. 6 in.=174 in.      2 ft.=24) 30276 (1261 (+)
                       174 in.                24
                       ---                    --
                       696                     62
                      1218                     48
                      174                      ---
  area of floor=30276 sq. in.                  147
                -----                          144
                                               ---
                                                 36
                                                 24
                                                 --
                                                 12
                                                 --
                                     in.   ft. in.
                                Ans. 1261=105  1=35 yds. 1 in.
```

Ex. 96.

(1.) Given area = 7 sq. yds. 6 ft. 96 in., and *length* = 5 yds. 2 ft. 5 in., find the *breadth*.

(2.) Given area = 10 sq. yds. 4 ft. 80 in., and *breadth* = 3 ft. 10 in., find the *length*.

(3.) I cut 300 inch-squares out of a piece of plank that was 2 ft. 1 in. long. What was its breadth?

(4.) I want to cut off an acre of land from a long narrow field, whose breadth is 55 yards. What length must I take of it?

(5.) A rectangular play-ground was 9 yards long, and contained 60 square yards. What was its breadth?

(6.) If 72 yards of carpet, 3 quarters wide, will cover a floor, how much of yard-wide carpet would be wanted for the same floor?

(7.) Reckoning each furrow as a foot in breadth, how many miles will a ploughman walk in ploughing an acre of land?

(8.) The width of a form is 9 inches, and its surface is 6 square feet. What is its length?

(9.) A school-room is 24 feet square. If one side were half as long again, what must be the length of the other, that it may be able to hold the same number of children?

(10.) It took me 32 yards of carpet to cover a parlour, 16 feet by 14. What was the width of the carpet?

(11.) What was the length of a room, whose breadth was 12 feet, and which it took 20 yards of yard-wide drugget to cover?

(12.) I want to paper the walls of a room, which measures 10 feet by $12\frac{1}{2}$, and whose height is 7 feet; and I have chosen a paper which is 2 ft. 6 in. wide. How many yards of it will be wanted? What will it cost, at $1\frac{1}{2}$*d.* a yard?

---

93. In like manner, since the number of cubic yards, feet, &c. in any space is found by multiplying together the three numbers which express the number of feet in its *length*, *breadth*, and *thickness*, if we have given the solid content and divide it by the product of the *length* and *breadth*, we shall get the *thickness*, or, if by the product of the *length* and *thickness*, we shall get the *breadth*, or, if by the product of the *breadth* and *thickness*, we shall get the *length*.

N.B. Remember, as before, to bring all the quantities to the same denomination before the division.

Ex. What is the length of a room, whose width is 9 ft. 4 in., and height 6 ft. 3 in., and which contains 875 cubic feet of air?

```
                                     c. ft.   c. in.
Here 9 ft. 4 in.=112 in.             875=1512000
     6 ft. 3 in.= 75 in.               in.     c. in.    in.
                  560                84,00) 15120,00 (180
                 784                         84
                 8400 sq. in.                672
                                             672

   Ans. 180 in.=15 ft.
```

Ex. 97.

(1.) What length must be cut off a piece of deal, which is cut so as to be 8 inches square all along, that the portion cut off may contain 4 solid feet of timber?

(2.) In digging a square pond, whose side was 24 feet, there were taken out 128 cubic yards of earth. How deep was it made?

(3.) The depth of a canal was 6 feet, the width 15 feet, and it contained 176,000 cubic yards of water. What was its length?

(4.) A log of wood contained 125 cubic feet of timber. Its length was 20 feet, and its thickness $2\frac{1}{2}$ feet. What was its breadth?

(5.) The bottom of a cistern is 12 square feet. What is its depth, if it contains 60 cubic feet of water? What would be the depth of a cistern containing the same quantity of water, but with its bottom 12 feet square?

(6.) Find the length of a beam, whose end is a foot square, which shall contain the same quantity of wood as one that is 16 in. wide, 18 in. deep, and 5½ ft. long.

(7.) In boring a circular well, whose mouth is 6 square feet, 18 cubic yards of earth were thrown out. What was its depth?

(8.) A plank of fir is 3 inches thick, and 18 inches broad. What length must be cut off it, so as to contain 6 cubic feet of timber?

(9.) What must be the size of a box to hold a work in 50 volumes, each of which is 9 inches long, 6 inches wide, and 2 inches thick?

(10.) If the bottom of the box in (9) be 15 inches square, what must be its depth?

(11.) If the length of the box in (9) be 2 ft. 1 in., and the breadth 1 ft., what must be the depth?

(12.) The bore of a waterpipe is 500 square inches. At what rate per hour must water be forced through it, so as to supply a town with 50,000 cubic feet of water each hour?

ARTIFICERS' WORK.

94. *Glazing* and *Masons' Flat Work* are charged by the square foot; *Painting*, *Plastering*, *Paving*, &c., by the square yard.

Ex. 98.

(1.) What is the cost of glazing 10 squares, each measuring 4 ft. by 2 ft. 6 in., at 1*s*. 3*d*. per square foot?

(2.) What will be the cost of a plate of glass, measuring 7 ft. by 5 ft. 6 in., at 4*s*. 6*d*. per foot?

(3.) A house had eight windows, each 4 feet high and 2 ft. 6 in. broad. What will the glazing come to, at 2*s*. 1*d*. a foot?

(4.) What will be the cost of a block of stone, 4 ft. 4 in. long, and 3 ft. broad, at 3*s*. 4*d*. per square foot?

(5.) There is a house with two stories. In the lower there are nine windows, each 3 ft. 6 in. high, and in the upper, seven windows, each 3 ft. high. The breadth of all the windows is the same, namely, 2 feet. What will the glazing cost at 1*s*. 6*d*. a foot?

(6.) Find the cost of a grave-stone, length 6 ft. 6 in., breadth 3 ft. 3 in., at 4*s*. per foot.

(7.) What is the value of a marble slab, whose length is 10 ft. 6 in., and breadth 3 ft. 6 in., at £2 10*s*. per foot?

(8.) What would be the expense of glazing a hall-window, containing 60 squares, each 1 ft. 6 in. long, and 1 ft. wide, at 3*s*. 6*d*. per foot?

(9.) What is the cost of a block of marble, 9 feet long, 4 feet broad, and 3 feet thick, at 15*s*. per solid foot?

(10.) What would be the cost of a stone slab, 8 feet long, 3 feet broad, and 9 inches thick, at 3*s*. 6*d*. per solid foot?

(11.) What is the value of a beam of timber, 20 feet long, 3 feet broad, and 2 ft. 3 in. thick, at 7*s*. 5½*d*. per cubic foot?

(12.) A plank of timber is 18 inches wide and 2 inches thick. What is the price of five such planks, each 8 feet long, at 2*s*. 2*d*. per solid foot?

(13.) What will it cost to pave a kitchen, 12 ft. by 10 ft. 6 in., at 10*s*. 6*d*. per yard?

(14.) What will be the charge for painting a room, 18 feet by 14 feet, and 8 feet high, at 2*s*. 3*d*. per yard?

(15.) What will be the cost of painting both sides of eight doors, each 6 ft. 6 in. high, and 3 ft. 6 in. broad, at 6¾*d*. per yard?

(16.) What will be the expense of paving a cellar, 15 feet by 12 ft. 8 in., at 6*s*. 4½*d*. per square yard?

(17.) A court-yard, 24 feet by 18½ feet, is to be paved at 4*s*. 6*d*. a yard. What will it cost to do it?

(18.) What is the cost of a skirting, 3 feet deep, for a room 20 feet by 16 feet, at 6*s*. 9*d*. the square yard?

(19.) There are two rooms, each 16 feet by 12 feet. What will it cost to have the ceilings plastered, at 6*d*. per yard?

(20.) If the height of the rooms in (19) be 8 feet, what will be the cost of washing the walls, at 2¼*d*. per yard, allowing in each room for two doors, each 7 ft. by 3 ft., and for two windows, each 4 ft. by 2½ ft.?

(21.) A court-yard, 15 yds. by 12 yds., is to be paved with pebbles at 3*s*. per yard, except a foot-way, 4 feet broad, which runs down the middle of the length of the court, and this is to be laid with Purbeck stone, at 3*s*. 6*d*. per yard. What will the whole come to?

(22.) What will be the cost of carpeting a floor, 18 feet square, with yard-wide carpet, at 5*s*. 6*d*. a yard?

(23.) If the carpet in (22) were only ¾-yd. wide, what would it cost at 3*s*. 6*d*. per (linear) yard?

(24.) What will it cost to paper a room, 16½ ft. by 12½ ft., and 8 ft. 6 in. high, with paper, 2 ft. wide, at 2½*d*. per (linear) yard?

95. *Flooring, Partitioning, Roofing, Tiling,* &c., are charged by the *square* of 100 square feet.

N.B. When the roof has, what is called, a *true pitch*, (that is, when the rafters are exactly *three-quarters* of the breadth of the building within the walls, and, consequently, the double rafters are *six quarters*, or one and a-half times of the breadth of the building), the measure of the roof may be found by taking $1\frac{1}{2}$ times the ground floor or *flat*.

Ex. Find the cost of roofing a house, which measures within the walls 26 ft. 4 in. long by 15 ft. 3 in. broad, at £1 per square,—roof true pitch.

$$
\begin{array}{rrl}
\text{Here } 26 \text{ ft. } 4 \text{ in.} = & 316 & \text{in.} \\
15 \text{ ft. } 3 \text{ in.} = & 183 & \text{in.} \\
 & 948 & \\
 & 2528 & \\
 & 316 & \\
\hline
\text{area of flat} = & 57828 & \text{sq. in.} \\
\frac{1}{2} \times \text{flat} = & 28914 & \text{sq. in.} \\
\hline
\text{area of roof} & 86742 & \text{sq. in.} = 602 \text{ sq. ft. } (+) \\
 & & = 6.02 \text{ squares.}
\end{array}
$$

Ans. £1 × 6.02 = £6.02 = £6 0*s.* 4*d.* (+).

$$
\begin{array}{r}
20 \\
.40 \\
12 \\
\hline
4.80 \\
\hline
\end{array}
$$

$$\text{Here } £.02 = £1 \times \frac{2}{100} = 20s. \times \frac{2}{100}$$

$$= \frac{40}{100} s. = .40s.\,;$$

$$.40s. = 12d. \times \frac{40}{100} = \frac{480}{100} d. = 4.80d.$$

Ex. 99.

(1.) What will be the cost of 175 feet long of partitioning, 10 feet high, at 30*s.* per square?

(2.) If a house measures within the walls 25 feet long by 16 feet broad, with a true-pitched roof, what will the roofing cost, at 18*s.* a square?

(3.) A roof was 60 feet long and 40 feet wide. What did it cost, at 9*s.* 6*d.* a square?

(4.) What will the tiling of a barn come to, at 7*s.* 6*d.* a square, the flat being 30 feet by 18 feet, and the roof true-pitched?

(5.) What will it cost to slate a roof, 30 feet long and 22 feet broad, at £1 13*s.* 4*d.* per square?

(6.) Calculate the cost, at 4 guineas a square, of laying the first floor of a house, which measured 21 ft. by 18 ft., allowing, for each of four fire-places, a space of 3 ft. by 2 ft. 6 in., and, for the well-hole of the staircase, a space of 9 ft. square.

96. *Bricklayers' Work* is charged by the *square rod*, the work being always reckoned at what is called the *standard measure* of $1\frac{1}{2}$ brick, or three half-bricks thick.

In order to estimate the value of such work, we must first find the number of square feet in it. Then, if the work be not of standard thickness, we must find how many feet of standard thickness are equivalent to the number of feet just found. This may be done by multiplying this number by the number of half-bricks in the thickness of the given work, and dividing the result by 3. Having found the number of square feet, standard measure, we must now reduce it to square rods, which is done by dividing by 272.

N.B. The divisor should, properly, be $272\frac{1}{4}$; for a square rod $=30\frac{1}{4}$ square yards $=30\frac{1}{4}\times 9$ square feet $=272\frac{1}{4}$; but the $\frac{1}{4}$ is usually given on the side of the workman.

It is common to reckon 68 sq. ft. = 1 quarter (rod), and so to divide first by 68, and then by 4, expressing the result in rods, qrs., and feet.

Ex. What is the charge for 330 sq. feet of brickwork, $2\frac{1}{2}$ bricks thick, at 50*s.* a rod?

Here 330 sq. ft. with a thickness of $2\frac{1}{2}$ bricks
= 330 „ „ „ 5 half-bricks
$=5\times 330$ „ „ „ 1 half-brick
$=\frac{5}{3}\times 330$ „ „ „ 3 half-bricks
= 550 sq. feet, standard measure.

sq. ft.
68) 550 (8 qrs. = 2 rods (+)
544
6

Ans. 50*s.* × 2 = £5.

Ex. 100.

(1.) Reduce 1368 square feet of brickwork, 2 bricks thick, to standard measure.

(2.) Reduce 2742 square yards, $2\frac{1}{2}$ bricks thick, to standard measure.

(3.) Reduce 3016 sq. rods, 3 bricks thick, to standard measure.

(4.) Reduce 317 sq. yds. 6 ft., $3\frac{1}{2}$ bricks thick, to standard measure.

(5.) How many rods of brickwork are there in a wall 100 yards long and 7 feet high, standard work?

(6.) How many rods are there in a wall, 56 yds. long and 6 feet high, $2\frac{1}{2}$ bricks thick?

(7.) A square garden, which is 200 yards round, has a wall 2 bricks thick and 8 ft. high. How many rods of standard brickwork are there in it?

(8.) I want to enclose a paddock with a 10-feet wall, $2\frac{1}{2}$ bricks thick, all but one side, which has iron palings, and is only half built up with brickwork. The sides are each 120 ft. long; but in one of them is a doorway, 7 ft. by 3 ft. What will the whole come to, at 35*s*. per rod, standard measure?

(9.) The cellars of a house contained 8 rods 2 qrs. 32 ft., 2 bricks thick. Find the cost, at 30*s*. per rod, standard measure.

(10.) In a violent gale, 25 yards of my garden-wall fell down, and I resolved to build it strongly, $2\frac{1}{2}$ bricks thick, and 7 feet high. What will it cost me, at 30*s*. a rod?

(11.) How many rods of standard brickwork are there in the gable end of a house, which is 40 feet long, and 28 feet above the ground to the bend of the roof, the foundation being 2 feet below, and the gable being 15 feet in height; the work being throughout $2\frac{1}{2}$ bricks thick?

(12.) Find the amount of brickwork in a school-room 50 feet long and 20 feet wide, the walls being 13 feet to the bend of the roof, the foundations a foot below the ground, and the height of the gable being 11 feet; the whole being $1\frac{1}{2}$ brick thick, and allowance being made for a door, 7 feet by 3 feet, and for four windows, each 4 feet by $2\frac{1}{2}$ feet. What also will be the cost, at 30*s*. a rod?

PART III.

97. One number is said to be a *measure* or a *factor* of another, when it divides it exactly without remainder.

Thus 1, 2, 3, 4, 6, 12, are all measures or factors of 12.

Unity, however, is not generally reckoned among the factors of a number.

98. Any number, which divides without remainder *each* of two or more numbers, is said to be a *common measure* or *common factor* of them; and, of course, the greatest number, which so divides them, is their *Greatest Common Measure* (G.C.M.).

99. Sometimes it is easy to see, by merely looking at them, what is the Greatest Common Measure of two numbers.

Thus it is plain that 6 is the G. C. M. of 12 and 18.

In other cases the G. C. M. of two given numbers is not evident at first sight.

Thus it would hardly be perceived that 113 will divide both 2599 and 4181.

The following method will enable us always to find the G. C. M. of two given numbers.

100. *To find the Greatest Common Measure of any two numbers.*

RULE. Divide the greater by the less, and the preceding divisor by the remainder, and so on, continually, until there is no remainder; then the *last divisor* will be the G. C. M. required.

Ex. Find the G.C.M. of 259 and 444.

```
259) 444 (1
     259
     185) 259 (1
          185
           74) 185 (2
               148
                37) 74 (2
                    74
```

Ans. 37.

The reason of the above process may be seen as follows:

Whatever number divides *both* 259 and 444, will also divide 444 − 259, or 185 : (thus, 37 goes 12 times in 444, and 7 times in 259, and, therefore, it goes (12−7) times, or 5 times, in 444−259, or 185 :) hence we see that whatever number divides *both* 259 and 444 will also divide *both* 185 and 259, and, therefore, the *Greatest* Common Divisor of 259 and 444 will be also *a Common Divisor* of 185 and 259.

Again, whatever number divides *both* 185 and 259 will divide also 259+185, or 444 : (for instance, 37 goes 7 times in 259 and 5 times in 185, and, therefore, it goes (7+5) times, or 12 times, in 259+185, or 444 :) hence we see that whatever number divides *both* 185 and 259 will also divide *both* 259 and 444, and, therefore, the *Greatest* Common Divisor of 185 and 259 will be also *a Common Divisor* of 259 and 444.

From the first of the above reasonings it appears that *all* the Common Divisors of 259 and 444 will be also Common Divisors of 185 and 259 ; and from the second it appears that all the Common Divisors of 185 and 259 will be also Common Divisors of 259 and 444. Putting these two conclusions together, it is plain that the Common Divisors of the one pair of numbers will be *exactly the same* as those of the other pair.

In like manner, it will follow that the Common Divisors of 185 and 259 will be the same as those of 74 and 185, and the Common Divisors of 74 and 185 will be the same as those of 37 and 74.

Hence it appears that all the Common Divisors of the first pair of numbers, 259 and 444, will be exactly the same as those of the last pair of numbers, 37 and 74 ; and, consequently, the *Greatest* Common Divisor of the last pair will be the same as that of the first pair.

But, if 37 divides 74 exactly, 37 itself is the Greatest Common Divisor of 37 and 74, and 37 is, therefore, also the Greatest Common Divisor (or G. C. M.) of 259 and 444.

Ex. 2. Find the G. C. M. of 279 and 4185, and of 17 and 36.

```
279) 4185 (15                 17) 36 (2
     279                          34
     ----                         --
     1395                          2) 17 (8
     1395                             16
     ----                             --
Ans. 279.                              1) 2 (2
                                          2
                                          -
```

Ans. 1 : that is, the numbers, 17 and 36, have no common divisor but unity.

Ex. 101.

Find the G. C. M. of

(1.) 350 and 1015.
(2.) 192 and 5120.
(3.) 112 and 1512.
(4.) 260 and 1365.
(5.) 2121 and 2626.
(6.) 1430 and 1859.
(7.) 2157 and 3741.
(8.) 1001 and 1056.
(9.) 931 and 532.
(10.) 765 and 2100.
(11.) 1008 and 2464.
(12.) 4941 and 1485.

---

101. One number is said to *contain*, or to be a *multiple of*, another, when it can be divided by it, without remainder.

Thus 12 contains each of 1, 2, 3, 4, 6, 12 ; and any number is a multiple of each of its measures.

102. A *Common Multiple* of two or more numbers is one which contains each of them ; and, of course, the *least* such number is their *Least Common Multiple* (L. C. M.).

Thus 12, 24, 36, 48, &c., are all common multiples of 2, 3, 4, 6, and 12; but 12 is their Least Common Multiple.

Of course, a common multiple of any given numbers may be formed by multiplying them all together, and any multiple of the number thus formed will also be a common multiple of the given numbers.

Thus a common multiple of 4, 6, and 9, is 4 × 6 × 9 or 216; and 2 × 216, 3 × 216, &c., or 432, 648, &c., are also common multiples of 4, 6, 9.

But, in practical arithmetic, we usually require the Least Common Multiple, especially in preparing Fractions for Addition and Subtraction.

103. *To find the Least Common Multiple of any given numbers.*

The method of doing this will be best understood by attending to the explanation of the following example.

Ex. 1. Find the L. C. M. of 24, 20, 16, 6, 4, 8, 10, 30, 12, 25.

$$\begin{array}{l} 12)\underline{24 . 20 . 16 . \cancel{6} . \cancel{4} . \cancel{8} . \cancel{10} . 30 . \cancel{12} . 25} \\ \qquad \cancel{2} . \cancel{5} . 4 . \qquad\qquad\qquad \cancel{5} . \quad 25 \end{array}$$

Ans. 12 × 4 × 25 = 1200.

We here want to find a number, that shall be divisible without remainder by 24, 20, 16, 6, 4, 8, 10, 30, 12, and 25. Now, if we find a number that is divisible by 24, it will, of course, be divisible by 6, 4, 8, and 12, which are themselves divisors of 24. We may, therefore, begin by striking out 6, 4, 8, and 12; and, for a similar reason, we may strike out 10, which is contained in 20: and the question is thus simplified, and reduced to finding the L. C. M. of 24, 20, 16, 30, and 25.

Now we take as a divisor some number, which is contained in one, at least, of these five remaining numbers, and which will, therefore, be a factor of the Least Common Multiple required. We have here taken 12, which is contained in 24, and will, consequently, be one of the factors of the L. C. M., which we are going to form: we set down 12 then, in the Answer, as the first factor of the L. C. M. Then, besides 12, what other factors shall we want, to form this L. C. M.—that is, to form a number, which shall contain each of the five numbers, 24, 20, 16, 30, 25, without remainder?

To ascertain this, we proceed to consider what factor besides 12, or one of the factors in 12, will be wanted to make up each one of these numbers in order. Thus, to make up 24, we shall want a factor 2, because 12 × 2 = 24; to make up 20, we shall want a factor 5, because there is a factor 4 in 12, and 4 × 5 = 20; to make up 16, we shall want a factor 4, because there is a 4 in 12, and 4 × 4 = 16; to make up 30 we shall want a factor 5, because there is a factor 6 in 12, and 6 × 5 = 30; and, lastly, to make 25, we shall want a factor 25, because there is no factor in 12 that will help towards making 25. In order, therefore, that the number formed may contain each of 24, 20, 16, 30, and 25, it must contain, besides the factor 12, factors

2, 5, 4, 5, and 25—

that is, the numbers given in the second line.

But to these numbers the same reasoning will apply. If we form a number, containing 4 as a factor, it is sure to contain 2, and we may, therefore, strike out the 2; and for a similar reason, we may strike out the 5 and 5, each of which is contained in 25. We have now left only the factors 4 and 25, and these have no factor in common. Since, therefore, the common multiple required must contain each of these, it must contain both of them, and it will, consequently, be formed by multiplying together $12 \times 4 \times 25 = 1200$.

N.B. Whenever 12, as in the above Example, *can* be taken as a divisor, it is a very good one to be chosen, because it is likely to assist so much (by its containing so many factors, 2, 3, 4, 6, 12) in the formation of the other numbers—reducing thus very much the labour of the operation. Now 12 can always be taken as a divisor, when it is seen that it must be a factor of the L. C. M.: that is, whenever either 12 itself occurs among the given numbers, whose L. C. M. we are to find, or when, as in the last Example, it divides one of them, or when, as will frequently happen, the factors, 3 and 4, which make up 12, are found as factors, 3 in one, and 4 in another, of the given numbers.

Ex. 2. Find the L. C. M. of 15, 16, 18, 20. Here 3 divides 15, and 4 divides 16; therefore, 3 and 4 must be factors of the L. C. M., that is, 12 must be a factor of it, and we may take 12 as divisor.

$$\begin{array}{r|l} 12 & 15\ .\ 16\ .\ 18\ .\ 20 \\ \hline & \cancel{5}\ .\ 4\ .\ 3\ .\ 5 \\ \hline \end{array}$$

Ans. $12 \times 4 \times 3 \times 5 = 720$.

Ex. 3. Find the L. C. M. of 16, 40, 50, 25, 8, 64.

$$\begin{array}{r|l} 4 & \cancel{16}\ .\ 40\ .\ 50\ .\ \cancel{25}\ .\ \cancel{8}\ .\ 64 \\ \hline 5 & 10\ .\ 25\ .\ \qquad 16 \\ \hline & \cancel{2}\ .\ 5\ .\ \qquad 16 \\ \hline \end{array}$$

Ans. $4 \times 5 \times 5 \times 16 = 1600$.

Ex. 102.

Find the L. C. M. of

(1.) 12, 16, 18.

(2.) 18, 24, 30.

(3.) 9, 3, 18, 27.

(4.) 2, 4, 10, 15.

(5.) 10, 14, 15, 21.

(6.) 8, 12, 20, 25.

(7.) 6, 9, 15, 18, 20.

(8.) 12, 18, 27, 30, 40.

(9.) 15, 40, 45, 60, 90.

(10.) 16, 42, 63, 14, 72.

(11.) 8, 10, 12, 25, 30, 36.

(12.) 15, 20, 25, 30, 40, 50.

Ex. 103.

(1.) The English divide the circumference of a circle into 360 *degrees*, the French into 400 *grades*. Find all the measures of each of these two numbers.

(2.) What is the greatest number that will divide both 360 and 400? and what is the least number which they will both divide without remainder?

(3.) Find the greatest number that is contained in each of the numbers 228, 304, and 342. Find also the least number which contains all three of them.

(4.) The number 12 will divide both 1428 and 2652. What larger number or numbers will divide each of them?

(5.) The number 10829 is divisible without remainder by each of 91 and 119. Will any smaller number be divisible by each of them?

(6.) Find the L. C. M. of the first ten *even* numbers.

(7.) What is the least sum of money which contains an exact number both of pounds and guineas? of florins and half-crowns?

(8.) I have before me a certain number of shillings, and I can arrange them either in four equal heaps, or in five, or in six, or in nine. What is the least number of shillings that will allow of my doing this?

(9.) Taking the number of shillings in (8) as the number of shillings on the table, in how many different ways might I arrange them in equal heaps?

(10.) In a village-school there are 36 boys and 42 girls. What is the smallest number of nuts that can be divided equally, without leaving any over, either among the boys or the girls?

(11.) The fore-wheel of a carriage is eight feet round, the hind-wheel is twelve feet. I observe that two particular spokes, one in each wheel, are pointing vertically upwards. How far will the carriage go, before the same two spokes will be again at the same moment in the same direction? and how often will this happen in going 1200 yards?

(12.) A market-woman sat counting her eggs; and she found that she could either sell them at two a penny, or at five for twopence, or nine for threepence, or twenty for sixpence, and have no remainder. Now the whole number of eggs was less than 200. How many were they?

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104. We have already (**32–35**) had occasion to touch upon fractions, and shall here proceed to notice them more at length.

A *Fraction* is a quantity, which represents any part or parts of an integer or whole.

A *Vulgar* (or Common) Fraction, in its simplest form, is expressed by means of two numbers, placed one over the other with a line between them.

The lower of these is called the *Denominator*, because it shows into how many equal parts the whole is divided, and, therefore, describes or *denominates* the nature of each of those parts. The upper is called the *Numerator*, because it shows how many of those parts are taken to form the fraction, and therefore, *numerates*, that is, numbers or counts them.

Thus $\frac{3}{4}$ denotes that the whole is divided into four equal parts, and that three of them are taken to form the fraction.

105. A *proper* fraction is one whose numerator is less than the denominator, in which case the fraction is really a *broken part* of the whole in question, being *less* than the whole: as $\frac{3}{8}$, $\frac{5}{7}$.

An *improper* fraction is one whose numerator is either *equal to* or *greater than* the denominator, in which case the fraction is either equal to the whole or greater than it, and is, therefore, *improperly* called a 'fraction' of it: as $\frac{8}{8}$, $\frac{11}{7}$.

A *mixed* number is one made up of a whole number and a fraction: as $2\frac{2}{3}$, $5\frac{3}{4}$.

A *compound* fraction is a fraction of a fraction: as $\frac{2}{3}$ of $\frac{3}{4}$, $2\frac{1}{2}$ of $\frac{3}{7}$ of $3\frac{1}{4}$.

A *complex* fraction is one in which either the numerator or denominator, or each of them, is a fraction: as $\frac{3\frac{1}{2}}{2}$, $\frac{2}{4\frac{2}{3}}$, $\frac{1\frac{1}{5}}{3\frac{3}{4}}$, $\frac{\frac{2}{3}\text{ of }5}{2\frac{1}{2}}$.

Every whole number may be considered as a fraction whose denominator is 1: thus 6 is $\frac{6}{1}$.

106. It must be particularly observed, as mentioned and explained in (**34**), that a fraction, such as $\frac{3}{4}$, may be regarded as expressing whichever we please of two things, namely,

(i.) *three-fourths* of *one unit*,
or (ii.) *one-fourth* of *three units*.

For instance, $\frac{3}{4}$ of £1, which is 15*s*., is the same as $\frac{1}{4}$ of £3, which is also 15*s*.

The reason of this is (see **34**) because it is the same thing whether we divide the *one* unit into four equal parts, and take *three* of these parts, or whether we divide the *three* units, each into four equal parts, and take *one* part out of each set.

107. *To express a whole number as a fraction with a given denominator.*

Rule. Multiply the number by the given denominator, and the result will be the numerator of the fraction required.

Ex. Express 5 as a fraction with denominator 6.

Ans. $5=\frac{5\times 6}{6}=\frac{30}{6}$.

For since 1 contains 6 sixth parts, therefore, 5 contains 5×6 or 30 sixth parts; that is, $5=\frac{30}{6}$.

108. *To reduce a mixed number to an improper fraction.*

Rule. Multiply the *whole number* of the mixed number by the denominator of the fractional part; add the result to the numerator of the fractional part for a new numerator, and retain the same denominator.

Ex. 1. $7\frac{2}{3}=\frac{23}{3}$. Ex. 2. $1\frac{1}{8}=\frac{9}{8}$. Ex. 3. $5\frac{4}{9}=\frac{49}{9}$.

For, in Ex. 1, $7=\frac{21}{3}$ by **107**, and, therefore, $7\frac{2}{3}=\frac{21}{3}+\frac{2}{3}=\frac{23}{3}$.

109. *To reduce an improper fraction to a whole or mixed number.*

RULE. Divide the numerator by the denominator: the quotient will be a whole number, and the remainder (if any) the numerator of the fractional part of the mixed number required.

Ex. 1. $\frac{45}{9}=5$. Ex. 2. $\frac{37}{37}=1$. Ex. 3. $\frac{116}{15}=7\frac{11}{15}$.

N.B. All improper fractions which occur in any answer should (except the contrary be desired) be expressed as whole or mixed numbers.

Ex. 104.

Express

(1.) 7 with denominators 3 and 5.
(2.) 9 with denominators 9 and 7.
(3.) 11 with denominators 11 and 13.
(4.) 12 with denominators 13 and 15.

Reduce to improper fractions

(5.) $2\frac{2}{3}$, $3\frac{3}{4}$, $4\frac{4}{5}$. (6.) $11\frac{1}{3}$, $13\frac{2}{7}$, $15\frac{3}{8}$.
(7.) $20\frac{10}{13}$, $31\frac{9}{11}$, $43\frac{8}{15}$. (8.) $101\frac{2}{11}$, $129\frac{5}{8}$, $143\frac{11}{12}$.

Reduce to whole or mixed numbers

(9.) $\frac{35}{5}$, $\frac{39}{5}$, $\frac{39}{7}$. (10.) $\frac{123}{8}$, $\frac{120}{15}$, $\frac{213}{13}$.
(11.) $\frac{1331}{16}$, $\frac{1000}{25}$, $\frac{1000}{27}$. (12.) $\frac{362}{17}$, $\frac{361}{19}$, $\frac{360}{23}$.

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110. If we *multiply* or *divide* the *numerator* of a fraction by any whole number, the fraction itself will be *multiplied* or *divided* by it.

Ex. 1. $\frac{2}{15}\times 7=\frac{14}{15}$. Ex. 2. $\frac{12}{5}\div 6=\frac{2}{5}$.

For (Ex. 1) *seven times two* fifteenths is *fourteen* fifteenths, and (Ex. 2) *twelve* fifths divided by *six* is *two* fifths.

111. If we *multiply* the *denominator* of a fraction by any whole number, the fraction itself will be *divided* by it; and if we *divide* the *denominator* by any number, the fraction itself will be *multiplied* by it.

Ex. 1. $\frac{2}{15}\times 3=\frac{2}{5}$. Ex. 2. $\frac{12}{5}\div 5=\frac{12}{25}$.

For (Ex. 1) since, in $\frac{2}{15}$, the unit is divided into *fifteen* equal parts, and, in $\frac{2}{5}$, it is divided into only *five* equal parts, it follows that each of the parts in the latter case will be three times as big as each in the former. But the *numerator* is the same in both fractions, that is, the same number of parts is to be taken in each case. Hence the latter fraction, $\frac{2}{5}$, will be three times as big as the former, $\frac{2}{15}$; or $\frac{2}{15}\times 3=\frac{2}{5}$.

So again (Ex. 2) each of the parts in $\frac{12}{5}$ is five times as big as each in $\frac{12}{25}$, because in the former fraction the unit is divided into *five* equal parts, and, in the latter, into *twenty-five*. But the same number of parts is taken in each fraction; and, consequently, $\frac{12}{5}$ is five times as big as $\frac{12}{25}$, that is, $\frac{12}{5} \div 5 = \frac{12}{25}$.

112. From the last two articles we obtain at once the following Rules:

(i.) To *multiply* a fraction by a whole number, either *multiply* the *numerator*, or *divide* the *denominator* by it.

(ii.) To *divide* a fraction by a whole number, either *divide* the *numerator*, or *multiply* the *denominator* by it.

N.B. It is always best, when possible, to *divide* the denominator for multiplication, and to *divide* the numerator for division, because then the resulting fraction will be most simple.

Ex. 1. $\frac{15}{28} \times 7 = \frac{105}{28}$; but $\frac{15}{28} \times 7 =$ (also) $\frac{15}{4}$, which is more simple.
Ex. 2. $\frac{15}{28} \div 5 = \frac{15}{140}$; but $\frac{15}{28} \div 5 =$ (also) $\frac{3}{28}$, " "

Ex. 105.

(1.) Multiply $\frac{45}{56}$ by 3, 4, 7, and 8.
(2.) Divide $\frac{45}{56}$ by 2, 3, 5, and 9.
(3.) Multiply $\frac{108}{125}$ by 5, 9, 12, and 25.
(4.) Divide $\frac{108}{125}$ by 5, 9, 12, and 25.
(5.) Multiply $\frac{100}{231}$ by 2, 3, 4, 5, 7, and 11.
(6.) Divide $\frac{100}{231}$ by 2, 3, 4, 5, 7, and 11.

---

113. If the numerator and denominator of a fraction be *both* multiplied, or *both* divided by the same number, the value of the fraction will not be altered.

Ex. 1. $\frac{5}{7} = \frac{10}{14} = \frac{15}{21} = \frac{20}{28} =$ &c. Ex. 2. $\frac{36}{48} = \frac{18}{24} = \frac{12}{16} = \frac{9}{12} = \frac{6}{8} = \frac{3}{4}$.

For it is the same thing whether we divide the unit into 7 equal parts and take 5 of them, or into twice as many equal parts, and take twice as many of them, &c.

114. *To reduce a fraction to lower terms.*

RULE. Divide both the numerator and denominator by any common factors they may contain.

Ex. 1. $^{5})\frac{270}{375} = {}^{3})\frac{54}{75} = \frac{18}{25}$. Ex. 2. $^{9})\frac{315}{378} = {}^{7})\frac{35}{42} = \frac{5}{6}$.

By **111**, the value of the fraction is not altered by this process.

When a fraction is reduced as much as possible by such division, it is said to be in its *lowest terms*; and all fractions that occur in any sum should (except when the contrary is expressed) be reduced to their lowest terms.

115. Sometimes it is easy to see at once what factors will divide both the numerator and denominator of a fraction, so as to bring it to lower terms; and for this purpose it will be useful to notice the following facts.

Any number is exactly divisible :—

(i.) By 5, if it ends in 5 or 0;

(ii.) By 2, 4, or 8, if the number formed by its last *one*, *two*, or *three* figures, respectively, is divisible by 2, 4, or 8;

(iii.) By 3 or 9, if the sum of its figures is divisible by 3 or 9;

(iv.) By 11, if the sum of its 1st, 3rd, 5th, &c., figures, when divided by 11, leaves the same remainder as the sum of its 2nd, 4th, 6th, &c., figures, when divided also by 11.

Thus 75 and 30 are each divisible by 5, since they end in 5 and 0, respectively;

1234 is div. by 2, because its last figure 4 is div. by 2; 45756 is div. by 4, because the number formed by its last two figures, 56, is div. by 4;

371528 is div. by 8, because the number formed by its last three figures, 528, is div. by 8;

6402041514 is divisible by 3 or by 9, because the sum of its figures is 27, which is div. by 3 or by 9;

And the same number is also div. by 11; because 6+0+0+1+1=8, which, when div. by 11, leaves a remainder 8, and 4+2+4+5+4=19, which, when div. by 11, leaves a remainder 8, also.

Ex. $^{11})\frac{11385}{16335}=^{9})\frac{1035}{1485}=^{5})\frac{115}{165}=\frac{23}{33}$.

Ex. 106.

Reduce to their lowest terms

(1.) $\frac{162}{360}$. (2.) $\frac{180}{216}$. (3.) $\frac{108}{132}$. (4.) $\frac{396}{1485}$.
(5.) $\frac{225}{405}$. (6.) $\frac{726}{1089}$. (7.) $\frac{2277}{3465}$. (8.) $\frac{715}{935}$.
(9.) $\frac{440}{1012}$. (10.) $\frac{1215}{2268}$. (11.) $\frac{1216}{1856}$. (12.) $\frac{2232}{2952}$.

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116. When no common factor can be seen, which will divide both numerator and denominator, the rule for finding their G. C. M. will always enable us to find such a factor, if there is any to be found.

Ex. $179)\frac{4117}{9487}=\frac{23}{53}$.

```
4117) 9487 (2
      8234
      ----
      1253) 4117 (3
            3759
            ----
             358) 1253 (3
                  1074
                  ----
                   179) 358 (2
                        358
                        ---
```

Ex. 107.

Reduce to their lowest terms

(1.) $\frac{259}{407}$. (2.) $\frac{689}{901}$. (3.) $\frac{703}{851}$. (4.) $\frac{529}{598}$. (5.) $\frac{806}{899}$. (6.) $\frac{1769}{1891}$.

Ex. 108.

(1.) In seventeen whole apples, how many sevenths of an apple are there? How many seventeenths? How many seventy-sevenths?

(2.) If I cut up a cake into six equal parts, and cut one of these into four equal parts, and give three of these parts to a boy, what fraction will he have of the whole cake? What fraction of it will now be left?

(3.) In $5\frac{1}{5}$ oranges, how many fifths of an orange? How many fifteenths?

(4.) How many tenths of a pound are there in £$8\frac{9}{10}$? How many twelfths of a penny in $10\frac{11}{12}$*d.*?

(5.) How many whole cakes would be wanted for a hundred and eleven children, if each had an eleventh of a cake? How many would be wanted for them, if each had a third of a cake?

(6.) How many whole oranges are there in $\frac{1001}{11}$ of an orange? How many in $\frac{1010}{111}$ of an orange?

(7.) A copy-book contains 48 pages. When a boy has written 20 pages of it, what fraction of it has he written? when he has written 32 pages of it, what fraction of it remains to be written?

(8.) In a regiment of soldiers there are 75 officers, 175 serjeants and corporals, and 1400 privates. What fraction of the whole regiment do the officers form, and what the serjeants and corporals?

(9.) There are 133 boys and girls in a school. The boys are 57 in number. What fraction do they form of the whole? What the girls?

(10.) Of the above children, 38 were absent, and an equal number of boys and girls. What part of those in actual attendance were boys, and what part girls?

(11.) There are 288 pages in a book, and I have just finished reading 180 pages of it. What portion have I read, and what remains to be read?

(12.) I had 117 nuts in my pocket, and I gave away 26 to *A*, 52 to *B*, and the rest to *C*. What portion of the whole did each of them get?

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117. *To multiply or divide a mixed number by a whole number.*

Rule. Express the mixed number as an improper fraction, and then multiply or divide as in (**112**).

Ex. 1. $6\frac{7}{8}\times 4=\frac{55}{8}\times 4=\frac{55}{2}=27\frac{1}{2}$.
Ex. 2. $6\frac{7}{8}\div 5=\frac{55}{8}\div 5=\frac{11}{8}=1\frac{3}{8}$.

Or such examples may be worked as follows:

Ex. 1. $6\frac{7}{8}\times 4=24+\frac{7}{2}=24+3\frac{1}{2}=27\frac{1}{2}$.
Ex. 2. $6\frac{7}{8}\div 5=1+1\frac{7}{8}\div 5=1+\frac{15}{8}\div 5=1+\frac{3}{8}=1\frac{3}{8}$.

Ex. 109.

(1.) $1\frac{2}{3}\times 2$. (2.) $2\frac{3}{4}\times 3$. (3.) $3\frac{4}{5}\times 4$. (4.) $4\frac{5}{6}\times 5$.
(5.) $1\frac{2}{3}\div 2$. (6.) $2\frac{3}{4}\div 3$. (7.) $3\frac{4}{5}\div 4$. (8.) $4\frac{5}{6}\div 5$.
(9.) How much is twenty times $\frac{5}{6}$? and thirty-nine times $1\frac{1}{5}$?
(10.) What is the seventh part of $11\frac{9}{10}$? and the tenth part of $9\frac{10}{11}$?
(11.) Take $9\frac{7}{9}$ nineteen times, and divide the result into eleven equal parts.
(12.) Find nine times the eleventh part of $10\frac{1}{12}$.

118.

	£	s.	d.
Ex.	23	13	$9\frac{7}{8}\times 35$
			5
	118	9	$1\frac{3}{8}$
			7
	829	3	$9\frac{5}{8}$

Here $5\times\frac{7}{8}=\frac{35}{8}=4\frac{3}{8}$; we set down $\frac{3}{8}$*d.*, and carry 4*d.*: so also $7\times\frac{3}{8}=\frac{21}{8}=2\frac{5}{8}$; we set down $\frac{5}{8}$*d.*, and carry 2*d.*

Ex. 110.

	£	s.	d.			£	s.	d.	
(1.)	1	2	$3\frac{5}{8}$	× 5.	(2.)	2	17	$3\frac{3}{5}$	× 8.
(3.)	3	0	$9\frac{1}{3}$	× 11.	(4.)		17	$2\frac{5}{6}$	× 14.
(5.)	5	11	$0\frac{1}{7}$	× 28.	(6.)	4	4	$4\frac{3}{10}$	× 30.
(7.)	3	13	$5\frac{4}{7}$	× 31.	(8.)	7	9	$7\frac{3}{8}$	× 43.
(9.)	6	16	$5\frac{5}{6}$	× 85.	(10.)	2	11	$11\frac{2}{5}$	× 102.
(11.)	3	13	$3\frac{5}{9}$	× 108.	(12.)		15	$5\frac{8}{15}$	× 135.

119.

		£	s.	d.
Ex. 1.	7)	37	14	8 ÷ 28
	4)	5	7	$9\frac{5}{7}$
		1	6	$11\frac{3}{7}$

Here, in the first division, there is 5*d.* over, to be divided by 7, which we set down as $\frac{5}{7}$*d.*, since (**106**) $\frac{1}{7}$ of 5*d.* $=\frac{5}{7}$ of 1*d.*

We might have brought these 5*d.* to 20*f.*, and then, dividing by 7, we should have had $2\frac{6}{7}$*f.*; but, as a farthing itself is only a *fraction* of a penny, it is usual, whenever the result does not come out a clear number of farthings, to express the whole below the pence as a fraction of a penny.

In the second division, there is $1\frac{5}{7}d.$ over, or $\frac{12}{7}d.$, which, divided by 4, gives $\frac{3}{7}d.$

Ex. 2.

$$\begin{array}{rrrl} & £ & s. & d. \\ 6) & 175 & 19 & 7\frac{1}{2} \div 48 \\ \hline 8) & 29 & 6 & 7\frac{1}{4} \\ \hline & 3 & 13 & 3\frac{29}{32} \\ \hline \end{array}$$

Here, in the first division, there is $1\frac{1}{2}d.$ over, $=\frac{3}{2}d.$, to be divided by 6, which gives $\frac{3}{12}d.=\frac{1}{4}d.$

In the second division, there is $7\frac{1}{4}d.$ over, $=\frac{29}{4}d.$, to be divided by 8, which gives $\frac{29}{32}d.$

The last example may also be worked by Long Division.

Ex. 3.

$$\begin{array}{rl} & \;\;£ \;\;\; s. \;\;\; d. \;\;\; £ \;\;\; s. \;\;\; d. \\ 48) & 175 \;\; 19 \;\; 7\frac{1}{2} \;\; (3 \;\; 13 \;\; 3\frac{29}{32} \\ & 144 \\ \hline & 31 \\ & \;\;20 \\ \hline & 639 \;(13s. \\ & 48 \\ \hline & 159 \\ & 144 \\ \hline & \;\;15 \\ & \;\;12 \\ \hline & 187 \;(3d. \\ & 144 \\ \hline & 43\frac{1}{2} \\ \hline \end{array}$$

Here there is $43\frac{1}{2}d.$ over. $=\frac{87}{2}d.$, to be divided by 48, which gives $\frac{87}{96}d.=\frac{29}{32}d.$, as before.

Ex. 4. Divide £251 18s. $6\frac{1}{2}d.$ by 1300.

$$\begin{array}{rl} & \;\;£ \;\;\; s. \;\;\; d. \;\;\; £ \;\;\; s. \;\;\; d. \\ 13) & 251 \;\; 18 \;\; 6\frac{1}{2} \;\; (19 \;\; 7 \;\; 6\frac{25}{26}. \\ & 13 \\ \hline & 121 \\ & 117 \\ \hline & \;\;\;4 \\ & \;\;20 \\ \hline & \;\;98 \;(7s. \\ & \;\;91 \\ \hline & \;\;\;\;7 \\ & \;\;\;12 \\ \hline & \;\;\;90 \;(6d. \\ & \;\;\;78 \\ \hline & \;\;\;12\frac{1}{2} \\ \hline \end{array}$$

Here $12\frac{1}{2}d.=\frac{25}{2}d.$, and $\frac{25}{2}d.\div 13=\frac{25}{26}d.$

$$\begin{array}{rl} & £ \;\;\; s. \;\;\; d. \\ 1{,}00) & 19 \;\;\; 7 \;\;\; 6\frac{25}{26} \\ & \;\;\;20 \\ \hline & 3{,}87 \\ & \;\;\;\;12 \\ \hline & 10{,}50\frac{25}{26} \\ \hline \end{array}$$

Here $50\frac{25}{26}d.=\frac{1325}{26}d.$ and $\frac{1325}{26}d.\div 100=\frac{1325}{2600}d.$ $=\frac{53}{104}.$

Ans. 3s. $10\frac{53}{104}d.$

Ex. 111.

	£	s.	d.		£	s.	d.
(1.)	4	7	$6\frac{1}{2} \div 5.$	(2.)	3	13	$4\frac{1}{3} \div 6.$
(3.)	7	10	$2\frac{1}{4} \div 8.$	(4.)	21	13	$2\frac{1}{2} \div 15.$
(5.)	37	11	$6\frac{3}{4} \div 24.$	(6.)	73	9	$9\frac{3}{4} \div 60.$
(7.)	23	11	$2\frac{3}{8} \div 100.$	(8.)	37	0	$7\frac{1}{2} \div 108.$
(9.)	43	16	$6\frac{1}{2} \div 110.$	(10.)	17	13	$1\frac{1}{2} \div 132.$
(11.)	35	2	$4\frac{1}{2} \div 144.$	(12.)	42	8	$7\frac{1}{2} \div 150.$

120. *To reduce a compound fraction to a simple one.*

RULE. Multiply together all the numerators for a new numerator, and all the denominators for a new denominator.

Ex. 1. $\frac{2}{3}$ of $\frac{4}{7}=\frac{8}{21}$. Ex. 2. $\frac{4}{3}$ of $\frac{2}{7}=\frac{8}{21}$.

For $\frac{1}{3}$ of $\frac{4}{7}$ is $\frac{4}{21}$ (**112**); therefore, $\frac{2}{3}$ of $\frac{4}{7}$, which must be twice as great, is $\frac{8}{21}$.

Ex. 3. $\frac{4}{3}$ of $5=\frac{4}{3}$ of $\frac{5}{1}=\frac{20}{3}=6\frac{2}{3}$.

Ex. 4. $2\frac{2}{3}$ of 3 of $4\frac{1}{2}=\frac{8}{3}$ of $\frac{3}{1}$ of $\frac{9}{2}=\frac{216}{6}=36$.

In such examples as the last, the work may be simplified by striking out any factors which are common to any one of the numerators, and also any one of the denominators.

Thus $2\frac{2}{3}$ of 3 of $4\frac{1}{2}=\frac{\overset{4}{\cancel{8}}}{\cancel{3}}$ of $\frac{\cancel{3}}{1}$ of $\frac{9}{\cancel{2}}=36$, as before.

Ex. 112.

(1.) $\frac{1}{4}$ of $\frac{5}{12}$ of 8. (2.) $\frac{2}{5}$ of $\frac{5}{7}$ of 14. (3.) $\frac{1}{3}$ of $\frac{2}{5}$ of $2\frac{1}{4}$.
(4.) $\frac{7}{8}$ of $\frac{1}{3}$ of $\frac{4}{5}$ of 30. (5.) $\frac{13}{14}$ of $\frac{9}{26}$ of $1\frac{5}{9}$.
(6.) $\frac{2}{3}$ of $1\frac{1}{4}$ of $1\frac{2}{5}$. (7.) $1\frac{1}{3}$ of $2\frac{2}{3}$ of $3\frac{3}{4}$.
(8.) $\frac{4}{5}$ of $1\frac{1}{2}$ of $1\frac{1}{4}$ of $\frac{2}{3}$. (9.) $\frac{3}{8}$ of $3\frac{1}{4}$ of $2\frac{2}{5}$ of 8.
(10.) $\frac{3}{4}$ of $\frac{6}{7}$ of $\frac{4}{15}$ of $\frac{11}{18}$ of $\frac{21}{22}$. (11.) $2\frac{3}{4}$ of $\frac{5}{6}$ of $1\frac{2}{3}$ of $2\frac{1}{22}$ of 8.
(12.) $1\frac{1}{2}$ of $2\frac{2}{3}$ of $3\frac{3}{4}$ of $4\frac{4}{5}$ of $5\frac{5}{6}$.

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121. *To reduce fractions to their least common denominator* (L. C. D.).

RULE. Find the L. C. M. of all the denominators, and take this for the common denominator; and for the new numerators, multiply the numerator of each fraction by the number obtained by dividing the common denominator by the denominator of that same fraction.

Ex. Reduce $\frac{7}{8}$, $\frac{5}{12}$, $\frac{11}{18}$, to their L. C. D.

The L. C. M. of 8, 12, 18, being found, as usual, to be 72, we have

$$\frac{7}{8}=\frac{7\times 9}{72}=\frac{63}{72},\ \frac{5}{12}=\frac{5\times 6}{72}=\frac{30}{72},\ \frac{11}{18}=\frac{11\times 4}{72}=\frac{44}{72},$$

where the factors 9, 6, 4, in the new numerators are obtained by dividing the common denominator 72 by the original denominators 8, 12, 18, respectively.

The reason of this is plain: since in any one of these fractions, the numerator and denominator have *both* been multiplied by the same number (namely, that which makes the denominator become 72), and, therefore, the value of the fraction is not altered.

Ex. 113.

Reduce to their L. C. D.

(1.) $\frac{1}{3}, \frac{3}{5}, \frac{5}{7}$. (2.) $\frac{3}{4}, \frac{4}{5}, \frac{8}{9}$. (3.) $\frac{1}{3}, \frac{1}{4}, \frac{1}{6}, \frac{1}{8}$.
(4.) $\frac{3}{8}, \frac{5}{9}, \frac{11}{16}, \frac{13}{18}$. (5.) $\frac{1}{4}, \frac{3}{8}, \frac{5}{16}, \frac{7}{32}$. (6.) $\frac{1}{6}, \frac{7}{8}, \frac{5}{9}, \frac{11}{24}$.
(7.) $\frac{1}{9}, \frac{4}{27}, \frac{7}{81}, \frac{10}{243}$. (8.) $\frac{3}{10}, \frac{5}{12}, \frac{7}{15}, \frac{9}{20}$. (9.) $\frac{11}{18}, \frac{13}{24}, \frac{17}{36}, \frac{19}{48}$.
(10.) $\frac{4}{25}, \frac{1}{30}, \frac{2}{45}, \frac{3}{50}$. (11.) $\frac{5}{24}, \frac{7}{27}, \frac{11}{36}, \frac{13}{40}$. (12.) $\frac{7}{12}, \frac{9}{16}, \frac{11}{20}, \frac{13}{24}$.

122. *Addition of Fractions.*

Rule. Reduce them (if necessary) to their L. C. D., and take the sum of the numerators, retaining the common denominator.

Ex. 1. $\frac{3}{5}+\frac{1}{5}=\frac{4}{5}$. **Ex. 2.** $\frac{2}{3}+\frac{3}{4}+\frac{4}{5}=\frac{40+45+48}{60}=\frac{133}{60}=2\frac{13}{60}$.

If any of the given quantities are whole or mixed numbers, it is best to add separately the whole and the fractional parts, and then add the two results together.

$$\text{Ex. 3. } 2\frac{3}{5}+3\frac{9}{10}+5\frac{5}{12}+4=2+3+5+4+\frac{36+54+25}{60}$$
$$=14+\frac{115}{60}=14+1\frac{11}{12}=15\frac{11}{12}.$$

Improper fractions must be reduced to mixed numbers, and compound fractions to simple ones, before the Rule can be applied.

$$\text{Ex. 4. } \frac{113}{8}+\frac{3}{5}\text{ of }\frac{10}{9}+2\frac{3}{4}\text{ of }2\frac{2}{11}\text{ of }\frac{5}{8}+5$$
$$=14\frac{1}{8}+\frac{2}{3}+3\frac{3}{4}+5=14+3+5+\frac{3+16+18}{24}$$
$$=22+\frac{37}{24}=22+1\frac{13}{24}=23\frac{13}{24}.$$

Ex. 114.

Find the value of

(1.) $\frac{3}{11}+\frac{4}{11}+\frac{7}{11}+\frac{10}{11}$. (2.) $1\frac{3}{5}+2\frac{2}{5}+3\frac{1}{5}+4$.
(3.) $1\frac{1}{2}+2\frac{1}{3}+3\frac{7}{8}+4\frac{5}{12}$. (4.) $3\frac{1}{2}+\frac{3}{4}+1\frac{5}{12}+2\frac{2}{9}$.
(5.) $1\frac{4}{5}+2\frac{1}{15}+4\frac{1}{10}+\frac{13}{15}$. (6.) $1\frac{3}{4}+4\frac{1}{6}+\frac{7}{8}+\frac{7}{9}$.
(7.) $1\frac{1}{2}$ of $\frac{2}{5}+2\frac{2}{3}+\frac{5}{6}$. (8.) $1\frac{3}{4}+\frac{3}{5}$ of $7\frac{1}{2}+5$.
(9.) $3\frac{7}{8}+5\frac{11}{12}+\frac{4}{5}$ of $\frac{3}{7}$ of $10\frac{1}{2}$. (10.) $5\frac{3}{4}+2\frac{1}{5}$ of $10+\frac{1}{2}$ of 3.
(11.) $2\frac{3}{4}$ of $3\frac{2}{3}+\frac{23}{4}+\frac{3}{4}$ of $\frac{1}{5}$ of $3\frac{1}{3}$.
(12.) $2\frac{3}{4}+3\frac{1}{3}$ of $2\frac{7}{8}$ of $\frac{3}{4}+\frac{1}{5}$ of $3\frac{1}{3}$.

123. *Subtraction of Fractions.*

Rule. Reduce them (if necessary) to their L. C. D., and take the difference of the numerators, retaining the common denominator.

Ex. 1. $\frac{4}{5}-\frac{1}{5}=\frac{3}{5}$.

Ex. 2. $\frac{5}{6}-\frac{4}{5}+\frac{3}{4}-\frac{2}{3}=\frac{50-48+45-40}{60}=\frac{7}{60}$.

If any of the given quantities are whole or mixed numbers, take separately the difference of the whole and fractional parts.

Ex. 3. $5\frac{5}{8}-2\frac{1}{2}=5-2+\frac{5}{8}-\frac{1}{2}=3+\frac{5-4}{8}=3\frac{1}{8}$.

Ex. 4. $5\frac{3}{8}-2\frac{1}{2}=5-2+\frac{3}{8}-\frac{1}{2}=3+\frac{3-4}{8}=3-\frac{1}{8}=2\frac{7}{8}$.

Ex. 5. $6-4\frac{3}{7}=2-\frac{3}{7}=1\frac{4}{7}$.

Improper fractions must be reduced to mixed numbers, and compound fractions to simple ones, before the Rule can be applied.

Ex. 6. $\frac{1}{5}$ of $2\frac{1}{2}$ of $16-1\frac{3}{7}$ of $5\frac{1}{2}=8-7\frac{6}{7}=1-\frac{6}{7}=\frac{1}{7}$.

Ex. 115.

Find the value of

(1.) $\frac{10}{11}-\frac{7}{11}+\frac{3}{11}-\frac{2}{11}$.
(2.) $\frac{19}{20}+\frac{11}{20}-\frac{17}{20}-\frac{9}{20}$.
(3.) $\frac{1}{2}-\frac{1}{3}+\frac{1}{4}-\frac{1}{5}$.
(4.) $3\frac{2}{3}-1\frac{1}{3}+2\frac{3}{4}-1\frac{1}{4}$.
(5.) $3-1\frac{1}{2}$.
(6.) $2\frac{3}{5}-\frac{9}{10}$.
(7.) $10-3\frac{3}{8}$.
(8.) $2\frac{1}{8}-1\frac{5}{8}$.
(9.) $4\frac{3}{5}-\frac{1}{2}$ of $1\frac{1}{2}$.
(10.) $3\frac{2}{5}-2\frac{5}{6}$.
(11.) $1\frac{1}{4}-\frac{5}{8}$.
(12.) $16\frac{5}{8}-3\frac{1}{4}$ of $3\frac{1}{5}$.

Ex. 116.

Add

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	17	13	$5\frac{5}{8}$	(2.)	22	12	$0\frac{1}{3}$	(3.)	25	7	$8\frac{3}{5}$
	3	17	$2\frac{1}{2}$		17	17	$7\frac{2}{9}$		16	11	$2\frac{1}{2}$
	33	15	$4\frac{3}{4}$		3	14	$6\frac{3}{4}$		19	9	$7\frac{3}{10}$
	9	0	$11\frac{1}{6}$		15	9	$10\frac{5}{6}$		8	18	$5\frac{7}{15}$

	cwt.	qrs.	lbs.		yds.	ft.	in.		A.	R.	P.
(4.)	22	2	$17\frac{1}{2}$	(5.)	55	2	$3\frac{1}{2}$	(6.)	17	3	$31\frac{2}{3}$
	18	3	$23\frac{4}{5}$		27	0	$11\frac{3}{8}$		19	2	$11\frac{1}{4}$
	11	0	$24\frac{2}{3}$		8	1	$4\frac{3}{5}$		73	0	$27\frac{7}{12}$
	19	3	$10\frac{1}{6}$		12	2	$10\frac{7}{10}$		35	3	$16\frac{4}{9}$

Subtract

	£	s.	d.		£	s.	d.		£	s.	d.
(7.)	19	0	$10\frac{1}{2}$	(8.)	27	1	$7\frac{3}{4}$	(9.)	32	15	$0\frac{5}{8}$
	11	18	$9\frac{2}{3}$		16	13	$11\frac{4}{5}$		22	18	$10\frac{5}{6}$

Subtract

	c. yds.	c. ft.	c. in.		qrs.	bus.	gals.		d.	hrs.	min.
(10.)	33	18	$100\frac{3}{5}$	(11.)	73	7	$5\frac{5}{12}$	(12.)	113	11	$23\frac{5}{8}$
	27	24	$1000\frac{2}{3}$		65	7	$7\frac{13}{16}$		97	19	$23\frac{11}{12}$

Ex. 117.

(1.) A man receives $\frac{1}{2}$ a loaf for himself, $\frac{1}{3}$ for his wife, and $\frac{1}{4}$ for each of his five children. How many loaves did he receive altogether?

(2.) What number must be taken from $5\frac{3}{4}$ to make $4\frac{2}{3}$?

(3.) What number must be added to $2\frac{3}{4}$ to make $3\frac{4}{5}$?

(4.) The sum of two numbers is $3\frac{1}{2}$, and the less is $1\frac{2}{3}$. Find the greater.

(5.) What is the difference between $\frac{1}{3}+\frac{2}{5}$, and $\frac{1}{3}$ of $\frac{2}{5}$?

(6.) I copied down by mistake $\frac{11}{30}$ instead of $\frac{11}{80}$. What is the amount of error I have made?

(7.) If I pay away $\frac{1}{2}$ of my money, and then $\frac{1}{2}$ of what remains, and then $\frac{1}{2}$ of what remains, what fraction of the whole will be left?

(8.) *A*, *B*, *C*, went out a-nutting. *A* brought home $1\frac{1}{2}$ of $2\frac{1}{3}$ of 10 nuts, *B*, $2\frac{1}{3}$ of $3\frac{1}{4}$ of 12 nuts, and *C*, $3\frac{1}{4}$ of $4\frac{1}{5}$ of 20 nuts. How many had they between them?

(9.) Which is the greater, $\frac{4}{5}$ of $1\frac{3}{14}$, or $\frac{5}{8}$ of $1\frac{5}{9}$, and by how much?

(10.) What number added to $\frac{2}{3}$, $\frac{3}{4}$, $\frac{4}{5}$, and $\frac{5}{6}$, will make the sum total, 4?

(11.) *A* bought $\frac{3}{5}$ of a stack of hay, and sold $\frac{2}{3}$ of it to *B*, who sold $\frac{3}{4}$ of what he had to *C*. What part of the stack had each of them now?

(12.) Take $\frac{4}{5}$ of a dozen, and add to it $\frac{5}{6}$ of a score, and subtract from the result $\frac{2}{9}$ of a hundred. What will be left?

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124. *Multiplication of Fractions.*

RULE. Multiply the numerators together for the new numerator, and the denominators for the new denominator.

Ex. 1. $\frac{2}{7}\times\frac{3}{4}=\frac{6}{28}=\frac{3}{14}$.

It will be seen that this Rule is the same as that already given in **120** for reducing a compound fraction to a simple one; so that to *multiply* $\frac{3}{4}$ by $\frac{2}{7}$ is the same thing as to take $\frac{2}{7}$ of $\frac{3}{4}$. The word *multiply* is here used in a wider sense than in the case where the multiplier is an integer; for then it means to *repeat* the multiplicand a certain number of times. or to take a certain *multiple* of it; whereas here it means to take any *part* or *parts* of it. But the Rule above given *includes* the case of integral multipliers.

Thus $7\times 3=\frac{7}{1}\times\frac{3}{1}=\frac{21}{1}=21$.

And it is for this reason, because the *general* method of taking *any part or parts* of a quantity includes the *particular* one, of taking any *multiple* of it, that the same name of multiplication, which properly belongs to the latter only, has been extended also to the former.

As before, in **120**, mixed numbers must be reduced to improper fractions before applying the Rule; and common factors may be struck out from any of the numerators and denominators of the fractions that are to be multiplied together.

Ex. 2. $2\frac{4}{5}\times 3\frac{1}{7}\times 1\frac{2}{3}$ of $8=\frac{\overset{2}{\cancel{14}}}{\cancel{5}}\times\frac{22}{\cancel{7}}\times\frac{\cancel{5}}{3}\times\frac{8}{1}=\frac{176}{3}=58\frac{2}{3}$.

125. *Division of Fractions.*

Rule. Invert the divisor, and multiply.

Ex. 1. $\frac{3}{7}\div\frac{5}{4}=\frac{4}{5}\times\frac{3}{7}=\frac{12}{35}$.

Here also the word *division* is used in a wider sense than before, to denote the finding that quantity which, when *multiplied* by the divisor, will produce the dividend, using now the word *multiplied* in the enlarged sense explained in **124**.

Hence in the above Example, where the divisor is $\frac{5}{4}$ and dividend $\frac{3}{7}$, we must have $\frac{5}{4}\times$ quotient $=\frac{3}{7}$: therefore, $\frac{1}{4}\times$ quotient $=\frac{1}{5}$ of $\frac{3}{7}$, and, therefore, the quotient itself $=\frac{4}{5}$ of $\frac{3}{7}=\frac{4}{5}\times\frac{3}{7}=\frac{12}{35}$, as above.

As in Multiplication, mixed numbers must be reduced to improper fractions before this rule can be applied.

Ex. 2. $2\frac{2}{3}\div 3\frac{3}{4}=\frac{8}{3}\div\frac{15}{4}=\frac{8}{3}\times\frac{4}{15}=\frac{32}{45}$.

Ex. 3. $2\frac{2}{3}$ of $3\frac{3}{4}\div 4\frac{1}{2}$ of $\frac{5}{7}$ of $\frac{4}{45}=\frac{8}{3}$ of $\frac{15}{4}\div\frac{9}{2}$ of $\frac{5}{7}$ of $\frac{4}{45}$

$$=\frac{\cancel{8}}{\cancel{3}}\times\frac{\overset{5}{\cancel{15}}}{\cancel{4}}\times\frac{\cancel{2}}{\cancel{9}}\times\frac{7}{\cancel{5}}\times\frac{\cancel{45}}{\cancel{4}}=35.$$

Observe that in Ex. 3, instead of reducing the divisor $\frac{9}{2}$ of $\frac{5}{7}$ of $\frac{4}{45}$, to a simple fraction, we have inverted *all* the fractions which compose it, and multiplied by them, thus inverted—which comes to the same thing, as if we had first brought the compound fraction to a simple one, and then inverted it, and multiplied by it.

Ex. 118.

Find the value of

(1.) $\frac{5}{6} \times 1\frac{1}{8} \div \frac{3}{16}$. (2.) $2\frac{2}{3} \times 3\frac{1}{5} \div 2\frac{2}{15}$. (3.) $(\frac{5}{12} \div 1\frac{7}{9}) \times 2\frac{2}{11}$.
(4.) $(1\frac{7}{9} \div 3\frac{2}{3}) \times 2\frac{1}{4}$. (5.) $(3\frac{1}{3} \div 2\frac{1}{2}) \div 3$. (6.) $1\frac{3}{8} \times 1\frac{3}{7} \div 2\frac{3}{4}$.
(7.) $(1\frac{1}{4}$ of $7\frac{1}{3}) \div (1\frac{3}{8}$ of $2\frac{2}{7})$. (8.) $(2\frac{1}{4}$ of $2\frac{2}{3}) \div (4\frac{1}{2}$ of $3\frac{1}{8})$.
(9.) $200 \div (1\frac{2}{3}$ of $2\frac{3}{4}$ of $3)$. (10.) $2\frac{3}{5} \div (2\frac{4}{11}$ of $2\frac{3}{4}$ of $5)$.
(11.) $(\frac{3}{5} \div 1\frac{1}{8}) \times (\frac{6}{7} \div 1\frac{1}{3}) \div 3\frac{3}{4}$.
(12.) $(2 \div 1\frac{1}{2}) \times (3 \div 2\frac{2}{3}) \times (4 \div 3\frac{3}{4})$.

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126. A *complex* fraction may be reduced to a simple one by a combination of the foregoing Rules, as shown in the following instances:

Ex. 1. $\dfrac{\frac{2}{3} \text{ of } 5}{\frac{3}{5} \text{ of } 1\frac{4}{5}} = \dfrac{\frac{10}{3}}{\frac{27}{25}} = \frac{10}{3} \div \frac{27}{25} = \frac{10}{3} \times \frac{25}{27} = \frac{250}{81} = 3\frac{7}{81}$.

From the above Example it appears that a complex fraction, when expressed in its simplest form, that is, like $\dfrac{\frac{10}{3}}{\frac{27}{25}}$, with its numerator and denominator both simple fractions, may be immediately turned into a simple fraction by multiplying together the two *outer* numbers or *extremes*, 10 and 25, for the new numerator, and the two *inner* numbers, 3 and 27, for the new denominator.

Ex. 2. $\dfrac{\frac{3}{7}}{\frac{4}{5}} = \frac{15}{28}$; $\dfrac{\frac{2}{3}}{3} = \dfrac{\frac{2}{3}}{\frac{3}{1}} = \frac{2}{9}$; $\dfrac{2}{2\frac{1}{5}} = \dfrac{\frac{2}{1}}{\frac{11}{5}} = \dfrac{10}{11}$; $\dfrac{2\frac{5}{8}}{3\frac{1}{3}} = \dfrac{\frac{21}{8}}{\frac{10}{3}} = \frac{63}{80}$.

Hence also, in a complex fraction expressed in its simplest form, any common factors that appear in *either one* of the extremes and *either one* of the means, may be struck out of both.

Ex. 3. $\dfrac{2\frac{5}{8}}{7} = \dfrac{\frac{\cancel{21}^{3}}{8}}{\frac{\cancel{7}}{1}} = \frac{3}{8}$; $\dfrac{5}{3\frac{3}{4}} = \dfrac{\frac{\cancel{5}}{1}}{\frac{\cancel{15}^{3}}{4}} = \frac{4}{3}$; $\dfrac{1\frac{7}{10}}{10\frac{1}{5}} = \dfrac{\frac{\cancel{17}^{1}}{\cancel{10}^{2}}}{\frac{\cancel{51}^{3}}{\cancel{5}^{1}}} = \frac{1}{6}$.

Ex. 119.

Reduce to simple fractions

(1.) $\dfrac{27}{2\frac{1}{4}}$. (2.) $\dfrac{3\frac{3}{8}}{9}$. (3.) $\dfrac{\frac{9}{25}}{2\frac{2}{5}}$. (4.) $\dfrac{\frac{14}{27}}{2\frac{1}{3}}$.

(5.) $\dfrac{1\frac{7}{9}}{2\frac{2}{3}}$. (6.) $\dfrac{3\frac{3}{4}}{2\frac{1}{2}}$. (7.) $\dfrac{6\frac{2}{3}}{5}$. (8.) $\dfrac{15}{2\frac{3}{4}}$.

(9.) $\dfrac{17}{1\frac{2}{3}+\frac{3}{5}}$. (10.) $\dfrac{1\frac{1}{2} \text{ of } 2\frac{2}{3}}{3\frac{3}{4} \text{ of } 4\frac{4}{5}}$. (11.) $\dfrac{2\frac{2}{3}-1\frac{1}{2}}{2\frac{2}{3}+1\frac{1}{2}}$. (12.) $\dfrac{2\frac{1}{5} \text{ of } 3}{3\frac{2}{3} \text{ of } 2}$.

Ex. 120.

(1.) What fraction must be multiplied by $3\frac{1}{5}$ of $3\frac{1}{8}$ to produce 3?

(2.) What number must be divided by the same fraction to produce the same result?

(3.) What will 83 yards of ribbon cost, at $\frac{4}{5}$*d.* per yard?

(4.) How many times is $\frac{4}{5}$ of a foot contained in 83 feet? how many times in $\frac{8}{3}$ of a foot?

(5.) A brick is $8\frac{8}{9}$ inches long: how many will there be in a length of 100 yards?

(6.) If the thickness of the brick in (5) be a fifth of its length, what will be the height of a wall which consists of 36 courses?

(7.) If $6\frac{3}{4}$ tons of coals be distributed among eighteen poor persons, what fraction of a ton will each receive?

(8.) How many times is $1\frac{2}{3}$*d.* contained in 30*d.*? how many times in $2\frac{1}{2}$*d.*?

(9.) From 36 yards of string I want to cut off a number of pieces, each $5\frac{1}{3}$ yards long. How many such pieces can I cut? and how many yards of the string will then be left?

(10.) *A* can walk $3\frac{1}{3}$ miles an hour, and *B* $3\frac{3}{4}$ miles. How many hours will each take to go a distance of 20 miles? If they started to meet each other from opposite ends of the 20 miles, how many hours will it be before they would meet?

(11.) One wheel of a carriage is $5\frac{1}{2}$ feet round, and another is $7\frac{1}{2}$ feet. How many times will the first revolve oftener than the second in going a mile?

(12.) How many complete suits of boys' clothes will a tailor make out of 25 yards of cloth, each taking $3\frac{3}{8}$ of a yard? How many yards will then be left?

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127. We shall now give examples of multiplying or dividing *concrete* quantities by fractions.

Ex. 1. Multiply £2 10*s.* 4*d* by $8\frac{5}{12}$.

$$\begin{array}{rrrrl} & \pounds & s. & d. & \\ & 2 & 10 & 4 & \times 8 \\ & & & 8 & \\ \hline & 7 & 11 & 0 & \\ \text{For } \frac{5}{12}, & 1 & 0 & 11\frac{2}{3} & \\ \hline & 8 & 11 & 11\frac{2}{3} & \\ \hline \end{array} \qquad \begin{array}{rrrrl} & \pounds & s. & d. & \\ & 2 & 10 & 4 & \times \frac{5}{12} \\ & & & 5 & \\ \hline 12) & 12 & 11 & 8 & \\ \hline & 1 & 0 & 11\frac{2}{3} & \\ \hline \end{array}$$

Ex. 2. Divide 1 ton 13 cwt. 15 lbs. by $1\frac{1}{3}$.

Here we have to *divide* by $1\frac{1}{3}=\frac{4}{3}$, that is, we have to multiply by $\frac{3}{4}$.

$$\begin{array}{rrrrr} & \text{ton} & \text{cwt.} & \text{qrs.} & \text{lbs.} \\ & 1 & 13 & 0 & 15 \\ & & & & 3 \\ \hline 4) & 4 & 19 & 1 & 17 \\ \hline & 1 & 4 & 3 & 11\frac{1}{4} \\ \hline \end{array} \qquad \text{or} \qquad \begin{array}{r|rrrr} & \text{ton} & \text{cwt.} & \text{qrs.} & \text{lbs.} \\ & 1 & 13 & 0 & 15 \\ \hline \text{for } \frac{1}{2} & & 16 & 2 & 7\frac{1}{2} \\ \text{for } \frac{1}{4} & & 8 & 1 & 3\frac{3}{4} \\ \hline & 1 & 4 & 3 & 11\frac{1}{4} \\ \hline \end{array}$$

In the second of the above methods, we have shown another way of multiplying by $\frac{3}{4}$, namely, by first dividing by 2, which gives $\frac{1}{2}$ of the given quantity, and then dividing this last result by 2, which gives $\frac{1}{4}$ of the given quantity; then, adding the two results, we have $\frac{1}{2}+\frac{1}{4}$, or $\frac{3}{4}$ of it.

Sometimes it is most convenient to reduce the given quantity, in such a case, to one denomination, before multiplying or dividing.

Ex. 3. Divide 7*s.* $1\frac{1}{2}$*d.* by $\frac{9}{115}$.

Here we have to multiply by $\frac{115}{9}=12\frac{7}{9}$, and 7*s.* $1\frac{1}{2}$*d.* = 342*f.*;

$$\begin{array}{rl} & 342\,f. \times 12 \\ & 12 \\ \hline & 4104 \\ \text{For } \frac{7}{9}, & 266 \\ \hline 4) & 4370\,f. \\ \hline 12) & 1092\frac{1}{2}d. \\ \hline 2,0) & 9,1s.\ 0\frac{1}{2}d. \\ \hline & £4\ 11s.\ 0\frac{1}{2}d. \\ \hline \end{array} \qquad \begin{array}{rl} & 342\,f. \times \frac{7}{9} \\ & 7 \\ \hline 9) & 2394 \\ \hline & 266 \\ \hline \end{array}$$

Ex. 121.

Find the value of

(1.) $\frac{5}{8}$ of £1 2*s.* 6*d.*

(2.) £1 2*s.* 6*d.* ÷ $\frac{5}{8}$.

(3.) £12 10*s.* × $\frac{3}{7}$.

(4.) £12 10*s.* ÷ $\frac{3}{7}$.

(5.) $\frac{5}{9}$ of £13 13*s.*

(6.) £13 13*s.* ÷ $\frac{5}{9}$.

(7.) $\frac{2}{11}$ of 3 tons 5 cwt. 101 lbs.

(8.) 3 tons 5 cwt. 101 lbs. ÷ $\frac{2}{11}$.

(9.) 23A. 3R. 6P. × $\frac{5}{14}$.

(10.) 23A. 3R. 6P. ÷ $\frac{5}{14}$.

(11.) $\frac{6}{25}$ of 15 c. yds. 20 ft. 140 in.

(12.) 15 c. yds. 20 ft. 140 in. ÷ $\frac{6}{25}$.

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123. Observe attentively the method of treating the following Examples:

Ex. 1. *A* has 156 nuts in his bag, and he has half as much again as *B*, and a third as much again as *C*. How many have they between them?

Here $A\text{'s} = B\text{'s} \times 1\frac{1}{2}$; therefore, $B\text{'s} = A\text{'s} \div 1\frac{1}{2} = 156 \div 1\frac{1}{2} = 156 \times \frac{2}{3} = \frac{2}{3}$ of $156 = 104$; and $A\text{'s} = C\text{'s} \times 1\frac{1}{3}$; therefore, $C\text{'s} = A\text{'s} \div 1\frac{1}{3} = 156 \div 1\frac{1}{2} = 156 \times \frac{3}{4}$ $= \frac{3}{4}$ of $156 = 117$; and $156 + 104 + 117 = 377$.

N.B. Notice in the above, that if
A's quantity $= \frac{3}{2}$ of *B*'s, then *B*'s quantity $= \frac{2}{3}$ of *A*'s.

Ex. 2. *A* can do a piece of work in 3 days, and *B* can do the same in 5 days. In what time would they do it, if set to work upon it together?

A does, in *one* day, $\frac{1}{3}$ of the work,
B „ $\frac{1}{5}$ „ ;

therefore, *A* and *B* together do, in *one* day, $\frac{1}{3} + \frac{1}{5} = \frac{8}{15}$ of the work: hence they do $\frac{1}{15}$ of the work in $\frac{1}{8}$ of a day, and the whole work in $\frac{15}{8}$ $= 1\frac{7}{8}$ of a day.

N.B. Notice in the above that if a man do $\frac{8}{15}$ of a work in 1 day, he will do the whole in $\frac{15}{8}$ of a day.

Ex. 3. Suppose *A* to work for 1 day, and then *B* to work for 2 days, how much longer will *A* take by himself to finish it?

In 1 day *A* does $\frac{1}{3}$ of the work, in 2 days *B* does $\frac{2}{5}$ of it; therefore, the two will have done $\frac{1}{3} + \frac{2}{5} = \frac{11}{15}$ of it; then there will remain $\frac{4}{15}$ of it to be done, and, since *A* does $\frac{1}{3}$ in one day, he will take $\frac{4}{15} \div \frac{1}{3} = \frac{4}{15} \times 3 = \frac{4}{5}$ of a day, to finish it.

Ex. 4. A cistern can be filled in 36 minutes by one pipe (*A*), and emptied in 48 minutes by another pipe (*B*). In what time would it be filled, if both were opened together?

(*A*) pours *in* $\frac{1}{36}$ of the whole in 1 minute,
(*B*) lets *out* $\frac{1}{48}$ „ ;

therefore, if both are opened together, there will be added to the water *in* the cistern $\frac{1}{36} - \frac{1}{48} = \frac{1}{144}$ of the whole, every minute: consequently, it will take 144 minutes to fill the cistern.

Ex. 5. If the pipe (*A*) be allowed to run for 15 minutes, how long will the pipe (*B*) be in emptying the cistern?

(*A*) pours *in* $\frac{1}{36}$ of the whole in 1 minute,
and, therefore, $\frac{15}{36} = \frac{5}{12}$ „ in 15 minutes;
(*B*) lets *out* $\frac{1}{48}$ of the whole in 1 minute,
and, therefore, $\frac{5}{12}$ of the whole in $\frac{5}{12} \div \frac{1}{48} = \frac{5}{12} \times 48 = 20$ minutes.

Ex. 122.

(1.) *A* has 15*s.* in his pocket, and he has half as much again as *B*, a third as much again as *C*, and a fourth as much again as *D*. What sum have they between them?

(2.) A gentleman paid two bills, one of them amounting to £1 7*s.* 6*d.*, and this was a fourth as much again as the other. Find the other bill.

(3.) If *A*'s money be $1\frac{2}{3}$ of *B*'s, and $1\frac{2}{5}$ of *C*'s, how much have the three between them, when *A* has just 8*s*. $0\frac{1}{4}$*d*.?

(4.) If *A*'s money be $2\frac{2}{3}$ of *B*'s, and *B*'s $3\frac{3}{4}$ of *C*'s, how much have they between them, when *B* has 3*s*. $5\frac{1}{4}$*d*.?

(5.) Some coals were distributed in a parish; and *A*, with his large family, received 3 cwt. 1 qr. 14 lbs., which was $1\frac{3}{4}$ of what his brother *B* received. What did they get between them?

(6.) *A* can mow a field of hay in 12 hours, and *B* in 16 hours. In how many hours would they mow it together?

(7.) Supposing *A* had mowed a third of it, and *B* came *then* to his help, how many hours longer would it take them to finish it?

(8.) Suppose *A* mowed by himself for 3 hours, and then *B* by himself for 3 hours, how many hours longer would *A* take by himself to finish it?. How many hours would *B*?

(9.) If three men of equal strength can reap a field of corn together in 12 hours, in what time would one of them reap it by himself? In what time would two of them reap it together? If one wrought by himself for six hours, and then two together for five hours, in how many hours more would the three together finish it?

(10.) *A* and *B* can do a piece of plastering together in three days of 12 hours each, working equally. *A* labours at it for a day by himself, and then they work together for half a day. How many days will *A* take to finish it?

(11.) A cistern can be filled in 20 minutes by one pipe (A), and in 30 minutes by another (B). In how many minutes would it be filled, if both were opened together?

(12.) Supposing the latter of the two pipes in (11) to be one by which the cistern can be *emptied* instead of filled, how many minutes would it take to fill the cistern, if both pipes were opened at once?

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129. *To express a given quantity as a fraction of another given quantity of the same kind.*

Rule. Reduce both to the same denomination, and take the result of the former for the numerator, and of the latter for the denominator, of the fraction required.

Ex. 1. Express 7*s*. 6*d*. as the fraction of £1 4*s*.

Since 7*s*. 6*d*. = 90*d*., and £1 4*s*. = 288*d*., the fraction required is $\frac{90}{288}$ $=\frac{5}{16}$.

For 1*d*. is $\frac{1}{288}$ of £1 4*s*., and, therefore, 7*s*. 6*d*., which = 90*d*., is $\frac{90}{288}$ of £1 4*s*.

Any common denomination, to which the two given quantities can be brought, will serve for the purpose of this Rule. But if we choose the *highest* of which they admit, the fraction will be expressed, without further reduction, in its lowest terms.

Ex. 2. Express half-a-crown as the fraction of half-a-guinea.

Since 2*s.* 6*d.* = 5 *sixpences*, and 10*s.* 6*d.* = 21 *sixpences*, the fraction required is $\frac{5}{21}$.

Ex. 123.

Express
(1.) 2*s.* 6*d.* as the fr. of £1. (2.) 3*s.* 9*d.* as the fr. of 30*s.*
(3.) 3*s.* 4*d.* „ 6*d.* (4.) 5*s.* 4*d.* „ £5.
(5.) 12*s.* 4*d.* „ £1 1*s.* 7*d.* (6.) £2 3*s.* 1*d.* „ 11*s.* 9*d.*
(7.) £2 1*s.* $9\frac{1}{2}$*d.* „ 14*s.* 9*d.* (8.) £2 2*s.* $5\frac{1}{4}$*d.* „ 14*s.* $1\frac{3}{4}$*d.*
(9.) £2 3*s.* $11\frac{1}{4}$*d.* 11*s.* $6\frac{3}{4}$*d.* (10.) £1 2*s.* $4\frac{1}{4}$*d.* 4*s.* $7\frac{1}{2}$*d.*
(11.) £6 15*s.* $9\frac{3}{4}$*d.* £1 3*s.* $2\frac{1}{4}$*d.* (12.) £8 16*s.* $5\frac{1}{2}$*d.* £1 5*s.* 8*d.*

Ex. 124.

Express
(1.) 1 cwt. 3 qrs. as the fr. of 3 cwt. 2 qrs.
(2.) 3 cwt. 0 qr. 25 lbs. „ 1 cwt. 2 qrs. 3 lbs.
(3.) 10 yds. 1 in. „ „ 2 yds. 1 ft. 11 in.
(4.) 7 po. 4 yds. „ „ 26 po. $1\frac{1}{2}$ yds.
(5.) 3A. 2R. 1P. „ „ 2A. 37P.
(6.) 1A. 9P. „ „ 1R. 25P.
(7.) 1 yd. 7 ft. 45 in. „ „ 1 sq. ft.
(8.) 1 cub. ft. 10 in. „ „ 1 cub. ft. 642 in.
(9.) 6 qrs. 6 bus. 3 gals. „ 3 qrs. 1 bus. 3 gals.
(10.) 6 bus. 1 pk. 3 qts. „ 3 gals. 2 qts. 1 pt.
(11.) 13 hrs. 15 min. 17 sec. „ 6 d. 1 hr. 48 min. 7 sec.
(12.) 27 d. 13 hrs. 22 min. 30 sec. 3 d. 20 hrs. 35 min. 33 sec.

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130. In the following Examples we are required to express a *fraction* of one given quantity as a fraction of another of the same kind.

Ex. 1. Express $\frac{2}{3}$*s.* as the fraction of £1.

Ans. $\frac{2}{3}$ of 1*s.* $=\frac{2}{3}$ of £$\frac{1}{20}$ $=$ £$\frac{1}{30}$.

Ex. 2. Express $3\frac{5}{9}$ of £1 $9\frac{3}{4}$*d.* as the fraction of £1 10*s.* 10*d.*

Here £1 $9\frac{3}{4}$*d.* $=999$*f.*, and £1 10*s.* 10*d.* $=1480$*f.*; $\therefore$ $3\frac{5}{9}$ of £1 $9\frac{3}{4}$*d.* $=3\frac{5}{9}$ of $\frac{999}{1480}$ of £1 10*s.* 10*d.*, as in **129**, where the required fraction $=3\frac{5}{9}$ of $\frac{999}{1480}=2\frac{2}{5}$.

Ex. 125.

Express

(1.) $1\frac{5}{9}$ of £2 12*s.* 6*d.* as a fraction of £1 3*s.* 4*d.*
(2.) $2\frac{3}{10}$ of 9*s.* $4\frac{1}{2}$*d.* „ „ $11\frac{1}{4}$*d.*
(3.) $3\frac{7}{8}$ of £5 „ „ £3 17*s.* 6*d.*
(4.) $4\frac{1}{5}$ of £2 5*s.* $3\frac{1}{4}$*d.* „ „ £4 12*s.* 9*d.*
(5.) $2\frac{1}{3}$ of £3 1*s.* 8*d.* „ „ £2 3*s.* 2*d.*
(6.) $5\frac{1}{4}$ of £1 10*s.* 10*d.* „ £4 17*s.* $1\frac{1}{2}$*d.*
(7.) $4\frac{3}{4}$ of 4 cwt. 2 qrs. 9 lbs. „ 6 cwt. 1 qr. 22 lbs.
(8.) $1\frac{2}{3}$ of 7 yds. 2 ft. 9 in. „ 13 yds. 7 in.
(9.) $2\frac{3}{4}$ of 2A. 3R. 4P. „ 15A. 1R. 2P.
(10.) $3\frac{1}{2}$ of 37 sq. yds. 8 ft. 55 in. 24 sq. yds. 1 ft. 35 in.
(11.) $1\frac{2}{5}$ of 33 qrs. 7 bus. 7 gals. 9 qrs. 4 bus. 1 gal.
(12.) $2\frac{1}{2}$ of 2d. 23 hrs. 29 min. 43 sec. 2d. 3 hrs. 4 min. 5 sec.

Ex. 126.

(1.) If nine boys between them ate 5 lbs. of cake at a school-feast, what fraction of an ounce did each of them eat?

(2.) Express a pound and an ounce Troy as fractions respectively of a pound and an ounce Avoirdupois.

(3.) Two farmers bought a crop of hay between them. One took $\frac{5}{8}$ of the crop, and paid £12 for it. What had the other to pay?

(4.) A field of $3\frac{1}{2}$ acres was divided into allotments, each containing $\frac{7}{32}$ of an acre. How many were there of these?

(5.) If there had been 28 allotments made of the field in (4), what fraction of an acre would each of them have been?

(6.) A father left $\frac{1}{4}$ of his property to his wife, $\frac{1}{5}$ to each of his three sons, $\frac{1}{10}$ to his daughter, and the rest to his nephew. What fraction of the whole was the nephew's share? And, if it was worth £100, what was the value of each of the other bequests?

(7.) In a contested election there were 342 votes, and *A* had a majority of 76 over *B.* What fraction of the whole number of votes did each get?

(8.) A man walked $11\frac{2}{3}$ miles in $3\frac{1}{2}$ hours. How many miles an hour did he walk? and how many hours would he be in going 20 miles at the same rate?

(9.) I took out of my pocket $\frac{1}{2}$ of my money, and then $\frac{1}{3}$ of what remained, and then $\frac{1}{4}$ of what remained, and now there was 6*d.* in my pocket. How much was there at first?

(10.) If $\frac{3}{7}$ of a cwt. of flour be worth $15\frac{3}{8}$*s.*, what is the value of $\frac{7}{3}$ of a cwt.?

(11.) A room measures $13\frac{1}{3}$ feet by $13\frac{1}{2}$ feet: what length of carpet, $\frac{5}{8}$ yard wide, will cover it?

(12.) A man can do in four days what his son can do in seven. How many hours will the father take to do what the son can do in a day, reckoning the day as twelve hours long?

(13.) Four men had between them a sum of money. *A* had $\frac{1}{3}$ of it, *B* had $\frac{1}{4}$, *C* had $\frac{1}{5}$, and *D* had the rest, which was 10*s.* $6\frac{3}{4}$*d.* What had they between them?

(14.) If $2\frac{1}{2}$ yards of cloth cost $3\frac{1}{3}$*s.*, what will be the cost of $4\frac{1}{4}$ yards?

(15.) What will be the area of a piece of ground, which is $55\frac{1}{2}$ yards square?

(16.) A pint contains $34\frac{2}{3}$ cubic inches. How many pints are there in a cubic foot of water?

(17.) How many eighths of a pound are there in $3\frac{2}{3}$*s.*? How many tenths of a shilling in $3\frac{2}{3}$*d.*?

(18.) The breadth of a play-ground is $55\frac{1}{8}$ feet. What must be the length that the area may be 168 square yards?

(19.) There was a yard, 10 feet square, paved with 1000 bricks. One side of each brick was $7\frac{1}{5}$ inches long. What was the other?

(20.) What is the difference between $\frac{7}{8}$ of a guinea and $\frac{8}{9}$ of a pound?

(21.) If $\frac{3}{5}$ of a ton of coals cost £$\frac{2}{3}$, what will $1\frac{1}{4}$ cwt. cost?

(22.) What would be the area of a rectangular room, which measured $12\frac{3}{4}$ feet by $10\frac{2}{3}$ feet? and how many yards of carpet, $\frac{3}{4}$ yard wide, would be required to cover it?

(23.) A straight plank is $3\frac{1}{8}$ inches thick, and $6\frac{3}{4}$ inches broad. What length must be cut off, so as to contain $3\frac{1}{8}$ cubic feet of timber?

(24.) A father left his son $\frac{4}{7}$ of his whole property. In the first year he squandered away $\frac{3}{8}$ of it, and, in the second, $\frac{3}{10}$ of what was left, and he had only now just 150 guineas remaining. What was the father's whole property?

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131. In common integers, the actual value of each figure depends, as we have seen, upon its position with respect to the place of *units*, and its value in any one position is *ten times* what it would be if it stood one place further to the *right*, or *one-tenth* of what it would be if it stood one place further to the *left*.

Now, if we continue the same method of notation to the *right* of the place of units, still reckoning the value of each figure to be *one-tenth* of what it would be if it stood one place further to the right, we shall obtain what are called *Decimal Fractions*, or, briefly, *Decimals*.

Thus setting, as is usual, a dot (or comma), called the decimal *point*, after the unit's place, in order to mark where it comes, the number 5403.045 &c. will denote 5 *thousands*, 4 *hundreds*, 0 *tens*, 3 *units*, 0 *tenths*, 4 *hundredths*, 5 *thousandths*, &c., or $5\times1000+4\times100+0\times10+3+\frac{0}{10}+\frac{4}{100}+\frac{5}{1000}+$&c.—all the figures *before* the point expressing integral quantities, and all the figures *after* the point expressing fractional quantities, whose denrs. are 10, 100, 1000, &c.

132. Any fraction whatever, having for its denominator 10, 100, 1000, &c., may be written down at once as a decimal, by setting down the numerator, and marking off by the point, from the right of it, as many figures as there are ciphers in the denominator—prefixing ciphers to it, if necessary, so as to make the number of figures after the point equal to the number of ciphers in the denominator.

Ex. 1. $\frac{347}{100}=3.47$; for $\frac{347}{100}=3\frac{47}{100}=3+\frac{40}{100}+\frac{7}{100}=3+\frac{4}{10}+\frac{7}{100}=3.47$.

Ex. 2. $\frac{13}{1000}=.013$; for $\frac{13}{1000}=\frac{10}{100}+\frac{3}{1000}=\frac{1}{100}+\frac{3}{1000}=.013$.

Ex. 3. $\frac{2125}{1000}=2.125$; $\frac{119}{10000}=.0119$; $\frac{27}{1000000}=.000027$.

Ex. 127.

Write down as decimal fractions

(1.) $\frac{9}{10}, \frac{235}{10}, \frac{76}{100}$.
(2.) $\frac{3}{100}, \frac{17}{1000}, \frac{39}{10000}$.
(3.) $\frac{1}{100}, \frac{11}{1000}, \frac{111}{1000000}$.
(4.) $\frac{2}{10000}, \frac{124}{10}, \frac{705}{100}$.
(5.) $\frac{117}{1000}, \frac{81}{10000}, \frac{73}{10}$.
(6.) $\frac{13}{1000}, \frac{301}{10}, \frac{123}{100000}$.
(7.) Five tenths + seven hundredths + fifty-seven millionths.
(8.) Thirty-one tenths + five thousandths + twenty-three millionths.
(9.) Seven hundred + three + seven hundredths + thirty-seven millionths.
(10.) One + eleven thousandths + sixty-nine millionths.
(11.) Two-hundred-and-three tenths + one thousandth + one-hundred-and-two millionths.
(12.) Seventeen tenths + seventy thousandths + seventy-seven millionths.

133. Conversely, any decimal may be converted at once into a vulgar fraction, by writing down its figures for the numerator, and for the denominator 10, 100, 1000, &c., according to the number of figures after the point.

Ex. 1. $2.03 = 2\frac{3}{100}$ or $\frac{203}{100}$. Ex. 2. $.379 = \frac{379}{1000}$.

Ex. 3. $42.037 = 42\frac{37}{1000}$ or $\frac{42037}{1000}$. Ex. 4. $15.001 = 15\frac{1}{1000}$ or $\frac{15001}{1000}$.

Sometimes the resulting fraction may admit of being reduced to lower terms.

Ex. 1. $13.75 = 13\frac{75}{100} = 13\frac{3}{4}$. Ex. 2. $23.0625 = 23\frac{625}{10000} = 23\frac{1}{16}$.

134. Any decimal is *multiplied* by 10, 100, 1000, &c., by moving the point one, two, three, &c. places to the *right*, and *divided* by moving it similarly to the *left*.

Ex. 1. $3.247 \times 10 = 32.47$: for $3.247 \times 10 = \frac{3247}{1000} \times 10 = \frac{3247}{100} = 32.47$.
Ex. 2. $3.247 \div 10 = .3247$: for $3.247 \div 10 = \frac{3247}{1000} \div 10 = \frac{3247}{10000} = .3247$.
Ex. 3. $3.247 \times 100 = 324.7$; $3.247 \div 100 = .03247$; $3.247 \times 1000 = 3247$.
Ex. 4. $2.3 \times 1000 = 2300$; $2.3 \div 1000 = .0023$; &c.

135. Observe, that setting ciphers *after* a decimal does not alter its value at all.

Thus .3, .30, .300, are all equal, representing $\frac{3}{10}$, $\frac{30}{100}$, $\frac{300}{1000}$, respectively, each of which $= \frac{3}{10}$.

But setting ciphers *before* a decimal, after the point, is equivalent to dividing it by 10, 100, &c.

Thus .3, .03, .003, are, respectively, $\frac{3}{10}$, $\frac{3}{100}$, $\frac{3}{1000}$.

Ex. 128.

Express as vulgar fractions
(1.) .059, .0003, 4.3. (2.) .225, .0075, 13.025.
(3.) .03125, 7.375, .625. (4.) .4125, .512, 1.024.
(5.) 40.96, 1.875, .0064. (6.) .0128, .00125, .256.

Find the value of
(7.) $.73 \times 10$, and 3.17×1000. (8.) $.73 \div 10$, and $3.17 \div 1000$.
(9.) $.0023 \times 10000$, and $2.3 \div 10000$.
(10.) 10.17×1000. and $.01017 \div 100$.
(11.) Multiply 26.025 by a hundred, ten thousand, and a million.
(12.) Divide 1394 by a hundred, ten thousand, and a million.

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136. *Addition* and *Subtraction of Decimals* are performed as in common integers—observing to place the points all in the same vertical column, so that the units' figures may fall under one another.

Ex. 1. Add together 2.8146, .0938, 8, and .875.

$$\begin{array}{r} 2.8146 \\ .0938 \\ 8.0000 \\ .8750 \\ \hline 11.7884 \\ \hline \end{array}$$

It is convenient to fill out the lines with ciphers, as in the third and fourth lines of this example. Then, adding the right-hand column, we get 8+6=14 *ten-thousandths* $=\frac{14}{10000}=\frac{10}{10000}+\frac{4}{10000}=\frac{1}{1000}+\frac{4}{10000}$: we set down the 4 *ten-thousandths*, and carry the *one-thousandth* to the next column; and so on.

Ex. 2. Find the difference of 2.418 and 1.2234.

$$\begin{array}{r} 2.4180 \\ 1.2234 \\ \hline 1.1946 \\ \hline \end{array}$$

There being no *ten-thousandths* in the upper line, we have to borrow 1 *thousandth*=10 *ten-thousandths*: taking 4 *ten-thousandths* from this, we have 6 *ten-thousandths* to be set down, and, for the 1 *thousandth* borrowed, we have either 7 *thousandths* (instead of 8) in the upper line, or (which comes to the same) we *carry* 1 *thousandth* to the lower line, and subtract 4 *thousandths* (instead of 3).

Ex. 129.

Add

(1.)	(2.)	(3.)	(4.)
2.731	11.8085	7.1426	29.786
.00356	3.7	11.	.1537
13.3	.00156	.009	2.00765
.46512	.00099	3.45678	.013
3.7342	.2	.0001	13.

(5.) 3.5723 + .00127 + 14.003 + .0762 + 54.001.
(6.) .0023 + 11.01 + 327.00345 + 17 + 1.26314.

Subtract

(7.)	(8.)	(9.)	(10.)
25.003	2.73	.2678	4.0001
18.789	.65747	.17895	3.98769

Find the value of

(11.) 32.0117 + 7.34 + .00127 − 3.15 − 15.0267.
(12.) 6.0808 + 5 + .19192 − 4.87078 − 4.78087.

---

137. *Multiplication of Decimals.*

Rule. Multiply the given decimals, as if they were common whole numbers, and mark off in the product as many decimal places as there are in the multiplier and multiplicand together.

Ex. 1. Multiply 1.0025 by 2.5.

$$\begin{array}{r} 10025 \\ 25 \\ \hline 50125 \\ 20050 \\ \hline 250625 \\ \hline 2.50625 \ \textit{Ans.} \end{array}$$

Here there are *four* decimal places in the multiplicand, and *one* in the multiplier; therefore, we mark off *five* in the product.

$$\text{For } 1.0025 \times 2.5 = \frac{10025}{10000} \times \frac{25}{10} = \frac{10025 \times 25}{100000} = \frac{250625}{100000} = 2.50625$$

Ex. 2. Multiply .0048 by .000012, and 1.005 × .005 × .0064.

$$\begin{array}{r} 48 \\ 12 \\ \hline 576 \\ .0000000576 \ \textit{Ans.} \end{array} \qquad \begin{array}{r} 1005 \\ 5 \\ \hline 5025 \\ 64 \\ \hline 20100 \\ 30150 \\ \hline 321600 \\ .0000321600 \end{array} \ \textit{Ans.} = .00003216.$$

Ex. 130.

(1.) 3.75 × 8. (2.) 37.5 × .8. (3.) .375 × .08.
(4.) 375 × .008. (5.) .0001 × 2.3. (6.) 21.25 × 1.4.
(7.) 5.002 × 1.5. (8.) 1.6 × 12.25. (9.) .1 × .2 × .3 × .4.
(10.) .01 × 2 × .003 × .4. (11.) 1.02 × 10.2 × .0102.
(12.) .017 × 5 × 1.3 × .0006.

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138. *Division of Decimals.*

RULE. Move the decimal point in the divisor to the right or left, as required, so as to bring it immediately after the last significant figure, and move the decimal point in the dividend (annexing ciphers if necessary) the same number of places (see **134**). Then divide as in the case of integers, annexing decimal ciphers to the dividend (see **135**) to any extent that may be required for carrying on the division; and point off in the quotient the same number of decimal places that there are in the dividend.

Ex. 1. Divide .805 by 2.3; or (moving the decimal point as directed in the rule) 8.05 by 23.

$$\begin{array}{r} 23)\ 8.05\ (.35 \\ 69 \\ \hline 115 \\ 115 \\ \hline \end{array}$$

Here there are two decimal places in the dividend: we mark off, *therefore*, two in the quotient.

Ans. .35.

$$\text{For } .805 \div 2.3 = \frac{805}{1000} \div \frac{23}{10} = \frac{805}{1000} \times \frac{10}{23} = \frac{805}{23} \times \frac{1}{100} = \frac{35}{100} = .35.$$

Ex. 2. Divide 2.22 by .37; or (moving the decimal point) 222. by 37.

$$\begin{array}{l}37)\ 222\ (6\\ \underline{222}\end{array}$$

Here there are no decimal places in the dividend, and therefore none in the quotient.

Ans. 6.

For $2.22 \div .37 = \frac{222}{100} \div \frac{37}{100} = \frac{222}{100} \times \frac{100}{37} = \frac{222}{37} = 6.$

Ex. 3. Divide 104.5 by .0011; or 1045000. by 11.

$$\begin{array}{r}11)\ \underline{1045000}\\ 95000\end{array}$$

For $104.5 \div .0011 = \frac{1045}{10} \div \frac{11}{10000} = \frac{1045}{10} \times \frac{10000}{11} = \frac{1045}{11} \times 1000 = 95000.$

Ex. 4. Hence, (taking the *figures* of the quotients from the above Examples,)

$$\frac{80.5}{2.3} \text{ or } \frac{805.}{23.} = 35,\quad \frac{.805}{23} = .035,\quad \frac{80.5}{.0023} \text{ or } \frac{805000.}{23.} = 35000.$$

$$\frac{.0222}{37} = .0006,\quad \frac{222}{.37} \text{ or } \frac{22200.}{37.} = 600,\quad \frac{2.22}{.037} \text{ or } \frac{2220.}{37.} = 60.$$

$$\frac{1.045}{1.1} \text{ or } \frac{10.45}{11.} = .95,\quad \frac{10.45}{.11} \text{ or } \frac{1045.}{11.} = 95.$$

Ex. 5. Divide 2.5 by .32; or 250. by 32.

$$\begin{array}{r}32)\ 250.0000\ (7.8125\\ \underline{224}\\ 260\\ \underline{256}\\ 40\\ \underline{32}\\ 80\\ \underline{64}\\ 160\\ \underline{160}\end{array}$$

Ans. 7.8125.

Here we have had to annex four decimal ciphers to the dividend: we therefore mark off four decimal places in the quotient.

Ex. 6. Divide 1 by .013; or 1000. by 13.

$$\begin{array}{r}13)\ 1000.000\ (76.923\\ \underline{91}\\ 90\\ \underline{78}\\ 120\\ \underline{117}\\ 30\\ \underline{26}\\ 40\\ \underline{39}\\ 1\end{array}$$

Ans. 76.923.

In this example the quotient will not terminate; but, by annexing decimal ciphers to the dividend, we may carry it on as far as we please. We have annexed three; therefore, we point off three decimal places in the quotient.

Obs. In the foregoing examples, when by moving the decimal point it has been brought after the last figure of the number, it has been inserted for distinctness sake, though in such cases it is usually omitted, the decimal in that case becoming simply an integer.

A little consideration will often save us the trouble of *counting* the decimal places of the dividend and divisor.

Ex. 7. Divide 15.95 by 2.75.

```
275) 15950 (58
     1375
      2200
      2200
Ans. 5.8.
```

Here, without counting, we may set at once the point after the 5 in the quotient, because it is plain that the divisor, whose whole number is 2, will go somewhere about 5 times in the dividend.

Ex. 131.

(1.) 56.25÷7.5; .05625÷.0075; 5.625÷.0000075; .05625÷.75.
(2.) 723.6÷22.5; 7.236÷.0225.
(3.) 1.464561÷44.1; 14645.61÷4.41.
(4.) .74151÷.0321; .074151÷321.
(5.) 75.04÷.032; .07504÷32.
(6.) .0602÷2; 60.2÷.002.
(7.) 123.4321÷1.01; 1234321÷10.1.
(8.) 2÷125; 200÷12.5. (9.) 7÷.0625; .007÷6.25.
(10.) 3.1÷.13. (11.) 22÷.0017. (12.) 1.01÷119.

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139. *To reduce any fraction to a decimal.*

Rule. If the denominator be 10, 100, &c., we may at once express it as a decimal. But, in other cases, if 10, 100, &c. be a *factor* of the denominator, divide the numerator by it first, and then divide the numerator as it now stands by the remaining factor (annexing ciphers to the numerator, if necessary), and the result will be the decimal required.

Ex. 1. $\frac{1}{400}=\frac{.01}{4}=.0025.$ Ex. 2. $\frac{37}{80}=\frac{3.7}{8}=.4625.$

Ex. 132.

Reduce to decimals

(1.) $\frac{3}{25}$. (2.) $\frac{13}{50}$. (3.) $\frac{47}{8}$. (4.) $\frac{11}{125}$. (5.) $\frac{100}{625}$. (6.) $7\frac{13}{16}$.
(7.) $\frac{7}{8}$ of $\frac{15}{16}$. (8.) $\frac{4}{5}$ of $\frac{24}{25}$. (9.) $5\frac{1}{16}$ of $2\frac{2}{5}$. (10.) $1\frac{1}{2}$ of $1\frac{1}{4}$ of $1\frac{1}{8}$.
(11.) $\frac{1}{3}$ of $2\frac{3}{5}$ of $\frac{3}{16}$. (12.) $2\frac{1}{3}$ of $3\frac{1}{4}$ of $4\frac{1}{5}$.

---

140. Sometimes the division will be found not to terminate, but the *same* figures will be repeated over again continually in the quotient.

Ex. Reduce $\frac{95}{90}=\frac{9.5}{9}$, $\frac{3}{1100}=\frac{.03}{11}$, and $\frac{4}{7}$, to decimals.

$$9)\underline{9.50000} \qquad 11)\underline{.03000000} \qquad 7)\underline{4.0000000}$$
$$\underline{1.05555}\ \&c. \qquad \underline{.00272727}\ \&c. \qquad \underline{.5714285}\ \&c.$$

Decimals of this kind, in which the same figures are repeated without end continually, are called *Circulating*, *Repeating*, or *Recurring* Decimals; and the part repeated is called the *Period* or *Repetend*.

It is usual to express any circulator by writing it down to the end of the first period, and setting dots over the first and last figures of the period; which dots will, of course, be on adjacent figures, when the period consists of only *two* figures, and will coalesce into one dot, when it consists of only one figure.

Thus the above results would be written $1.0\dot{5}$, $.00\dot{2}\dot{7}$, $.\dot{5}7142\dot{8}$.

A *pure* circulator is one in which the period begins immediately after the point; all others are called *mixed* circulators.

141. It may be shewn (see Colenso's *Arithmetic*, pp. 63, 64), that when the denominator of any fraction, *reduced to its lowest terms*, contains only *powers* of 2 or 5, or both of them, as factors (where by a *power* of 2 is meant any number formed by multiplying 2 by itself any number of times, such as 2 × 2 or 4, 2 × 2 × 2 or 8, &c.), then any such fraction can always be reduced to a *finite* or *terminating* decimal.

But if the denominator contains any factors, besides 2, or 5, or their powers, (such as 3, 7, 9, 11, for instance), then the corresponding decimal will be sure to circulate, and the number of figures in the period *may* be as large as one less than the denominator, but cannot be larger, and will, generally, be much less.

Thus, $\frac{6}{7}=.\dot{8}5714\dot{2}$, where there are 6 figures in the period;

$\frac{1}{19}=.\dot{0}5263157894736842\dot{1}$, where there are 18 figures;

but $\frac{3}{22}=.1\dot{3}\dot{6}$, $\frac{1}{13}=.\dot{0}7692\dot{3}$, &c.

Ex. 133.

Reduce to decimals

(1.) $\frac{8}{9}$. (2.) $\frac{111}{180}$. (3.) $\frac{27}{22}$. (4.) $\frac{6}{55}$. (5.) $\frac{23}{14}$. (6.) $\frac{101}{110}$.
(7.) $\frac{110}{111}$. (8.) $3\frac{3}{440}$. (9.) $\frac{20}{21}$. (10.) $\frac{103}{13}$. (11.) $\frac{105}{99}$. (12.) $\frac{119}{28}$.

142. *To reduce a pure circulator to a fraction.*

Since $\frac{1}{9}=.111111$ &c., it follows that $\frac{2}{9}=.2222$ &c., $\frac{5}{9}=.5555$ &c.; so that any pure circulator, having *one* figure in the period, may be expressed as a fraction with that figure in the numerator and 9 in the denominator.

Again $\frac{1}{99}=\frac{1}{9}\div 11=.111111$ &c. $\div 11=.010101$ &c.; hence $\frac{2}{99}=.020202$ &c., $\frac{7}{99}=.070707$ &c., $\frac{53}{99}=.535353$ &c.; so that any pure circulator, having *two* figures in the period, may be expressed as a fraction with those figures in the numerator, and 99 in the denominator.

In like manner, since

$\frac{1}{999}=\frac{1}{9}\div 111=.001001$ &c., $\frac{1}{9999}=\frac{1}{9}\div 1111=.00010001$ &c., and so on, it will follow that any pure circulator whatever may be expressed as a fraction, with the period itself in the numerator, and in the denominator as many 9's as there are figures in the period.

Ex. $.\dot{3}7\dot{8}=\frac{378}{999}=\frac{14}{37}$, $.\dot{0}37\dot{8}=\frac{378}{9999}=\frac{42}{1111}$, &c.

143. *To reduce a mixed circulator to a fraction.*

If we have a pure circulator with any figures *before* the dot, we may either keep these to form a mixed number, as $3.\dot{4}=3\frac{4}{9}$, $5.\dot{4}\dot{3}=5\frac{43}{99}$, or (as will be seen) we may bring the whole to an improper fraction, with the same denominator as before, by writing for the numerator all the figures to the end of the first period, *subtracting the figures before the point.*

Ex. $3.\dot{4}=\frac{34-3}{9}=\frac{31}{9}$; $5.\dot{4}\dot{3}=\frac{543-5}{99}=\frac{538}{99}$.

For $3\frac{4}{9}=\frac{3\times 9+4}{9}=\frac{3(10-1)+4}{9}=\frac{30+4-3}{9}=\frac{34-3}{9}=\frac{31}{9}$;

$5\frac{43}{99}=\frac{5\times 99+43}{99}=\frac{5(100-1)+43}{99}=\frac{500+43-5}{99}=\frac{543-5}{99}=\frac{538}{99}$; &c.,

and so we may reason in all similar cases.

Now suppose the point moved any number of places to the left: this is equivalent to dividing the decimal by 10, 100, &c., and we must therefore annex to the denominator, as already found, as many ciphers as there are figures now brought between the point and the first circulating figure.

Ex. $.03\dot{4}=\frac{31}{900}$; $.5\dot{4}\dot{3}=\frac{538}{990}=\frac{269}{495}$; $.62\dot{7}=\frac{627-62}{900}=\frac{565}{900}=\frac{113}{180}$.

If there should be any figures of a mixed circulator still left standing before the point, it will be best to leave these, as they are to form a mixed number.

Ex. $2.4\dot{6}=2\frac{46-4}{90}=2\frac{42}{90}=2\frac{7}{15}$, the same as $\frac{246-24}{90}=\frac{222}{90}=2\frac{42}{90}$.

The above results may be thus stated, as a rule for reducing any circulator to a fraction:

Consider only the figures *after* the point;

For the numerator, write down the decimal to the end of the first period, subtracting from it (if any) the figures which do *not* circulate;

For the denominator, write as many 9's as there are figures circulating, followed by as many 0's as there are figures not circulating.

Ex. 134.

Reduce to vulgar fractions

(1.) $.\dot{9}$. (2.) $2.0\dot{3}$. (3.) $3.0\dot{1}\dot{5}$. (4.) $4.0\dot{2}\dot{7}$.
(5.) $.003\dot{6}$. (6.) $5.\dot{0}90\dot{9}$. (7.) $1.04\dot{4}\dot{9}$. (8.) $3.9\dot{8}\dot{4}$.
(9.) $.306\dot{8}\dot{1}$. (10.) $17.30\dot{5}$. (11.) $.0291\dot{6}$. (12.) $4.\dot{8}5714\dot{2}$.

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144. Arithmetical operations in which circulating decimals are concerned, may often be performed, with sufficient accuracy for all practical purposes, by repeating the periods several times.

Ex. 1. Add together $13.\dot{5}$, $2.0\dot{2}\dot{5}$, $111.00\dot{4}$, $2.\dot{0}2\dot{4}$, $3.\dot{1}4159$, correctly to *four* decimal places.

```
 13.555555
  2.025252
111.004444
  2.024024
  3.141594
----------
131.750869
```

Here the last two figures are not correct; but as far as four decimal places the Answer is the true one, namely, 131.7508.

In such a case, it is generally sufficient to carry out the periods to *two* or *three* places more than the number required to be accurate.

Ex. 2. From $1.02\dot{3}4\dot{1}$ take $.62\dot{8}$, correct to 4 places.

```
1.023413
 .628888
--------
 .394525
```

Ans. .3945.

It is sometimes best to reduce the circulators to fractions, especially for the purpose of *multiplying* or *dividing* one circulator by another; and then the resulting fraction, after the multiplication or division, may be reproduced in the decimal form.

Ex. 3. $.2\dot{3}\times.\dot{3}\dot{6}=\frac{2}{90}\times\frac{36}{99}=\frac{14}{165}=.0\dot{8}\dot{4}$.

Ex. 4. $.1\dot{6}\div.00\dot{2}\dot{7}=\frac{15}{90}\div\frac{27}{9900}=\frac{550}{9}=61.1$.

Ex. 135.

Find the value, correct to *four* places of decimals, of

(1.) $7.\dot{4} + .0\dot{3}\dot{7} + 5.\dot{6}1\dot{9} + 23.\dot{2}3\dot{7} + .\dot{0}23\dot{5}$.
(2.) $29.\dot{3} + 13.\dot{3}17\dot{7} + .\dot{2}08\dot{9} + 17.\dot{0}7\dot{6} + 1.5$.
(3.) $13.\dot{5}0\dot{2} - 7.\dot{3}$. (4.) $2.0\dot{2}3\dot{5} - 1.1\dot{6}$.
(5.) $23.0\dot{7}\dot{8} - 19.\dot{1}4\dot{4}$.
(6.) $37.\dot{2}9\dot{8} + 21.0\dot{6}3\dot{7} + 19.\dot{2}3\dot{4} - 11.4\dot{7} - .6\dot{5}3\dot{8}$.

Find the value of

(7.) $3.\dot{5} \times .8$. (8.) $3.5. \times .0\dot{8}$. (9.) $3.5 \times .\dot{0}\dot{8}$.
(10.) $.\dot{3} \times .\dot{2}\dot{7}$. (11.) $3.\dot{6} \div 2.4$. (12.) $3.6 \div 2.\dot{4}$.

---

145. *To find the value of any decimal of a given quantity.*

RULE. As in common Reduction, multiply the given decimal by the number of units of the next lower denomination, which make one of the given denomination; the *integral* part (if any) of the product so obtained will be so many units of the lower denomination, and the *fractional* part may now be reduced in the same manner to a lower denomination: and so on.

Ex. 1. Find the value of £.36875, and 3.215 of a ton.

£.36875
20
7.37500*s.*
12
4.500*d.*
4
2.0*f.*

Ans. 7*s.* 4½*d.*

3.215 tons
20
4.300 cwt.
4
1.2 qrs.
28
5.6 lbs.
16
9.6 oz.

Ans. 3 tons 4 cwt. 1 qr. 5 lbs. 9 oz. (+)

If the given quantity be expressed in more than one denomination, it should be reduced to one denomination, before applying the rule.

Ex. 2. Find the value of .07 of £2 10*s.*, and of .7365 of 1 cwt. 10 lbs.

Here £2 10*s.* = 50*s.*
.07
50
3.50*s.*
12
6.00*d.*

Ans. 3*s.* 6*d.*

And 1 cwt. 10 lbs. = 122 lbs.
.7365
122
14730
14730
7365
89.8530 lbs.
16
51180
8530
13.6480 oz.

Ans. 89 lbs. 13 oz. (+)

It is often best, as in (**144**), to express a circulator in such a case by means of its equivalent fraction, and so find its value.

Ex. 3. Find the value of $3.2\dot{7}$ of a ton.

$3.2\dot{7}=3\frac{25}{90}=3\frac{5}{18}$; and $3\frac{5}{18}$ of a ton $=3$ tons 5 cwt. 2 qrs. 6 lbs. $3\frac{5}{9}$oz.

Ex. 136.

Find the value of

(1.) £.75. (2.) £.13125. (3.) £6.975. (4.) £3.46875.
(5.) 1.1625 of 5*s*. (6.) .58125 of 6*s*. 8*d*.
(7.) 1.74375 of 13*s*. 4*d*. (8.) 43.725 of 2*s*. 6*d*.
(9.) $3.\dot{2}$ of £6. (10.) $2.3\dot{6}$ of £6 1*s*. 3*d*.
(11.) $3.\dot{3}4\dot{5}$ of £4 11*s*. 8*d*. (12.) $2.0\dot{1}3\dot{6}$ of £2 1*s*. 3*d*.

Ex. 137.

Find the value of

(1.) .975 of a ton. (2.) .2625 of a mile.
(3.) $3.2\dot{3}$ of $1\frac{1}{2}$ acre. (4.) 3.125 of a cub. yd.
(5.) .1875 of a bushel. (6.) 3.1725 of a day.
(7.) $2.3\dot{2}$ of 2 cwt. 1 qr. 18 lbs. (8.) 1.125 of 7 yds. 1 ft.
(9.) 1.4125 of 10 sq. yds. (10.) 1.625 of $3\frac{1}{2}$ cub. yds.
(11.) $3.\dot{6}$ of 4 qrs. 4 bus. (12.) 21.325 of a year.

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146. *To reduce a given quantity to the decimal of another given quantity.*

RULE. Begin with the term of lowest denomination in the first given quantity, and reduce it to a decimal of the next higher denomination.

Prefix to this decimal the term (if any) of this higher denomination, which is found in the first given quantity, and reduce the result to a decimal of the next higher denomination: and so on.

Ex. 1. Reduce £3 17*s*. $6\frac{3}{4}$*d*. to the decimal of £5.

4) 3.00
12) 6.7500
20) 17.562500
5) 3.878125
.775625

Here we first reduce $\frac{3}{4}$*d*. to the decimal of 1*d*.; the result is .75, that is, $\frac{3}{4}$*d*. = .75*d*.; we now prefix the 6*d*., and reduce 6.75*d*. to the decimal of 1*s*.; and so on.

Sometimes we cannot thus pass directly through successive denominations from the first to the second given quantity, and then it will be necessary to express the first as a fraction of the second, and then to reduce this fraction to a decimal.

Ex. 2. Reduce 2*s.* $9\frac{3}{4}$*d.* to the decimal of 7*s.* $9\frac{3}{4}$*d.*

Here $\frac{2s.\ 9\frac{3}{4}d.}{7s.\ 9\frac{3}{4}d.}=\frac{135f.}{375f.}=\frac{9}{25}=\frac{36}{100}=.36.$

Ex. 138.

Express

(1.) 3*s.* 9*d.* as a decimal of £1.
(2.) 6*s.* $6\frac{3}{4}$*d.* „ „ £10.
(3.) 2*s.* 6*d.* „ „ 6*s.* 8*d.*
(4.) 8*s.* $7\frac{1}{2}$*d.* „ „ 15*s.*
(5.) £1 13*s.* 9*d.* „ „ 10*s.*
(6.) 7*s.* $10\frac{1}{2}$*d.* „ „ 13*s.* 4*d.*
(7.) 9*s.* $11\frac{1}{4}$*d.* „ „ 13*s.* 3*d.*
(8.) £1 12*s.* 1*d.* „ „ 1*s.* 2*d.*
(9.) £1 11*s.* $1\frac{1}{2}$*d.* „ „ £2
(10.) 6*s.* 8*d.* „ „ 5*s.*
(11.) £5 3*s.* $1\frac{1}{2}$*d.* „ „ £3
(12.) £12 6*s.* $3\frac{1}{2}$*d.* „ „ £1 10*s.*

Ex. 139.

Express

(1.) 4 oz. 9 dr. as a decimal of 1 lb.
(2.) 1 fur. 11 yds. „ „ 1 mile.
(3.) 1 sq. ft. 18 in. „ „ 1 sq. yd.
(4.) 8 cub. ft. „ „ 1 cub. yard.
(5.) 3 gals. 1 qt. „ „ 1 bushel.
(6.) 1 d. 1 hr. 50′ 42″ „ „ 1 day.
(7.) 3 cwt. 2 qrs. 14 lbs. „ „ 5 tons.
(8.) 7 inches „ „ 1 furlong.
(9.) 1R. 2P. „ „ half an acre.
(10.) 7 c. ft. 100 in. „ „ 1000 c. in.
(11.) 5 qrs. 3 bus. „ „ 1 coom.
(12.) 9 d. 3 h. „ „ 1 week.

Ex. 140.

(1.) Add together three and thirteen-thousandths, two and five-hundredths, seventy-five ten-thousandths, and a hundred and nineteen millionths.

(2.) What must be added to 1, .3, .13, and .013, to make 13?

(3.) The globe has .7614 of its surface covered with water. What portion of the surface is land?

(4.) Atmospheric air consists of nitrogen and oxygen. Out of 100 cub. ft., 79.19 c. ft. are nitrogen. How many are oxygen? Express the quantities of each in cubic feet and inches.

(5.) There are 120 children in a school. The first class contained 15, the second 16, the third 24, the fourth 40, and the rest made up the fifth class. Express the numbers in each class as decimals of the whole.

(6.) What decimal was the *first* class in (5) of the *last?* and what the *last* of the *first?*

(7.) The highest mountain in the world is Dhawalagiri, in the Himalayas, which is 28000 feet above the level of the sea. Express this as a decimal of the earth's radius, 4000 miles.

(8.) A grocer bought tea at 3*s.* 6*d.* per lb., and sold it at 4*s.* 6*d.* per lb. Express his profit as a decimal, (i.) of the *selling* price, (ii.) of the *cost* price.

(9.) Find the value of .2 of .3 of .4 of £.5.

(10.) What decimal of £1 is a guinea? and what decimal of a guinea is £1?

(11.) In a parish of 450 people, .44 of the whole were men and boys, .38 were women and girls, and the rest were children. How many were there of each?

(12.) A shilling is .0568 inches thick. What is the thickness of a pile of 500 shillings?

(13.) There is a number, which, when multiplied by 2.5 and divided by 5.2, produces 1. Find it.

(14.) If a pound of sugar cost .03515625 of 32*s.*, what is the value of .3125 cwt.?

(15.) There are 360 children in a school. What are the numbers corresponding, respectively, to .6, $.\dot{6}$, and $.3\dot{6}$ of the whole?

(16.) From a lb. Troy of standard gold are coined 46.725 sovereigns. Find the weight in grains of each sovereign.

(17.) If a bag of gold weighs 10 lbs. 8 oz. 8 dwts. 5 grs., and each sovereign weighs 123.274 grains, how many were there in the bag?

(18.) The circumference of any circle is usually taken as $3\frac{1}{7}$ × its diameter. But, more accurately, it should be 3.14159 × its diameter. What is the difference, expressed decimally, between these two multipliers? And what length of string should I want, on each supposition, to go round a circle that is 875 yards across through the centre?

(19.) Show that the fraction $\frac{355}{113}$ differs very little from the true value of the multiplier in (18).

(20.) What space does the extremity of the minute-hand of a school-clock travel over in a day, reckoning its length as $4\frac{1}{2}$ inches, and using the true multiplier?

(21.) A tree measured $3\frac{1}{2}$ feet across the centre of its section: and the area of a circle $= 3.14159 \times$ area of the square upon its radius. What amount of error should I make, in calculating the superficial measure of the circular section, if I take the multiplier $3\frac{1}{7}$, instead of 3.14159?

(22.) The imperial gallon contains 277.274 cubic inches. What is the size of a pint pot in cubic measure? and what is the weight of water it would hold, if one cubic foot of water weighs 1000 ounces?

(23.) The true length of a year is 365.242218 days. Express this in days, hours, minutes, and seconds.

(24.) If *every* year were reckoned as 365 days, how many days' error would arise in 100 years? And, if *every* fourth year were kept as a leap year, what would be the error in 400 years? Show that this error may be obviated by *not* keeping as leap year the years which end the centuries, when the number of centuries is not divisible by 4; and that the error still left will amount to more than a day in 4000 years.

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147. The *square* or *square power* of any number is that number multiplied by itself.

Ex. 1. The square of 1 is $1 \times 1 = 1$, of 2 is $2 \times 2 = 4$, of 3 is $3 \times 3 = 9$, &c.

Ex. 2. The square of $\frac{1}{2}$ is $\frac{1}{2} \times \frac{1}{2} = \frac{1}{4}$, of $5\frac{1}{2}$ is $\frac{11}{2} \times \frac{11}{2} = \frac{121}{4} = 30\frac{1}{4}$.

Ex. 3. The square of .1 is $.1 \times .1 = .01$, of 1.05 is $1.05 \times 1.05 = 1.1025$.

The *cube* or *cube power* of any number is that number multiplied twice by itself.

Ex. 1. The cube of 1 is 1, of 2 is 8, of 3 is 27, of 10 is 1000.

Ex. 2. The cube of $\frac{1}{4}$ is $\frac{1}{4} \times \frac{1}{4} \times \frac{1}{4} = \frac{1}{64}$, of $2\frac{1}{5}$ is $\frac{11}{5} \times \frac{11}{5} \times \frac{11}{5} = \frac{1331}{125} = 10\frac{81}{125}$.

Ex. 3. The cube of .1 is $.1 \times .1 \times .1 = .001$, of 1.2 is $1.2 \times 1.2 \times 1.2 = 1.728$.

The square and cube of any number are called also, respectively, the *second* and *third* powers of the number; and in like manner we might go on to form the *fourth* power, the *fifth* power, &c. by mere multiplication.

148. The *square root* or *cube root* of any number is that number whose square or cube power is equal to the given number.

Ex. 1. The square root of 1 is 1, because $1 \times 1 = 1$;
„ of 4 is 2, because $2 \times 2 = 4$;
„ of 16 is 4, because $4 \times 4 = 16$;
„ of $2\frac{7}{9}$ or $\frac{25}{9}$ is $\frac{5}{3}$ or $1\frac{2}{3}$, because $\frac{5}{3} \times \frac{5}{3} = \frac{25}{9}$;
„ of .04 is .2, because $.2 \times .2 = .04$.

Ex. 2. The cube root of 1 is 1, of 8 is 2, of 27 is 3, &c.
„ of $\frac{8}{27}$ is $\frac{2}{3}$, of $3\frac{3}{8}$ or $\frac{27}{8}$ is $\frac{3}{2}$ or $1\frac{1}{2}$, of .125 is .5.

In the above examples it was easy to discover the square and cube roots required, because very simple numbers were concerned; but in other cases it requires the application of rules which, though easy enough in practice, require some knowledge of algebra to understand the reason of them. We shall here explain the methods of these two Rules, and refer for the proof of them to Colenso's Algebra, part i. pp. 37 and 43.

149. *Extraction of the Square Root.*

Ex. 1. Find the square root of 186624, 77841, 10291264.

186624 (432	77841 (279	10291264 (3208
16	4	9
83) 266	47) 378	62) 129
249	329	124
862) 1724	549) 4941	6408) 51264
1724	4941	51264

Here we first place a dot over the last figure, and then over every *second* figure, reckoning from it. By this means the given number will be divided into *periods* of two figures each, except the first, which (when the number of figures in the given number is *odd*) will evidently consist of only one figure.

We then look for the nearest square number not greater than the first period. This is 16, in the first of the above examples; and we set its square root, 4, as the first figure in the root.

We then subtract its square, 16, and bring down the next period, 66.

We now set the double of the first figure in the root, 8, in a loop, as divisor, to the left of the remainder, regarding it, however, as standing for 80, not for 8, since we shall presently have to set another figure after it. Dividing the remainder by this divisor, 80, we set the quotient, 3, as the second figure both in the root and also in the divisor.

Then multiplying the 83 by 3, we subtract the product, and take down the remaining period, 24.

To form the next divisor, we double the last figure of the preceding one, making 86, which (as before) we regard as 860, and proceed exactly in the same manner; and if finally, as here, we find there is *no* remainder, we may conclude that we have found the exact square root.

N.B. In the second of the above examples, observe (i.) that the second remr. 49 is greater than the divr. 47: this may sometimes happen, but no difficulty can arise from it, as it will be found that if, instead of 7, we took 8 for the second figure, the subtrahend would be 384, which is too large; (ii.) that the last figure 7 of the first divisor, being doubled in order to make the second divisor, and thus becoming 14, causes 1 to be added to the preceding figure, 4, which now becomes 5.

In the third example we have the case of a cipher in the root.

Ex. 2. Extract the square roots of 1000, 2, 1.6, .002.

$1\dot{0}0\dot{0}.0\dot{0}$ (31.6 &c.	$\dot{2}$ (1.41 &c.	$\dot{1}.6\dot{0}$ (1.26 &c.	$.0\dot{0}2\dot{0}$ (.0447 &c.
9	1	1	16
61) 100	24) 100	22) 60	84) 400
61	96	44	336
626) 3900	281) 400	246) 1600	887) 6400
3756	281	1476	6209
144	119	124	191

In the first of these examples we find a remainder, 39, when we have made use of the last period of the given number, 1000.

But we may continue the operation as long as we please in such a case, by setting a decimal point after the given number, and annexing ciphers as decimal places; and, for every period of two ciphers thus formed, we shall obtain a decimal figure in the root.

The same is true of the second example, except that we have not taken the trouble to set down the extra ciphers at the end of the given number, though we have taken them down as required, and set the decimal point in the root.

In the third example, it is to be noticed that the first dot must always be placed on the last figure of the *integral* part of any number, that is, *on the figure next before the dot*, and then upon every second figure on each side of it. In the fourth example, this integral figure, over which the first dot is to be put, is a cipher, and is omitted. In both these instances, we have had to annex a cipher to the original decimal, in order to complete its points.

In all such cases as the above, where a remainder is left, the square root can never be exactly obtained; though, by annexing pairs of ciphers, it may be ascertained to as many decimal places as we please. Such roots are called in algebra *irrational* or *surd* roots.

Ex. 141.

Obtain the square roots of

(1.) 229441. (2.) 917764. (3.) 20649.69.
(4.) 11242609. (5) .257049. (6.) 1028196.
(7.) 231.3441. (8.) 12595401.

(9.) Extract, to five figures, the square roots of 2 and .2̇.

(10.) Extract, to five figures, the square roots of 30 and .03̇.

(11.) Find the side of a square field, which shall contain exactly an acre.

(12.) The area of a circle being 3.14159 times the square upon its radius, what should be the radius of a circle, to contain exactly an acre of ground?

150. *Extraction of the Cube Root.*

Ex. Find the cube root of 80677568161.

		80̇677̇568̇161̇ (4321
		64
123	4800	16677
	369	
	5169	15507
1292	554700	1170568
	2584	
	557284	1114568
12961	55987200	56000161
	12961	
	56000161	56000161

Here we first divide the number into periods by placing a dot over the last figure, and then over every *third* figure beginning from it. We then look for the nearest cube number not greater than the first period 80. This is 64, and we set its cube root, 4, as the first figure in the root. Then we subtract its cube 64, and bring down the next period, 677̇.

We now set the *triple* of the first figure of the root, 12, at some distance to the left of the remainder; (there is 123 in the sum, but the 3 will be accounted for by and by;) then we multiply this triple by the first figure of the root, and place the product 48 between the 12 and the remainder, annexing two ciphers to it.

We now divide the remainder by this 4800, and set the quotient, 3, as the second figure in the root, and *also* after the 12, making 123. Now we multiply this 123 by 3, the second figure in the root, set the product 369 under 4800, add them up, multiply the sum 5169 by the second figure in the root, and subtract the product, 15507. We bring down the next period, 568, and have now to form the two quantities to the left of it.

The first is obtained by tripling the last figure, 3, of 123, which thus becomes 129; (the final 2 in 1292 will be accounted for, when the next figure in the root is found); and the other quantity, 5547, is found by adding 9, the square of the second figure in the root, to the two preceding middle lines, $\begin{matrix}369\\5169\end{matrix}$. We now add two ciphers, and repeat the whole process described in this paragraph.

The remarks made above with respect to pointing the last figure of the integer, and also with respect to *surd* square roots, apply also to cube roots. Thus .01, 24.1 would be pointed for the cube root of $.01\dot{0}$, $2\dot{4}.10\dot{0}$.

Ex. 142.

Obtain the cube roots of

(1.) 16194277. (2.) 138188413. (3.) 437.245479.
(4.) 2024284.625. (5.) 363994344. (6.) 2911954752.
(7.) 1.105507304. (8.) 308915776.

(9.) Extract, to four figures, the cube roots of 20 and .002.

(10.) Extract, to four figures, the cube roots of 300 and .3.

(11.) What must be the length of a cubical box, which shall have room for 10 cubic feet?

(12.) What must be the side of a cubical cistern, which shall contain exactly 1000 gallons of water, if a gallon contains 277.274 cubic inches?

---

151. In *Decimal* Arithmetic, the value of any figure is increased *ten*-fold by its being moved one place to the left, or diminished *ten*-fold by its being moved one place to the right.

In *Duodecimal* Arithmetic, the value of any figure is increased or diminished *twelve*-fold by its being moved one place to the left, or right, respectively. So that 12 is, to numbers which are written in duodecimal notation, just exactly what 10 is to common numbers.

Ex. 1. In common numbers, 37.5 denotes 3 *tens*+7 *ones*+5 *tenths*; but, in duodecimal notation, 37.5 would denote 3 *twelves*+7 *ones*+5 *twelfths*.

Ex. 2. In common numbers, 8765 would denote $8\times1000+7\times100+6\times10+5$; or, since 100 is the *square* of 10, 1000 the *cube* of 10, &c., if we denote the square (or second power) of 10 by 10^2, the cube (or third power) of 10 by 10^3, and so on, we may say that, in common numbers, 8765 denotes

$$8\times10^3+7\times10^2+6\times10+5:$$

but, in duodecimal notation, 8765 would be used to denote

$$8\times12^3+7\times12^2+6\times12+5,$$

which, in common numbers, is equivalent to

$$8\times1728+7\times144+6\times12+5=14909.$$

152. From the last example it is seen, that it is very easy to convert a duodecimal whole number to a common number. But one remark must here be made. In order to write down any number in the decimal notation, we require the use of the *nine digits and zero*, that is, we require symbols for the *nine* numbers less than 10, and for zero, and for these we use the figures 1, 2, 3, 4, 5, 6, 7, 8, 9, 0. So, likewise, in the duodecimal notation, we shall require symbols for the *eleven* numbers less than 12 and for zero: and, as there is no symbol in common use for the numbers 10 and 11, it is usual to represent them by the letters *t* and *e*.

Ex. The duodecimal number *te*10 means

$$10 \times 12^3 + 11 \times 12^2 + 1 \times 12 + 0 = 10 \times 1728 + 11 \times 144 + 1 \times 12 + 0 = 18876.$$

153. So, too, a common number may be easily converted to a duodecimal, as follows:—

Ex. Reduce 67890 to a duodecimal.

$$\begin{array}{r l} 12)\,\underline{67890} & \\ 12)\,\underline{5657} & \ldots\ 6 \\ 12)\,\underline{471} & \ldots\ 5 \\ 12)\,\underline{39} & \ldots\ 3 \\ \underline{3} & \ldots\ 3 \end{array}$$

Here, dividing by 12, we find there are 5657 *twelves* in the given number, and 6 *units* over. Then, dividing this number of twelves by 12, we find that there are 471 *hundred-and-forty-fours* in the number, and 5 *twelves* over: and so on.

Ans. 33356 duodecimal; which, in common numbers,

$$\begin{aligned} &= 3 \times 12^4 + 3 \times 12^3 + 3 \times 12^2 + 5 \times 12 + 6 \\ &= 3 \times 20736 + 3 \times 1728 + 3 \times 144 + 5 \times 12 + 6 \\ &= 62208 + 5184 + 432 + 60 + 6 = 67890, \text{ as it should be.} \end{aligned}$$

Ex. 143.

Reduce to decimal or common numbers the following duodecimals:

(1.) 10. (2.) 100. (3.) 1000.
(4.) 1853. (5.) teet. (6.) eeee.

Reduce to duodecimals the following decimals:

(7.) 10. (8.) 100. (9.) 1000.
(10.) 1853. (11.) 8998. (12.) 9999.

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154. Our examples have hitherto been only of whole numbers; but fractions are also used in duodecimals, as well as in common numbers.

Ex. In common numbers, .359 would be used to denote $\frac{3}{10} + \frac{5}{100} + \frac{9}{1000}$ or $\frac{3}{10} + \frac{5}{10^2} + \frac{9}{10^3}$; and so, in duodecimals, .359 would be used to denote $\frac{3}{12} + \frac{5}{12^2} + \frac{9}{12^3}$.

Fractions may also be converted from one scale to another, without any real difficulty but, as the process is sometimes a little more complicated, and not much required in practice, we shall only illustrate it here by an example of each kind.

Ex. 1. $.359 \text{ (duodecimal)} = \frac{3}{12} + \frac{5}{144} + \frac{9}{1728} \text{ (decimal)},$

$$= \frac{3 \times 144 + 5 \times 12 + 9}{1728} = \frac{502}{1728} = \frac{251}{864}.$$

We have here added the three fractions together, by the usual process of finding their L. C. D., 1728; but it would be simpler to reason thus:

$$.359 \text{ (duod.)} = \frac{359}{1000} \text{ (duod.)} = \frac{3 \times 144 + 5 \times 12 + 9}{1728} \text{ (dec.)} = \frac{502}{1728}.$$

Here we have only converted the numerator and denominator of the duodecimal fraction to common numbers.

Ex. 2. $.379 \text{ (dec.)} = \frac{379}{1000} \text{ (dec.)} = \frac{257}{6e4}$, by the process of **153**,

$= .4315$ &c., by Ex. 4 of **155**.

155. It will be very useful to observe that, whereas in decimals, $\frac{1}{2}$ is expressed by .5, in duodecimals $\frac{1}{2}$ or $\frac{6}{12}$ is expressed by .6; so $\frac{1}{4} = \frac{3}{12} = .3$, $\frac{3}{4} = \frac{9}{12} = .9$, $\frac{1}{6} = \frac{2}{12} = .2$, &c.

156. Duodecimals are to be added, subtracted, &c., just like other numbers, only observing to borrow and carry 12, instead of 10, when necessary.

Ex. 1.

$$\begin{array}{r} 73t4 \\ 206e \\ t321 \\ te89 \\ \hline 26541 \end{array}$$

The first column $= 9+1+e+4 = 25$ (dec.), and 12 is in 25 *twice*, and 1 over; we set down 1, and carry 2; then $2+8+2+6+t = 28$ (dec.), and 12 is in 28 *twice*, and 4 over; we set down 4, and carry 2; and so on.

Ex. 2.

$$\begin{array}{r} 660452 \\ 271e43 \\ \hline 3tt50e \end{array}$$

Here 3 from 2, I cannot: I borrow 1, that is, 1×12, or 12; then 3 from 12 leaves 9, and 2 (of the upper line) makes 11; I set down *e*, and carry 1; and so on.

Ex. 3.

$$\begin{array}{r} e3t564 \\ e4t \\ \hline 9728734 \\ 3935t14 \\ t467098 \\ \hline t91594474 \end{array}$$

Ex. 4.

$$\begin{array}{r|l} 6e4) & 257.0000 \; (.4315 \text{ \&c.} \\ & \underline{2394} \\ & 1980 \\ & \underline{18t0} \\ & t00 \\ & \underline{6e4} \\ & 3080 \\ & \underline{2t88} \\ & 1e4 \end{array}$$

157. In consequence of the *foot* containing 12 inches, duodecimals are often convenient for workmen in measuring their work. But, besides the foot, the *inch* also is supposed, for this purpose, to be divided into 12 equal parts, called *seconds*, and marked ′, the *second* into 12 *thirds*, marked ″, &c.;

(9.) Extract, to five figures, the square roots of 2 and .2.

(10.) Extract, to five figures, the square roots of 30 and .03.

(11.) Find the side of a square field, which shall contain exactly an acre.

(12.) The area of a circle being 3.14159 times the square upon its radius, what should be the radius of a circle, to contain exactly an acre of ground?

150. *Extraction of the Cube Root.*

Ex. Find the cube root of 80677568161.

		$8\dot{0}67\dot{7}56\dot{8}16\dot{1}$	(4321
		64	
123	4800	16677	
	369		
	5169	15507	
1292	554700	1170568	
	2584		
	557284	1114568	
12961	55987200	56000161	
	12961		
	56000161	56000161	

Here we first divide the number into periods by placing a dot over the last figure, and then over every *third* figure beginning from it. We then look for the nearest cube number not greater than the first period 80. This is 64, and we set its cube root, 4, as the first figure in the root. Then we subtract its cube 64, and bring down the next period, $67\dot{7}$.

We now set the *triple* of the first figure of the root, 12, at some distance to the left of the remainder; (there is 123 in the sum, but the 3 will be accounted for by and by;) then we multiply this triple by the first figure of the root, and place the product 48 between the 12 and the remainder, annexing two ciphers to it.

We now divide the remainder by this 4800, and set the quotient, 3, as the second figure in the root, and *also* after the 12, making 123. Now we multiply this 123 by 3, the second figure in the root, set the product 369 under 4800, add [illegible] up, multiply the sum 5169 by the second figure in th[illegible] and subtract the product, 15507. We bring [illegible] period, 568, and have now to form the two [illegible] left of it.

The first is obtained by tripling th[illegible] which thus becomes 129; (the final [illegible] counted for, when the next figure [illegible] the other quantity, 5547, is found by adding 9, the square of the second figure in the root, to the two preceding middle lines, $\begin{matrix}369\\5169\end{matrix}$. We now add two ciphers, and repeat the whole process described in this paragraph.

The remarks made above with respect to pointing the last figure of the integer, and also with respect to *surd* square roots, apply also to cube roots. Thus .01, 24.1 would be pointed for the cube root of $.01\dot{0}$, $2\dot{4}.10\dot{0}$.

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Obtain the cube roots of

(1.) 16194277. (2.) 138188413. (3.) 437.245479.
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(10.) Extract, to four figures, the cube roots of 300 and .3.

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---

151. In *Decimal* Arithmetic, the value of any figure is increased *ten*-fold by its being moved one place to the left, or diminished *ten*-fold by its being moved one place to the right.

In *Duodecimal* Arithmetic, the value of any figure is increased or diminished *twelve*-fold by its being moved one place to the left, or right, respectively. So that 12 is, to numbers which are written in duodecimal notation, just exactly what 10 is to common numbers.

Ex. 1. In common numbers, 37.5 denotes 3 *tens*+7 *ones*+5 *tenths*; but, [illegible] notation, 37.5 would denote 3 *twelves*+7 *ones*+5 *twelfths*.
[illegible] numbers, 8765 would denote $8\times1000+7\times100+6$
[illegible] is the *square* of 10, 1000 the *cube* of 10, &c.,
[illegible] power) of 10 by 10^2,
[illegible] power) of 10 by 10^3, aud so on, we may
[illegible]65 denotes
[illegible] $10^2+6\times10+5$;
[illegible] be used to denote
[illegible] $\times12+5$.
[illegible] 14909.

(4.) A court-yard is 20 ft. 3¼ in. square. What will it take to pave it, at 4*s*. 3*d*. per square yard?

(5.) A grave-stone was charged at 4*s*. 8*d*. a foot. What did it cost, being 6 ft. 7 in. long, and 3 ft. 2½ in. broad?

(6.) Find the cost of laying a floor, which is 24 ft. 4 in. broad, and 96 ft. 6 in. long, at 1¾*d*. per square foot.

(7.) What is the breadth of a room, whose length is 23 ft. 10 in., and which it takes 34 sq. yds. 5 ft. 118 in. of carpet to cover?

(8.) The side of a rectangular building measures 13 yds. 2½ in., and its area contains 170 sq. yds. 7 ft. 42¼ in. Show that it is square.

(9.) How many yards of carpet, 2 ft. 4 in. broad, will be wanted for a room that is 13 ft. 4½ in. by 17 ft. 6¾ in.?

(10.) I find that it will take 144 yards of paper, 2 ft. 8 in. wide, to cover the walls of my parlour. But there is another paper, 2 ft. 3 in. wide. How many yards of this will it take?

(11.) What will be the cost of a drugget, 2 ft. 5½ in. wide, to cover a square room, whose side is 37 ft. 8 in., at 2*s*. 10½*d*. per yard?

(12.) If the room in (11) be 14 ft. 6 in. high, how many yards of paper, 2 ft. 5 in. wide, would paper it—not allowing for the doors and windows?

(13.) Find the solid content of a log of timber, that is 15 ft. long, 2 ft. 4 in. broad, and 1 ft. 8 in. thick.

(14.) How many cubic feet of water are there in a vessel with square base, whose side is 3 ft. 3 in., and height 2 ft. 6 in.?

(15.) Find the solid content of a cube, whose side is 3 ft. 3⅓ in.

(16.) In making a square pond, whose side was 68 yards, there were taken out 10000 cubic yards of earth. How deep was it made?

(17.) What must be the length of a trench, 4½ feet deep, and 10¾ feet wide, that it may contain exactly 5000 cubic feet?

(18.) The depth of a canal is 6 ft. 4 in., the width 19 ft. 8 in., and the length 20 miles. How many feet of water will it hold?

(19.) What must be the height of a room, whose length is 20 ft. 8 in., and breadth 14 ft., that it may contain 3000 cubic feet of air?

(20.) Find the solid content of a box, 6 ft. 2¼ in. long, 4 ft. 3½ in. broad, and 2 ft. 4⅓ in. deep.

(21.) A block of granite is 7 ft. 8 in. long, 3 ft. 4 in. broad, and 3 ft. 10 in. thick. What is its cost, at 6*s.* 6*d.* per foot?

(22.) What would be the expense of digging a cellar, $16\frac{3}{4}$ ft. long, $12\frac{1}{2}$ ft. broad, and $8\frac{1}{4}$ ft. high, at $13\frac{1}{2}$*d.* per yard?

(23.) A log of mahogany is 46 ft. $7\frac{1}{4}$ in. long, 4 ft. $9\frac{1}{2}$ in. broad, and 3 ft. $11\frac{3}{4}$ in. thick. What is its solid content?

(24.) A rectangular cistern, whose length is 13 ft. $7\frac{1}{4}$ in., and breadth 6 ft. $5\frac{1}{2}$ in., contains $294\frac{3}{4}$ cubic feet of water. What is the depth of the cistern?

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159. When we wish to compare together the magnitudes of two numbers, we may do so in two different ways.

(i.) We may compare them by means of *subtraction*, that is, by observing by how much one of them exceeds or falls short of the other.

Thus 24 is greater than 8 by 16, and 12 is less than 18 by 6.

(ii.) We may compare them by means of *division*, that is, by observing how many times one of them contains the other, or, in other words, by expressing one of them as a fraction of the other.

Thus 24 is *three times* of 8, for $\frac{24}{8}=3$ or $\frac{3}{1}$.
and 12 is *two-thirds* of 18, for $\frac{12}{18}=\frac{2}{3}$.

160. Now the last of the above ways is that in which we naturally, and almost unconsciously, compare the magnitudes of quantities. Thus the mere *difference* between 999 and 1000 is 1, the same as the difference between 1 and 2: but no one would hesitate to say that 999 is much greater, *compared with* 1000, than 1 is, *compared with* 2. Why? Because, in fact, we reason immediately within ourselves, (whether we are conscious of it or not,) that 999 is a greater fraction of 1000 than 1 is of 2.

161. The fraction, which thus enables us to compare the magnitudes of two numbers, is called their *Ratio.* So that the Ratio of one number to another is the number *which expresses what fraction the former is of the latter.*

We usually employ the mark :, placed between the two numbers compared, to express their ratio.

Ex. The ratio of 24 : 8 (which is read 24 *to* 8) is $\frac{24}{8}$ or $\frac{3}{1}$, and the ratio of 12 : 18 is $\frac{12}{18}$ or $\frac{2}{3}$.

162. It is plain that all ratios are equal, which can be expressed by the same fractions.

Thus the ratios of 12 : 18, 24 : 36, 6 : 9, 2 : 3, $\frac{2}{3}$: 1, 1 : $\frac{3}{2}$, &c., are all equal, because each of them can be expressed by the same fraction, $\frac{2}{3}$.

163. Hence it follows that the two terms of any ratio may always be *both* multiplied or divided by the same number, whatever that number may be, without altering the value of the ratio, because the fraction by which it is expressed will remain the same.

Ex. 12 : 18 = 3 : $4\frac{1}{2}$, or = $1\frac{1}{3}$: 2, or = $\frac{1}{2}$: $\frac{3}{4}$, &c., where the equivalent ratios have been formed by dividing both terms of the given one by 6, 9, 24, respectively.

The former term in any ratio is called the *antecedent*, the latter the *consequent*.

164. When two ratios are equal, they are said to form a *Proportion*, and the four quantities are called *Proportionals*. Instead of the sign =, it is usual to put the mark :: between the two equal ratios.

Ex. 12 : 18 = 2 : 3, or 12 : 18 :: 2 : 3, which is read 12 *is to* 18 *as* 2 *is to* 3.

N.B. Notice attentively that the thing really stated by any such proportion is this: the first term (12) is the *same fraction* of the second term (18), that the third term (2) is of the fourth term (3).

165. In any numerical proportion, the product of the two *outer* terms (the *first* and *fourth*) is always equal to the product of the two *middle* terms (the *second* and *third*); or, as it is commonly expressed,

the product of the extremes = the product of the means.

Ex. 12 : 18 :: 2 : 3, and 12 × 3 = 36 = 18 × 2;
the reason of which is plain; for $\frac{12}{18}=\frac{2}{3}$, and, therefore, multiplying each fraction by the product of the denominators (18 × 3), we get

$$\frac{12\times18\times3}{18}=\frac{18\times3\times2}{3}, \text{ or } 12\times3=18\times2.$$

166. Conversely, if the product of any two numbers = the product of any two others, the four will make a proportion, the factors in one product being set as the *extremes*, and those of the other as the *means*.

Ex. 5 × 16 = 80 = 4 × 20.
Now if we divide each of these equals by 5 × 4, we get $\frac{16}{4}=\frac{20}{5}$,
whence 16 : 4 :: 20 : 5, or 20 : 5 :: 16 : 4;
but, if $\frac{16}{4}=\frac{20}{5}$, then also, $1\div\frac{16}{4}=1\div\frac{20}{5}$, or $\frac{4}{16}=\frac{5}{20}$,
whence 4 : 16 :: 5 : 20, or 5 : 20 :: 4 : 16.

Again, dividing each of the given products by 5×20, we get

$\frac{16}{20} = \frac{4}{5}$, whence $16 : 20 :: 4 : 5$, or $4 : 5 :: 16 : 20$;
and $\frac{20}{16} = \frac{5}{4}$, whence $20 : 16 :: 5 : 4$, or, $5 : 4 :: 20 : 16$.

It will be seen that, in each of the above proportions, the terms of the one product form the two extremes, and those of the other, the two means.

167. Hence it follows that if four numbers, arranged in any order, are proportionals, they will also be proportionals when arranged in any other order which allows of the same pairs of quantities coming together as extremes and means.

Ex. $12 : 18 :: 2 : 3$, whence (**165**) we have $12 \times 3 = 18 \times 2$; and, consequently, by **166**, we may have any of the proportions,

$12 : 2 :: 18 : 3$, or $3 : 18 :: 2 : 12$,
or $2 : 3 :: 12 : 18$, or $18 : 12 :: 3 : 2$, &c.,

in each of which 12 and 3 go together, (and, therefore, also 18 and 2,) either as extremes or as means.

168. By means of the property in **165**, we can always, having given any three terms of a proportion, determine the remaining term, whichever it may be.

For 1st × 4th = 2nd × 3rd: hence we have

$$\text{1st} = \frac{\text{2nd} \times \text{3rd}}{\text{4th}}, \qquad \text{2nd} = \frac{\text{1st} \times \text{4th}}{\text{3rd}},$$

$$\text{3rd} = \frac{\text{1st} \times \text{4th}}{\text{2nd}}, \qquad \text{4th} = \frac{\text{2nd} \times \text{3rd}}{\text{1st}}.$$

Ex. 1. Find a number which shall be to 5 as 6 is to 7.

Ans. The number $= \frac{5 \times 6}{7} = \frac{30}{7} = 4\frac{2}{7}$; so that $4\frac{2}{7} : 5 :: 6 : 7$.

Ex. 2. Find a fourth proportional to $\frac{1}{2}$, $\frac{1}{3}$, and $\frac{1}{4}$.

Ans. The number $= \dfrac{\frac{1}{3} \times \frac{1}{4}}{\frac{1}{2}} = \frac{1}{6}$; so that $\frac{1}{2} : \frac{1}{3} :: \frac{1}{4} : \frac{1}{6}$.

Ex. 3. Find a mean proportional to 9 and 16.

This expression, *mean proportional*, is used to denote the middle term, when *three* quantities only are proportional, in such a way that the 1st : the 2nd :: the 2nd : the 3rd; in which case, by **165**, 2nd × 2nd, or the *square* of 2nd, = 3rd × 4th.

Here then the square of 2nd $= 9 \times 16 = 144$, and the mean term $= 12$;
so that $9 : 12 :: 12 : 16$.

Ex. 146.

Find a number which shall be

(1.) *First* in a proportion with 2, 3, 4.
(2.) . . . . with 3, 5, 7.
(3.) . . . . with $1\frac{1}{2}$, $2\frac{2}{3}$, $3\frac{3}{4}$.
(4.) *Second* in a proportion with 2, 6, 9.

(5.) *Second* in a proportion with 1, 6, $1\frac{1}{2}$.
(6.) *Third* in a proportion with $1\frac{1}{4}$, 2, 3.
(7.) . . . . with 3, 2, 5.
(8.) *Mean* in a proportion with 2 and $3\frac{1}{8}$.
(9.) . . . . with $2\frac{2}{3}$, and $3\frac{3}{8}$.
(10.) *Fourth* in a proportion with 4, 5, 8.
(11.) . . . . with 5, 7, 9.
(12.) . . . . with $2\frac{2}{3}$, $3\frac{3}{4}$, $4\frac{4}{5}$.

---

169. Hitherto we have spoken only of the ratios of mere *abstract* numbers. But exactly the same reasoning (with the exception noted below) applies also to the ratios of *concrete* quantities.

Thus the ratios of £12 : £18, of 24 cwt. : 36 cwt., of 36 apples : 54 apples, of 6 cows : 9 cows, of 2 feet : 3 feet, &c., are all equal, since each may be expressed by the same fraction $\frac{2}{3}$, the former of the two quantities being in each case $\frac{2}{3}$ of the latter.

But, of course, in any such ratio, the two terms compared must be of the *same kind:* otherwise it would be absurd to speak of a ratio between them.

Thus there can be no ratio of 12 cows : 18 cwt.; one of these quantities not being able to be expressed as a fraction of the other.

Moreover, if the two terms in such a ratio are of the same kind, but not of the same denomination, they must be reduced to the same denomination before their ratio can be expressed as a fraction.

Thus the ratio of 1*s.* : 1*s.* 6*d.* = that of 12*d.* : 18*d.* $=\frac{12}{18}$ or $\frac{2}{3}$; or we might have said, 1*s.* : 1*s.* 6*d.* = 1*s.* : $1\frac{1}{2}$*s.* $= 1 : 1\frac{1}{2} = 2 : 3$.

170. When two such ratios are equal, they form, as before, a proportion.

Thus £12 : £18 :: 2 feet : 3 feet.

Only we cannot here, as in **167**, change the order of the terms, unless the change be such as still leaves the two ratios possible.

Thus we can write the above proportion *inversely*, and say

£18 : £12 :: 3 feet : 2 feet;

but we cannot write it *alternately*, and say

£12 : 2 feet :: £18 : 3 feet,

because £12 can have no ratio to 2 feet, nor £18 to 3 feet.

171. For the like reason, we cannot say of such a proportion that *the product of the extremes = the product of the means*, unless we understand by the *extremes* and *means* only the *numbers* in each term, and not the concrete quantities themselves.

Thus, in the above proportion, it *is* true that 18 × 2 = 12 × 3, though it would be absurd to talk of multiplying £18 × 2 feet, and £12 × 3 feet.

With this understanding, however, that is, if we consider only the *numerical value* of each term, the property of **165** will still be true; and we can apply it, as before, having given any three terms of such a proportion, to find the *numerical value* of the fourth—which we must then set down of the same kind and denomination, as the other term of the ratio to which it belongs.

Thus, to find a fourth proportional to £45, £36, and 20 cwt., we have the numerical value of 4th term $=\frac{36\times 20}{45}=16$, which must mean 16 cwt., of the same kind and denomination as 20 cwt.;
so that £45 : £36 :: 20 cwt. : 16 cwt.

172. The method just exemplified, by which we may find the fourth proportional to three given quantities, namely, by multiplying together the 2nd and 3rd, and dividing by the 1st, is commonly called the *Rule of Three.*

A very large class of Examples under this Rule consists of those, in which, the *cost* of a given *quantity* of some article being given, we are required to find what will be the cost of another given *quantity*, or what quantity can be bought for another given *cost.* Here, then, are four proportional quantities; for it is plain that the former cost will be the *same fraction* of the latter cost that the former quantity is of the latter quantity. And, accordingly, the above method may be applied in such cases.

Ex. If 18 cwt. of sugar cost £54, what will 56 cwt. cost?

cwt. cwt. £
Here 18 : 56 :: 54 : the *Answer*, whose numerical value we obtain by multiplying together the 2nd and 3rd *numbers*, dividing by the 1st. The result is 168, which must be of the same kind and denomination as the third term, that is, it must be £168.

$$\begin{array}{r} 56 \\ \underline{54} \\ 224 \\ \underline{280} \\ 18)\,3024\,(168 \\ \underline{18} \\ 122 \\ \underline{108} \\ 144 \\ \underline{144} \end{array}$$

Ans. £168.

It will be remembered, that we have already solved such questions as these (see **36, 47, 88**) in a very simple manner as follows:—

18 cwt. cost £54,
therefore, 1 cwt. „ $\frac{1}{18}$ of £54,
and 56 cwt. „ $\frac{56}{18}$ of £54 = £168.

It is plain that the two processes are really identical, since, in this latter case, we reason that the required cost will be the *same fraction* of the given cost, that the quantity corresponding to the former is of the quantity corresponding to the latter.

We shall here only give examples of this kind of question, to show the different points to be noticed, in treating these and other questions by the Rule of Three: and refer the Pupil back to the Exercises already given, under the above-mentioned articles, as examples for practice.

173. Since the Answer $= \dfrac{\text{2nd} \times \text{3rd}}{\text{1st}}$, and the value of this fraction is not altered by striking common factors out of its numerator and denominator, we may often simplify the operation by striking out (before we multiply and divide according to the Rule) a common factor, either from the 1st and 2nd terms, or from the 1st and 3rd terms.

Ex. If 275 reams of paper cost £266 15*s*., what would 900 reams cost?

reams		reams		£ *s.*
25) 275	:	5) 990	::	266 15
11) 11		198		20
1		97		5) 5335
		1386		11) 1067
		1782		97
		2,0) 1920 6*s.*		
		£960, 6*s.*		

Here we have first reduced the 3rd term, for convenience, to shillings; then we have struck a factor 25 out of the first term, and a factor 5 out of *each* of the second and third terms, which is equivalent to striking twenty-five out of their *product*; then we have struck 11 out of the first term and 11 out of the third, and have now only to multiply the second term by 97, the divisor being thus reduced to 1. The result will be of the same kind as the third term, namely, shillings, which we then bring into £'s.

174. The first and second terms must always be reduced (**169**) to the same denomination, before the Rule can be applied.

Ex. **If 14 tons of bar-iron cost £35 10*s.* 6*d.*, how much may be had for £97?**

```
       £  s.  d.         £             tons
       35 10  6    :     97      ::    7) 14
       20                40               --
      ---              ----                2
      710              3880
        2                 2
     ----              ----
  7) 1421        203) 7760 (38 46/203 tons, Ans.
     ----              609
      203              ----
                       1670
                       1624
                       ----
                         46
```

Here we have first reduced the two terms of the first ratio to the same common denomination, *sixpences*: then we strike a factor, 7, out of the first and third terms, and proceed as before.

See Exercises **21**, **39**, **92**.

175. Besides the above examples of *cost* and *quantity*, there are many other cases of Proportion, to which the same method of the Rule of Three may be applied, in order to find the fourth term, when the other three are given. Before, however, we can proceed to apply it, we must always be sure that the case under consideration *is* really one of Proportion—that is, that the first quantity will be the same fraction of the second that the third is of the fourth. In most cases, this will be apparent upon a moment's reflection. Thus, since the cost of most common articles is at so much for every pound, there is no difficulty in perceiving that one cost will be the same fraction of another cost, as the quantity bought for the first will be of the quantity (of the same goods) bought for the second. This is expressed by saying that the cost of such goods is *proportional* to the quantity of it.

But so, too, the *rent* of a house will be proportional to the *time* for which it is occupied,—a man's *wages* will be proportional to the *time* for which he works,—the distance run by a train, moving uniformly, in a given time, will be proportional to the *rate* at which it moves, &c.

In other cases we can only know of the fact of four quantities being proportionals, from philosophical experiment and reasoning. For instance, it may be shown that the length of the shadow cast by a vertical rod at any given hour of the day, will be proportional to the length of the rod,—or that the velocity acquired by a heavy body, falling freely from rest by the action of gravity, will be proportional to the *time* during which it has been falling. To all such cases of Proportion the Rule of Three may be applied.

Unless we are *sure*, however, that the case before us *is* one of Proportion, we have no right to apply the Rule of Three, however tempting the example may appear to be for the employment of this rule. For instance, it might be *guessed* that, because a heavy body, falling from rest through *one* foot, would acquire a speed of 8 feet in a second, therefore, it would be likely, by falling through *two* feet, to acquire a speed of 16 feet in a second, and so on,—in other words, that the velocity acquired by a heavy body, falling from rest, is proportional to the *space* through which it has fallen. But experience shows that this is *not* the case: and we should be entirely wrong in applying the Rule of Three to such a case.

176. It is desirable, then, to have some simple *test* by which we may satisfy ourselves, in any question of doubt, as to whether we have a right to treat the case before us as one of Proportion, and perhaps the following may be most readily employed for this purpose.

Whenever a question is put, appearing to be one for the Rule of Three, that is, when we have given one pair of quantities of the same kind, and another quantity of the same or a different kind, in order to find a fourth of the same kind as the third, consider for a moment whether we have reason for saying that, if one of the two terms in the first or given pair were the *double*, *triple*, *half*, *third*, &c. of the other, then the corresponding quantity in the second pair would be also *double*, *triple*, *half*, *third*, &c. of the other. If this be so, then the case *is* one of Proportion, and the Rule of Three may be applied, which may be stated more fully as follows.

RULE. Set third or *last* the single given term, of the same kind as the answer, and set *first* that term of the other given pair which corresponds to the third term;

Reduce (if necessary) the first and second terms to the same denomination, and the third to any denomination that may be thought most convenient;

Strike out any common factors from the first and second, or from the first and third;

Multiply together the second and third, and divide by the first, when the quotient will give the answer in the same denomination as that to which we have reduced the third term.

177. The cases hitherto given have been examples of (what is called) *Direct* Proportion. But there is a class of questions which are also solved by the Rule of Three, though it is generally better to treat them as in **37.** These are those in which it appears that, if one of the quantities in the first pair were *double*, *triple*, &c. of the other, the corresponding quantity of the second pair will be the *half*, the *third*, &c. of the other. Thus, if any given number of men would do a piece of work in a certain time, it is plain that *double* that number would do it in *half* the time. In this case the four quantities will still be proportional, but with the terms of the second ratio in *inverted* order, since the first term of one pair of quantities will be the same fraction of the second that the second term of the other pair is of the first.

Here too, then, the Rule of Three may be applied, setting last, as before, the single given quantity, but setting first that term of the other given pair which does *not* correspond to the third term.

Ex. 1. A person completed a journey in 32 days, travelling 8 hours a day; how long would he take to do the same, travelling only 6 hours a day?

$$\begin{array}{r}
\text{hrs. hrs. d.}\\
6 : 8 :: 32\\
\underline{8}\\
6)\overline{256}\\
\underline{42\tfrac{2}{3}}\text{ days.}
\end{array}$$

Here it is plain that if we *doubled* the number of hours in each day, the number of days required for the journey would be only *half* of what it was before. The case, then, is one of *Inverse* Proportion; and we state the sum accordingly, setting last the single term, 32 days, as before, and for the *second* term that one, namely 8 hrs., of the other given pair, which corresponds to the 32 days.

But the question might be more simply solved as follows. It takes $32 \times 8 = 256$ hours for the journey: and, therefore, if the day be 6 hours long for travelling, the number of days required will be $\frac{256}{6} = 42\frac{2}{3}$ days.

Ex. 2. If 72 sheep can be grazed in a field for 9 days, how long might 108 sheep be grazed in the same field?

$$\begin{array}{r}
\text{sheep sheep days}\\
108 : 72 :: 9\\
\underline{9}\\
108)\overline{648}\text{ (6 days.}\\
\underline{648}
\end{array}$$

Here, if the number of sheep were *doubled*, they could be kept only *half* the number of days in the same field; so that the number of sheep is *inversely* proportional to the number of days. The single term is 9 days, and we set the corresponding term of the given pair, namely, 72 sheep, as the *second* term.

But we might reason thus:

The field will supply 72 sheep for 9 days, or 1 sheep for $9 \times 72 = 648$ days, or 108 sheep for $\frac{648}{108} = 6$ days.

See EXERCISE **22**.

178. When we are satisfied that the case before us is one of Proportion, either direct or inverse, we may relieve ourselves from some of the care required in stating the sum by the following general Rule, which includes both cases, and is that commonly given as the

RULE OF THREE.

Set last the single given term which corresponds to the Answer, and set the *greatest* or *least* of the other given pair of terms second, according as it is seen that the Answer will be *greater* or *less* than the third term. Then multiply together the second and third, and divide by the first.

The reason of this is plain. For, if the three quantities *do* form the first three terms of a Proportion, the single term must be set third, since it belongs to the ratio, of which the Answer is the other term. And then, as we know that the Answer will be found by multiplying this term by one of the two remaining terms, and dividing by the other, it is plain that, if the Answer is to be *greater* than the third term, this must arise from our multiplying by the greater of the two, and dividing by the less, that is, from our putting the *greater* of them second. And similarly if the Answer is to be *less* than the third term.

The above explanation, however, is only intended to show that the Rule will enable us to make the same statement of the sum as we should have done by the proper considerations, and so to get the correct result. It does not at all profess to give the true *reason* for so stating it, which depends on the foregoing considerations.

Ex. If $10\frac{5}{7}$ lbs. of salt cost $4\frac{11}{16}$*s.*, what will $3\frac{2}{3}$ cwt. cost?

$$\overset{\text{lbs.}}{10\tfrac{5}{7}} : \overset{\text{lbs.}}{3\tfrac{2}{3}} \times 112 :: \overset{s.}{4\tfrac{11}{16}} : \textit{Ans.}\text{, which} = \frac{3\frac{2}{3} \times 112 \times 4\frac{11}{16}}{10\frac{5}{7}}\,s.$$
$$= £8\ 19s.\ 8d.$$

Ex. 147.

(1.) A bankrupt owes £2363 5*s.*, and his effects are worth £590 16*s.* 3*d.* How much will he be able to pay in the £?

(2.) If $4\frac{5}{8}$ shares in a speculation are worth £33 18*s.* 4*d.*, what will 10 shares be worth?

(3.) If the poor-rates on a rental of £525 7*s.* 6*d.* be £61 5*s.* $10\frac{1}{2}$*d.*, what is that in the £?

(4.) What is the height of a steeple, whose shadow was 170 ft. 8 in., at the same time that the shadow of a rod 6 ft. 6 in. high was 5 ft. 4 in.?

(5.) If the carriage of $12\frac{1}{4}$ cwt. for 50 miles cost 3*s.* $9\frac{1}{2}$*d.*, what ought to be the charge for $17\frac{1}{2}$ cwt. for the same distance?

(6.) If $18\frac{2}{9}$ oz. Av. cost $35\frac{7}{8}$*s.*, what will $34\frac{1}{6}$ lbs. cost?

(7.) *A* offers to give £2 3*s.* 4*d.* to a charity for every pound *B* gives to it. *B* gives £5 6*s.* 8*d.* What will *A* have to give?

(8.) Which is the cheaper, a hat which costs 15*s.*, and will last 9 months, or one which costs 12*s.*, and will last 7 months? and how much will a man save in 21 years, who wears the cheaper kind of hat?

(9.) If 51 tenpenny loaves are made out of a quarter of flour, what will be the price of a loaf, when 68 are made out of the same quantity?

(10.) The shadow of a church-tower extends 57 yards from its base. Find its height, it being observed that a two-foot rule held perpendicularly cast a horizontal shadow of 4 ft. 9 in.

(11.) Find a fourth proportional to $4\frac{1}{16}$, $6\frac{1}{2}$, and 4A. 3R. $2\frac{1}{2}$P.

(12.) How many feet in 100 must a road rise, which is to be a mile and a half long, and to be carried from the plain to the top of a hill 462 feet in perpendicular height?

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179. Suppose we had the question, If 9 men can reap 30 acres of wheat in 10 days of 6 hours each, how many men would reap 40 acres *in the same time?* This would be an instance of common Direct Proportion; and we should have

$$30\text{A.} : 40\text{A.} :: 9 \text{ men} : \frac{40}{30} \times 9 = 12 \text{ men}.$$

But now suppose that, instead of '*in the same time*,' the question had said, 'in 15 days *of the same length*.' Here, after finding, as above, the number of men, 12, who would reap 40A. in 10 days, we must still have another Proportion, to find the number who will reap the *same* number of acres in 15 days. Thus we shall have (the case being here one of Inverse Proportion)

$$15 \text{ days} : 10 \text{ days} :: 12 \text{ men} : \frac{10}{15} \times 12 = 8 \text{ men}.$$

Once more, suppose that, instead of 15 days *of the same length*, the question had said '15 days of 8 hours each.' Here, after having found, as above, the number of men, 8, who would reap the 40A. in 15 days of 6 hours each, we must still have a third Proportion, to find the number of men who will reap the *same* number of acres in the *same* number of days of 8 hours each. Thus we shall have (the case being also one of Inverse Proportion)

$$8 \text{ hrs.} : 6 \text{ hrs.} :: 8 \text{ men} : \frac{6}{8} \times 8 = 6 \text{ men}.$$

180. Now the above is an instance of *Compound* Proportion, whereas the preceding examples have all been instances of *Simple* Proportion. And in such cases we may either work by means of successive Simple Proportions, as above, or we may obtain the Answer at once by means of (what is called) the Double Rule of Three, which arises from the following considerations.

Taking the numerical value of the 1st Answer in (**179**) in its original form, $\frac{40}{30} \times 9$, we shall have the second Answer, $\frac{10 \times 40}{15 \times 30} \times 9$, and the third and final Answer, $\frac{6 \times 10 \times 40}{8 \times 15 \times 30} \times 9$ (which would of course reduce itself to 6, that is, 6 *men*). But this is the same result as we should get, if we made only one statement, in which we set down last the single given term, as usual, and, for the first and second terms, the *products* respectively of the numerical values of the first and second terms of the three separate Proportions.

The same reasoning will apply to all similar cases. It is best to set down, one under another, the numerical values of the first Ratio of each of these Proportions, stating each of them separately with reference to the third term. Then we may multiply these together, (striking out common factors, as before, from the first or second, or from the first and third,) and finally multiplying together the second and third terms of the resulting compound statement, and dividing by the first.

Ex. If 8 men set up a work of 6 sheets in 10 days of 9 hours long, how many will set up a work of 8 sheets in 16 days of 12 hours long ?

6 sheets : 8 sheets :: 8 men.
16 days : 10 days.
12 hours : 9 hours.

We have stated above the three Proportions, at full length, in order to show how we do it. First, we set down the single term, 8 men, last. Then it will take *more* men for a work of 8 sheets than for one of 6 sheets, and we set the 8 sheets second; it will take *fewer* men for 16 days than for 10 days, and we set the 10 days second; it will take *fewer* men when the days are 12 hours long, than when they are 9 hours long, and we set the 9 hours second. The first Proportion then is *direct*, and the other two are *inverse*.

In *common* practice, it would not be necessary to make the above complete statements, but we should only set down the numerical values of the different terms, and work the sum as follows:

6 : 8 :: 8 men.
16 : 10
12 : 9

Whence, striking out common factors in the first and second terms, we have the compound statement,

$$8 : 5 :: 8 \text{ men} :: \frac{5 \times 8}{8} = 5 \text{ men}.$$

Ex. 148.

(1.) If 2 loads of hay will serve 3 horses for 4 weeks, how long will 5 loads serve 6 horses?

(2.) If 5 men receive £5 18*s.* wages for 12 days, what will be the wages of 16 men for 20 days?

(3.) If 8 men build 24 square yards of walling in 6 days, how many will build 18 square yards in 3 days?

(4.) If £7 16*s.* pay 8 labourers for 18 days, how many at the same rate will £17 11*s.* pay for 12 days?

(5.) If the 8*d.* loaf weighs 48 oz. when wheat is at 54*s.* the quarter, what should be the price of wheat when the 6*d.* loaf weighs $32\frac{2}{5}$ oz.?

(6.) If 19 men can build 38 yds. of wall in 12 days, how many will build 96 yds. 2 ft. 3 in. in $21\frac{1}{2}$ days?

(7.) If a man can travel 360 miles in 12 days of 8 hours each, how many hours a day must he walk at the same rate, to complete 450 miles in 20 days?

(8.) If 14 men can mow 168 acres in 12 days of $8\frac{1}{4}$ hours each, how many acres can be mowed by 20 men in 11 days of $7\frac{4}{5}$ hours each?

(9.) If 5 needlewomen can finish a certain quantity of work in $10\frac{5}{8}$ days of $9\frac{3}{5}$ hours each, how long would it take 3 needlewomen to do twice the same work, reckoning 10 hours to the day?

(10.) If 12 men build a wall, 60 feet long, 4 feet thick, and 20 feet high, in 24 days, working 12 hours a day, how many will it take to build a wall, 100 feet long, 3 thick, and 12 high, in 18 days, working 8 hours a day?

(11.) If three men, working 11 hours a day, can reap 20 acres in 11 days, how many men, working 12 hours a day, will reap a field, 360 yards long and 320 yards broad, in 4 days?

(12.) Five men can reap a field in 4 days, working 8 hours a day. But, after the first day, one of them leaves his work, and, bad weather coming on, the other four can afterwards work only 6 hours a day. When will the work be finished?

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181. The principles of Proportion may be applied to solve many questions under the head of *Profit and Loss*, as in the following instances.

Ex. 1. If tea be bought at 5*s.* 6*d.* per lb., and sold at 6*s.* 5*d.*, what is the gain per cent.?

Here the gain on the *prime cost*, 5*s*. 6*d*., is 11*d*.: hence we have

5*s*. 6*d*. : £100 :: 11*d*. : the Answer,

which is found in the usual way to be £16$\frac{2}{3}$=£16 13*s*. 4*d*.

Ex. 2. If bar-iron, which cost in making £2 1*s*. 8*d*. per cwt., be sold at a loss of £5$\frac{3}{8}$ per cent., what price did it fetch per cwt.?

Here iron, which cost £100, was sold for £100−£5$\frac{3}{8}$=£94$\frac{5}{8}$; hence we have

£100 : £2 1*s*. 8*d*. :: £94$\frac{5}{8}$: the Answer, which=£1 19*s*. 5$\frac{1}{8}$*d*.

Ex. 3. If 5 per cent. be gained by selling 125 yards of cloth for £95, what was the prime cost per yard?

Here, if the cloth had sold for £105, the prime cost would have been £100: we have, therefore,

£105 : £95 :: £100 : the prime cost of 125 yards,

which=£90$\frac{10}{21}$, and the prime cost of each yard is 14*s*. 5$\frac{5}{7}$*d*.

Ex. 4. If 4 per cent. be lost by selling linen at 2*s*. 9*d*. a yard, at what price must it be sold to gain £10 per cent.?

[Here, cloth which would have cost £100, would have been sold for £96 at the first price, and for £110 at the second; we have, therefore,

£96 : 2*s*. 9*d*. :: £110 : second price=3*s*. 1$\frac{13}{16}$*d*. Ed.]

Ex. 149.

(1.) Eggs are sold at the rate of five for 2*d*. At what price must they have been bought to bring a profit of 20 per cent.?

(2.) An article was sold for £5, at a profit of 20 per cent. on the cost price. What did it cost?

(3.) How much per cent. is 2$\frac{1}{2}$*d*. on a shilling?

(4.) How much per cent. is 2*s*. 9*d*. on the pound?

(5.) I bought goods for £12, and I sold them so as to make a profit of 10 per cent. on the selling price. What profit did I get upon them?

(6.) At what price must tea be sold, which cost 3*s*. 9*d*. per lb., so as to make a profit of 20 per cent. upon the cost price?

(7.) If cheese, which was bought at 50*s*. per cwt., be sold at 6$\frac{1}{2}$*d*. per lb., what was the gain per cent.?

(8.) If 5 per cent. be gained upon the cost price by selling butter at £5 12*s*. per cwt., what will be gained per cent. by selling it at 16*d*. per lb.?

(9.) A grocer mixes two kinds of sugar, at 8*d*. and 11*d*., taking two pounds of the first to one of the second. At what price per lb. should he sell the mixture, to gain a profit of 25 per cent.?

(10.) If 336 yards of cambric are bought at 7*s*. 10$\frac{1}{2}$*d*. per yard, and sold, one-fourth at 10*s*. 3*d*., one-third at 8*s*. 6*d*., and the remainder at 7*s.* per yard, what will be the loss or profit per cent. upon the whole outlay?

(11.) Bought £13 6*s.* 8*d.* worth of apples, at 3*s.* 4*d.* per bushel, part of which I found to be damaged and worthless. For the rest, which were sold at 50 per cent. profit, I received £16. How many bushels were damaged?

(12.) Bought quills at 4*s.* 7*d.* the hundred, and sold them so as to gain $\frac{3}{8}$ of the selling price. What is the selling price? and what is the profit per cent. upon the cost price?

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182. *To divide a given quantity into two parts which shall have to each other a given ratio.*

RULE. Form fractions whose common denominator is the *sum* of the two terms of the ratio, their numerators being the separate terms themselves. Take these fractions of the given quantity; they will be the parts required.

Ex. Divide 75 into two parts, which shall have the ratio of 2 : 3.

Here the fractions are $\frac{2}{5}$ and $\frac{3}{5}$, and the parts required are $\frac{2}{5}$ of $75=30$, and $\frac{3}{5}$ of $75=45$, which are plainly in the given ratio.

The reason of the Rule is evident, since the sum of the numerators makes up the denominator, and therefore the sum of the fractions makes up *unity*, and the sum of the parts makes up the *whole* of the number, while the fractions themselves, having a common denominator, are in the ratio of their numerators, and so, consequently, are the corresponding parts of the whole.

183. Similar reasoning may be applied when the whole is to be divided into more than two parts, having given ratios to each other.

Ex. 1. Gunpowder is composed of 76 parts of nitre, 14 of charcoal, and 10 of sulphur: how much of each of these will be required for a cwt. of powder?

Here the fractions are $\frac{76}{100}=\frac{19}{25}$, $\frac{14}{100}=\frac{7}{50}$, $\frac{10}{100}=\frac{1}{10}$, and the parts are 3 qrs. $1\frac{3}{25}$ lbs., $15\frac{17}{25}$ lbs., and $11\frac{1}{5}$ lbs. respectively.

Ex. 2. Divide £1000 among *A*, *B*, *C*, so that *A* may have half as much again as *B*, and *B* $\frac{1}{3}$ as much again as *C*.

Here, representing *C*'s part by 1, *B*'s is $1\frac{1}{3}$, and *A*'s $1\frac{1}{3}+\frac{1}{2}$ of $1\frac{1}{3}=2$; and therefore the parts are to be as the numbers 2, $1\frac{1}{3}$, 1, or 6, 4, 3. Hence the fractions will be $\frac{6}{13}$, $\frac{4}{13}$, $\frac{3}{13}$; and the parts required

£461 10*s.* $9\frac{3}{13}$*d.*, £307 13*s.* $10\frac{2}{13}$*d.*, £230 15*s.* $4\frac{8}{13}$*d.*

N.B.—It will be found most convenient, where there are many fractions with the same den^{r}, to find the part corresponding to that den^{r} with num^{r} *unity*, and then multiply this successively by the num^{rs} of the different fractions; thus we should find $\frac{1}{13}$ of £1000, and then multiply this by 6, 4, 3, respectively.

Ex. 3. *A*, *B*, and *C* form a joint capital for conducting a business, of which *A* contributes £500, *B* £650, and *C* £700. At the end of a year the profits are £555; what share should each receive?

Their shares should evidently be in the ratio of their contributions of capital, i.e. in the ratio of 500, 650, 700, or of 10, 13, 14; hence the fractions are $\frac{10}{37}$, $\frac{13}{37}$, $\frac{14}{37}$, and since $\frac{1}{37}$ of £555 = £15, we have the shares required, £150, £195, £210.

Ex. 4. *A* begins business with a capital of £800, and at the end of 3 months takes *B* into partnership, with a capital of £1000; at the end of another 6 months they divide their profits, £330; what should each receive?

Here *A* contributes £800 for 9 months, and *B* £1000 for 6 months: and the interest of £800 for 9 months = interest of 9 × £800 for 1 mo., and the interest of £1000 for 6 months = interest of 6 × £1000 for 1 month; hence the value of *A*'s and *B*'s outlay may be represented by the products 9 × 800 and 6 × 1000, or 7200 and 6000 respectively, and their shares of the profits must be in this ratio = that of 6 : 5;
hence *A*'s share = $\frac{6}{11}$ of £330 = £180, and *B*'s share = $\frac{5}{11}$ of £330 = £150.

N.B.—It appears, as in the above Ex., that the *values* of sums employed in business, &c., for different times are proportional to the products of the sums by the times, or rather of their numerical values, the sums being expressed in the same den^{n}, and so also the times.

Ex. 5. *A* and *B* enter into partnership, *A* contributing £500 and *B* £300; at the end of 9 months they take in *C* as partner, who brings into the concern a capital of £1000. The profits, £2000, being divided at the end of another 9 months, what shares did they each receive?

Here, as in Ex. 4, at the end of 18 months, the shares of capital supplied by *A*, *B*, *C*, respectively, may be measured by the numbers 500 × 18, 300 × 18, 1000 × 9, or 5, 3, 5, respectively: hence the fractions will be $\frac{5}{13}$, $\frac{3}{13}$, $\frac{5}{13}$; and since $\frac{1}{13}$ of £2000 = £153 16*s*. 11$\frac{1}{13}$*d*., their shares of profit will be £769 4*s*. 7$\frac{5}{13}$*d*., £461 10*s*. 9$\frac{3}{13}$*d*., £769 4*s*. 7$\frac{5}{13}$*d*., respectively.

Ex. 150.

(1.) Divide 441 into parts, which shall be in the ratio of 1, 3, 5.

(2.) Divide 207 into parts, which shall be in the ratio of 1, $\frac{1}{3}$, $\frac{1}{5}$.

(3.) *A*, *B*, *C*, traded together, investing respectively £225, £325, and £425. Their profits were £585. What portion should each receive of this sum?

(4.) Gunpowder is made of 75 parts of nitre, 10 of sulphur, and 15 of charcoal. What weight of each will there be in 5 cwt.?

(5.) Divide £32 1*s.* 8*d.* among 4 persons, in the proportion of the fractions $\frac{1}{2}$, $\frac{1}{3}$, $\frac{1}{4}$, $\frac{1}{5}$.

(6.) *A*, *B*, *C*, rent a pasture for £28 10*s.* *A* puts in 5 sheep for 4 months, *B*, 8 sheep for 5 months, *C*, 9 sheep for 6 months. What must each pay of the rent?

(7.) How much copper and tin will be wanted for a cannon weighing 15 cwt. 3 qrs. 12 lbs., gun-metal being composed of 100 parts of copper, and 11 of tin?

(8.) *A* works regularly 9 hours a day; *B* works 8 hours for each of 3 days, and 7 hours for each of the other 3 days of the week. The work they had done between them was worth £2 1*s.* 3*d.* What should each receive of this sum?

(9.) *A* and *B* rent a field for £35 10*s.* *A* puts in six horses for the whole year. *B* puts in five horses for eleven months, and three more for five months. How much should each pay of the rent?

(10.) The standard silver coin of this realm is made of 37 parts of pure silver, and 3 of copper; and a lb. Troy of this metal yields 66 shillings. What weight of pure silver is there in half-a-crown?

(11.) The rent of a piece of land is £92 14*s.* *A* feeds on it eight cows for $4\frac{1}{2}$ months, *B*, five cows for $7\frac{1}{2}$ months, *C*, nine cows for 9 months. What should they each pay of the rent?

(12.) *A* and *B* join their capitals, which are as 2 : 3. At the end of 7 months *A* withdraws $\frac{1}{2}$ of his capital, and at the end of 9 months *B* withdraws $\frac{1}{4}$ of his. What portion of the whole profits, £132 12*s.*, should each take at the end of the year?

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184. *Stock* is the name given to money, lent to some trading company, or to our own or some foreign Government, at some given rate of interest.

Thus, if the Government were to borrow to the amount of £5,000,000 at 4 per cent., and *A* had lent £100 of this sum, *A* would be said to have £100, 4 *per cent. stock*, and would be entitled to receive the interest (viz. £4) upon this stock, from year to year, until the Government chose to repay the principal, and put an end to the debt.

The source from which the interest in such a case is paid is called the 'Public Funds,' being, however, only an imaginary property, representing the credit of the country itself, which is pledged to raise taxes from year to year, sufficient to pay the debts contracted by its Government. The interest is paid half-yearly, and the right to receive it may be sold from one party to another, just like any other kind of property.

Let it be distinctly noticed, then, that when a person is said to have £100 or £1000 stock, it is not meant that he has £100 or £1000 of *money*, but only the *right to receive interest* at a certain rate, say at 4 per cent., *as if* he had £100 or £1000 of money—that is, the right to receive £4 or £40 a-year respectively. Now, this right may be worth either more or less than £100 or £1000. For instance, suppose that money was likely to become very *scarce*, and was very much wanted in the country, as at a time when war was threatening, or that there was a great deal of speculation going on, then it would be easy to obtain higher interest than 4 per cent. for £100 of money; and, therefore, £100 stock, or *the right to have* 4 *per cent.*, would not be worth £100 of money, but would be sold for something less, say for £87 7*s.* 6*d.*, or £87⅜. In this case, the funds would be said to be *below par*, and selling at 87⅜. On the other hand, supposing that money were very *plentiful*, as when the country was very prosperous, with no great demand for money for any particular purpose, then it would not be so easy to get 4 per cent. for £100, and the funds would *rise* in value, and £100 stock be worth perhaps more than £100 of money, say £102 2*s.* 6*d.*, or 102⅛, in which case the funds would be said to be *above par*, and selling at 102⅛.

The best way of treating questions on stocks is to regard a *cent.* (or £100 of stock) as an article to be bought or sold, like any other article, as a *cwt.* of goods, for instance. Then, with the help of fractions, most questions of this kind can be easily solved.

Ex. 1. How much 3½ per cent. stock can I buy for £3625, the stock selling at 98⅝?

I can buy one *cent.* for £90⅝; therefore, for £3625, I can buy $\frac{3625}{90}$ $=3625\times\frac{8}{725}=40$ *cents.* = £4000 stock.

Ex. 2. If *money* were making in the market 4 per cent., would the above be a profitable way of investing my £3625?

At 4 per cent., £3625 (money) would produce £145 per annum; at 3½ per cent., £4000 (stock) would produce £140; so that I should lose £5 a-year by the transaction.

Ex. 3. Suppose the price of stock to rise at the end of the year to 96¾ and I were then to *sell out*, how much money should I have gained by the proceeding?

As I have 40 *cents.* of stock, and I sell each for £$96\frac{3}{4}$, I shall receive $40 \times$ £$96\frac{3}{4}$ = £3870 of money for my £4000 stock; and, as I had at first £3625 of money, I shall have gained £245; but I have lost £5 of interest; and, therefore, my whole gain is £240.

Obs. If Brokerage be charged for buying or selling stock, the *buyer* must pay the brokerage over and above the market price, and the *seller* will receive the market price less the brokerage. It follows that in cases of *buying* stock, the brokerage must be *added* to the market price, and in cases of *selling* it must be *deducted* from the market price.

Ex. 151.

(1.) What quantity of stock will £1022 purchase in the $3\frac{1}{2}$ per cents. at 73?

(2.) Find the quantity of stock that £8622 17*s.* 6*d.* will purchase in the 4 per cents. at $85\frac{3}{8}$.

(3.) If £4706 be invested in the three per cents. at $90\frac{3}{8}$, what amount of stock will it produce, brokerage being charged at $\frac{1}{8}$ per cent.?

(4.) Find the value in sterling money of £2300 in the 4 per cent. stock at 97.

(5.) What is the sterling value of £3775 stock if sold out from the three per cents. at $91\frac{3}{8}$?

(6.) What sum of sterling money will the sale of £3230 5*s.* Bank stock produce if sold at $188\frac{7}{8}$, brokerage $\frac{1}{8}$ per cent.?

(7.) Find the yearly income arising from investing £2016 in the 4 per cents. at 84.

(8.) A person invests £2394 in the three-and-a-half per cents. at $104\frac{7}{8}$, brokerage being charged at $\frac{1}{8}$ per cent. Required his income.

(9.) What sum must be invested to produce an income of £149 in the 4 per cents. at 74?

(10.) What sum invested in the 4 per cents. at $93\frac{1}{4}$ (brokerage being charged at $\frac{1}{8}$ per cent.) will produce an income of £25?

(11.) A person buys £1000 stock at $93\frac{3}{8}$, and sells it at $101\frac{1}{4}$. What is his gain by the transaction?

(12.) A person transfers £2500 stock from the 4 per cents. at 90 to the 3 per cents. at 72. Find the alteration of his income.

For additional examples on Stocks, reference may be made to the author's 12mo. Arithmetic, as well as for a very large collection of miscellaneous examples in all parts of arithmetic.

tions with the same den^{r}, to find the part corresponding to that den^{r} with num^{r} *unity*, and then multiply this successively by the num^{rs} of the different fractions; thus we should find $\frac{1}{13}$ of £1000, and then multiply this by 6, 4, 3, respectively.

Ex. 3. *A*, *B*, and *C* form a joint capital for conducting a business, of which *A* contributes £500, *B* £650, and *C* £700. At the end of a year the profits are £555; what share should each receive?

Their shares should evidently be in the ratio of their contributions of capital, i.e. in the ratio of 500, 650, 700, or of 10, 13, 14; hence the fractions are $\frac{10}{37}$, $\frac{13}{37}$, $\frac{14}{37}$, and since $\frac{1}{37}$ of £555=£15, we have the shares required, £150, £195, £210.

Ex. 4. *A* begins business with a capital of £800, and at the end of 3 months takes *B* into partnership, with a capital of £1000; at the end of another 6 months they divide their profits, £330; what should each receive?

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hence *A*'s share=$\frac{6}{11}$ of £330=£180, and *B*'s share=$\frac{5}{11}$ of £330=£150.

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Here, as in Ex. 4, at the end of 18 months, the shares of capital supplied by *A*, *B*, *C*, respectively, may be measured by the numbers 500 × 18, 300 × 18, 1000 × 9, or 5, 3, 5, respectively: hence the fractions will be $\frac{5}{13}$, $\frac{3}{13}$, $\frac{5}{13}$; and since $\frac{1}{13}$ of £2000=£153 16*s.* 11$\frac{1}{13}$*d.*, their shares of profit will be £769 4*s.* 7$\frac{5}{13}$*d.*, £461 10*s.* 9$\frac{3}{13}$*d.*, £769 4*s.* 7$\frac{5}{13}$*d.*, respectively.

Ex. 150.

(1.) Divide 441 into parts, which shall be in the ratio of 1, 3, 5.

(2.) Divide 207 into parts, which shall be in the ratio of 1, $\frac{1}{3}$, $\frac{1}{5}$.

(3.) *A*, *B*, *C*, traded together, investing respectively £225, £325, and £425. Their profits were £585. What portion should each receive of this sum?

(4.) Gunpowder is made of 75 parts of nitre, 10 of sulphur, and 15 of charcoal. What weight of each will there be in 5 cwt.?

Ex. II.

Find the price of

			£	s.				£	s.
(1.)	1 gross at 3*d*.	*Ans.*	1	16	(2.)	1 gross at $4\frac{1}{4}d$.	*Ans.*	2	11
(3.)	1 gross at $5\frac{1}{2}d$.	*Ans.*	3	6	(4.)	1 gross at $6\frac{3}{4}d$.	*Ans.*	4	1
(5.)	1 gross at 7*d*.	*Ans.*	4	4	(6.)	1 gross at $8\frac{1}{4}d$.	*Ans.*	4	19
(7.)	1 gross at $9\frac{1}{2}d$.	*Ans.*	5	14	(8.)	1 gross at $10\frac{3}{4}d$.	*Ans.*	6	9
(9.)	1 gross at $11\frac{1}{4}d$.	*Ans.*	6	15	(10.)	1 gross at 1*s*. $0\frac{1}{2}d$.	*Ans.*	7	10
(11.)	1 gross at 1*s*. $3\frac{1}{4}d$.	*Ans.*	9	3	(12.)	1 gross at 2*s*. $1\frac{3}{4}d$.	*Ans.*	15	9

187. (iii.) *To find the price per score, the price of one being given.*

RULE. Reckon each shilling in the given price as a pound, each 3*d*. as 5*s*., 1*d*. as 1*s*. 8*d*., $\frac{1}{4}d$. as 5*d*.

Ex. 20 at 3*s*. 10*d*. = (£3 + 15*s*. + 1*s*. 8*d*. =) £3 16*s*. 8*d*.
For 20 at 1*s*. is £1, 20 at 3*d*. is 5*s*., 20 at 1*d*. is 1*s*. 8*d*., &c.

Ex. III.

Find the price of

			£	s.	d.				£	s.	d.
(1.)	20 at 3*s*.	*Ans*	3	0	0	(2.)	20 at 4*s*. 6*d*.	*Ans.*	4	10	0
(3.)	20 at 5*s*. 3*d*.	*Ans.*	5	5	0	(4.)	20 at 7*s*. 9*d*.	*Ans.*	7	15	0
(5.)	20 at 8*s*. $1\frac{1}{2}d$.	*Ans.*	8	2	6	(6.)	20 at 9*s*. $7\frac{1}{2}d$.	*Ans.*	9	12	6
(7.)	20 at 11*s*. $4\frac{1}{2}d$.	*Ans.*	11	7	6	(8.)	20 at 12*s*. $10\frac{1}{2}d$.	*Ans.*	12	17	6
(9.)	20 at 13*s*. 1*d*.	*Ans.*	13	1	8	(10.)	20 at 14*s*. 5*d*.	*Ans.*	14	8	4
(11.)	20 at 11*d*.	*Ans.*		18	4	(12.)	20 at 17*s*. $2\frac{1}{2}d$.	*Ans.*	17	4	2

188. (iv.) *To find the price per hundred, the price of one being given.*

RULE. Reckon, for every farthing in the given price, as many pence, and twice as many shillings.

Ex. 100 at $1\frac{1}{2}d$. = 12*s*. 6*d*.
For to take a penny for every farthing is the same as to multiply by 4, and to take 2*s*. for every farthing is the same as to multiply by 96; and 96 + 4 = 100.

Ex. IV.

Find the price of

			£	s.	d.				£	s.	d.
(1.)	100 at 1*d*.	*Ans.*		8	4	(2.)	100 at $1\frac{1}{4}d$.	*Ans.*		10	5
(3.)	100 at $2\frac{1}{2}d$.	*Ans.*	1	0	10	(4.)	100 at $3\frac{3}{4}d$.	*Ans.*	1	11	3
(5.)	100 at 4*d*.	*Ans.*	1	13	4	(6.)	100 at $5\frac{1}{4}d$.	*Ans.*	2	3	9
(7.)	100 at $6\frac{1}{2}d$.	*Ans.*	2	14	2	(8.)	100 at $7\frac{3}{4}d$.	*Ans.*	3	4	7
(9.)	100 at $8\frac{1}{4}d$.	*Ans.*	3	8	9	(10.)	100 at $9\frac{1}{2}d$.	*Ans.*	3	19	2
(11.)	100 at $10\frac{1}{2}d$.	*Ans.*	4	7	6	(12.)	100 at $11\frac{3}{4}d$.	*Ans.*	4	17	11

189. (v.) *To find the price of one article (and therefore, by multiplication, the price of any number of articles), the price per dozen being given.*

RULE. Reckon each shilling in the given price as a penny, 6*d.* as $\frac{1}{2}$*d.*, 3*d.* as $\frac{1}{4}$*d.*

Ex. The price of 1, at 11*s.* 9*d.* per dozen = 11$\frac{3}{4}$*d.*

For to reckon each shilling as a penny, and 3*d.* as $\frac{1}{4}$*d.*, is to divide by 12.

Ex. V.

Find the price of

	s.	*d.*		*s.*	*d.*
(1.) 1 at 7*s.* 6*d.* per doz. *Ans.*		7$\frac{1}{2}$	(2.) 1 at 8*s.* 3*d.* per doz. *Ans.*		8$\frac{1}{4}$
(3.) 1 at 4*s.* 9*d.* . . *Ans.*		4$\frac{3}{4}$	(4.) 1 at 6*s.* 6*d.* . . *Ans.*		6$\frac{1}{2}$
(5.) 1 at 9*s.* 9*d.* . . *Ans.*		9$\frac{3}{4}$	(6.) 1 at 11*s.* 3*d.* . . *Ans.*		11$\frac{1}{4}$
(7.) 1 at 12*s.* 6*d.* . . *Ans.*	1	0$\frac{1}{2}$	(8.) 1 at 13*s.* 3*d.* . . *Ans.*	1	1$\frac{1}{4}$
(9.) 2 at 14*s.* 9*d.* . . *Ans.*	2	5$\frac{1}{2}$	(10.) 3 at 15*s.* 6*d.* . . *Ans.*	3	10$\frac{1}{2}$
(11.) 4 at 16*s.* 3*d.* . . *Ans.*	5	5	(12.) 5 at 17*s.* 9*d.* . . *Ans.*	7	4$\frac{3}{4}$

190. (vi.) *To find the price of one article, the price per gross being given.*

RULE. Reckon each shilling in the given price as a penny, and divide by 12.

Ex. The price of 1 at 27*s.* per gross, is 2$\frac{3}{4}$*d.*

For to reckon each shilling as a penny is to divide by 12, and to divide again by 12 is equivalent to dividing altogether by 144.

Ex. VI.

Find the price of

	s.	*d.*		*s.*	*d.*
(1.) 1 at 9*s.* per gross. *Ans.*		0$\frac{3}{4}$	(2.) 1 at 15*s.* per gross. *Ans.*		1$\frac{1}{4}$
(3.) 1 at 24*s.* . . *Ans.*		2	(4.) 1 at 33*s.* . . *Ans.*		2$\frac{3}{4}$
(5.) 1 at 45*s.* . . *Ans.*		3$\frac{3}{4}$	(6.) 1 at 54*s.* . . *Ans.*		4$\frac{1}{2}$
(7.) 1 at 63*s.* . . *Ans.*		5$\frac{1}{4}$	(8.) 1 at 75*s.* . . *Ans.*		6$\frac{1}{4}$
(9.) 2 at 87*s.* . . *Ans.*	1	2$\frac{1}{2}$	(10.) 3 at 93*s.* . . *Ans.*	1	11$\frac{1}{4}$
(11.) 4 at 102*s.* . . *Ans.*	2	10	(12.) 5 at 117*s.* . . *Ans.*	4	0$\frac{3}{4}$

191. (vii.) *To find the price of one article, the price per score being given.*

RULE. Reckon each pound in the given price as a shilling, 10*s.* as 6*d.*, 5*s.* as 3*d.*, 2*s.* 6*d.* as 1$\frac{1}{2}$*d.*, 1*s.* 8*d.* as 1*d.*, 5*d.* as $\frac{1}{4}$*d.*

Ex. The price of 1, at £5 3*s.* 4*d.* per score, is 5*s.* 2*d.*

For to reckon each pound as a shilling, &c., is to divide by 20.

Ex. VII.

Find the price of

	s.	*d.*
(1.) 1 at £4 per score. *Ans.*	4	0
(2.) 1 at £5 10*s.* . . *Ans.*	5	6
(3.) 1 at £6 5*s.* . . . *Ans.*	6	3

				s.	d.
(4.)	1 at £7 15s.	per score	Ans.	7	9
(5.)	1 at £8 10s.		Ans.	8	6
(6.)	1 at £9 2s. 6d.		Ans.	9	1½
(7.)	1 at £11 12s. 6d.		Ans.	11	7½
(8.)	1 at £13 17s. 6d.		Ans.	13	10½
(9.)	1 at £15 1s. 8d.		Ans.	15	1
(10.)	1 at £17 3s. 4d.		Ans.	17	2
(11.)	1 at £18 16s. 8d.		Ans.	18	10
(12.)	1 at £20 18s. 4d.		Ans.	20	11

192. (viii.) *To find the price of any large number of articles, the price of one being given.*

Rule. Reckon the number of shillings (or pence) in the given price as the number of articles, and the number of articles as the number of shillings (or pence) in the price, and so multiply them.

Ex. The price of 48 articles at 5d. = price of 5 articles at 48d. or 4s. = 20s.
The price of 77 articles at 5s. = 5 × 77s. = £19 5s.
For we plainly get the same number of pence in the result, whether we multiply 5d. by 48 or 48d. by 5.

Ex. VIII.

Find the price of

			£	s.	d.				£	s.	d.
(1.)	19 at 4d.	Ans.		6	4	(2.)	29 at 5d.	Ans.		12	1
(3.)	37 at 7d.	Ans.	1	1	7	(4.)	43 at 8d.	Ans.	1	8	8
(5.)	41¾ at 3d.	Ans.		10	5¼	(6.)	52½ at 9d.	Ans.	1	19	4½
(7.)	19 at 4s.	Ans.	3	16	0	(8.)	29 at 5s.	Ans.	7	5	0
(9.)	37 at 7s.	Ans.	12	19	0	(10.)	43 at 8s.	Ans.	17	4	0
(11.)	41¾ at 3s.	Ans.	6	5	3	(12.)	52½ at 9s.	Ans.	23	12	6

193. (ix.) *To find the amount in a year of a given number of pence per day.*

Rule. Take, as in (viii.) the number of days in a year, 365, as pence, = £1 10s. 5d., and multiply this sum by the number of pence per day.

Ex. 365 days at 3d. per day = £4 11s. 3d.

If the year, omitting the Sundays, be reckoned as only 313 days, then multiply 313d. = £1 6s. 1d. by the number of pence per day.

If there are odd farthings in the sum per day, add for every farthing 7s. 7¼d. or 6s. 6¼d., according as there are to be reckoned 365 or 313 days in the year.

Ex. 313 days at 11¾d. per day = £15 6s. 5¾d.
For 365f. = 91¼d. = 7s. 7¼d., and 313f. = 78¼d. = 6s. 6¼d.

Ex. IX.

Find the value of

				£	s.	d.
(1.)	365 days at	4*d.*	*Ans.*	6	1	8
(2.)	365 . .	5*d.*	*Ans.*	7	12	1
(3.)	365 . .	7*d.*	*Ans.*	10	12	11
(4.)	365 . .	9*d.*	*Ans.*	13	13	9
(5.)	365 . .	$11\frac{1}{4}$*d.*	*Ans.*	17	2	$2\frac{1}{4}$
(6.)	365 . .	$14\frac{1}{2}$*d.*	*Ans.*	22	1	$0\frac{1}{2}$
(7.)	313 . .	6*d.*	*Ans.*	7	16	6
(8.)	313 . .	8*d.*	*Ans.*	10	8	8
(9.)	313 . .	10*d.*	*Ans.*	13	0	10
(10.)	313 . .	14*d.*	*Ans.*	18	5	2
(11.)	313 . .	$15\frac{1}{4}$*d.*	*Ans.*	19	17	$9\frac{1}{4}$
(12.)	313 . .	$17\frac{1}{2}$*d.*	*Ans.*	22	16	$5\frac{1}{2}$

194. (x.) *To find the price of a pound, the price of an ounce* (*Av.*) *being given.*

Rule. Reckon each farthing in the given price as a shilling, and divide by 3.

Ex. The price of 1 lb., at $3\frac{1}{2}$*d.* per oz., is 4*s.* 8*d.*

For to take each farthing as a shilling is to multiply by 48, and then to divide by 3 is equivalent to multiplying altogether by 16, the number of ounces in a lb.

Ex. X.

Find the price of

			s.	d.				s.	d.
(1.)	1 lb. at $1\frac{1}{4}$*d.*	per oz. *Ans.*	1	8	(2.)	1 lb. at $2\frac{1}{2}$*d.*	per oz. *Ans.*	3	4
(3.)	1 lb. at $3\frac{3}{4}$*d.*	per oz. *Ans.*	5	0	(4.)	1 lb. at 4*d.*	per oz. *Ans.*	5	4
(5.)	1 lb. at $5\frac{1}{2}$*d.*	per oz. *Ans.*	7	4	(6.)	1 lb. at $6\frac{1}{4}$*d.*	per oz. *Ans.*	8	4
(7.)	1 lb. at $7\frac{3}{4}$*d.*	per oz. *Ans.*	10	4	(8.)	1 lb. at $9\frac{1}{2}$*d.*	per oz. *Ans.*	12	8
(9.)	1 lb. at 11*d.*	per oz. *Ans.*	14	8	(10.)	1 lb. at $13\frac{1}{4}$*d.*	per oz. *Ans.*	17	8
(11.)	1 lb. at $14\frac{1}{2}$*d.*	per oz. *Ans.*	19	4	(12.)	1 lb. at $15\frac{3}{4}$*d.*	per oz. *Ans.*	21	0

195. (xi.) *To find the price of an ounce, the price of a pound being given.*

Rule. Multiply the given price by 3, and reckon each shilling in the result as a farthing.

Ex. The price of 1 oz., at 4*s.* 6*d.* per lb., is 13*f.* = $3\frac{1}{4}$*d.* (+).

For to reckon each shilling as a farthing is to divide by 48: and to multiply by 3, and divide by 48, is equivalent altogether to dividing by 16.

Ex. XI.

Find the price of

	d.		d.
(1.) 1 oz. at 2*s.* per lb.	*Ans.* $1\frac{1}{2}$	(2.) 1 oz. at 3*s.* 4*d.* per lb.	*Ans.* $2\frac{1}{2}$
(3.) 1 oz. at 4*s.* 8*d.* per lb.	*Ans.* $3\frac{1}{2}$	(4.) 1 oz. at 5*s.* 6*d.* per lb.	*Ans.* 4
(5.) 1 oz. at 6*s.* 9*d.* per lb.	*Ans.* 5	(6.) 1 oz. at 7*s.* 8*d.* per lb.	*Ans.* $5\frac{3}{4}$
(7.) 1 oz. at 8*s.* 7*d.* per lb.	*Ans.* $6\frac{1}{4}$	(8.) 1 oz. at 9*s.* 2*d.* per lb.	*Ans.* $6\frac{3}{4}$
(9.) 1 oz. at 10*s.* 6*d.* per lb.	*Ans.* $7\frac{3}{4}$	(10.) 1 oz. at 11*s.* 10*d.* per lb.	*Ans.* $8\frac{3}{4}$
(11.) 1 oz. at 12*s.* 9*d.* per lb.	*Ans.* $9\frac{1}{2}$	(12.) 1 oz. at 13*s.* 8*d.* per lb.	*Ans.* $10\frac{1}{4}$

196. (xii.) *To find the price of a hundred-weight, the price per pound being given.*

RULE. Take the number of lbs. in a cwt. (112) as pence (= 9*s.* 4*d.*), or as shillings (= £5 12*s.*), and multiply by the number of pence, or of shillings, respectively, in the given price per pound.

Ex. The price of 1 cwt. at 2*d.* per lb. (= 2 × 9*s.* 4*d.*) is 18*s.* 8*d.*
1 cwt. at 2*s.* per lb. (= 2 × £5 12*s.*) is £11 4*s.*

If the price be given in pence, and there are odd farthings in it, add (112*f.* =) 2*s.* 4*d.* for every farthing.

If the price be given in shillings, and there are odd pence in it, add, as above, 9*s.* 4*d.* for every penny.

Ex. XII.

Find the price of

		£	s.	d.
(1.) 1 cwt. at 3*d.* per lb.	*Ans.*	1	8	0
(2.) 1 cwt. at 4*d.* per lb.	*Ans.*	1	17	4
(3.) 1 cwt. at 5*d.* per lb.	*Ans.*	2	6	8
(4.) 1 cwt. at 7*d.* per lb.	*Ans.*	3	5	4
(5.) 1 cwt. at $9\frac{1}{4}$*d.* per lb.	*Ans.*	4	6	4
(6.) 1 cwt. at $11\frac{1}{2}$*d.* per lb.	*Ans.*	5	7	4
(7.) 1 cwt. at 3*s.* per lb.	*Ans.*	16	16	0
(8.) 1 cwt. at 4*s.* per lb.	*Ans.*	22	8	0
(9.) 1 cwt. at 5*s.* 6*d.* per lb.	*Ans.*	30	16	0
(10.) 1 cwt. at 7*s.* 10*d.* per lb.	*Ans.*	43	17	4
(11.) 1 cwt. at 8*s.* $8\frac{1}{2}$*d.* per lb.	*Ans.*	48	15	4
(12.) 1 cwt. at 9*s.* $2\frac{3}{4}$*d.* per lb.	*Ans.*	51	13	8

Since 1 ton = 20 cwt., the price per ton may be found from the price per cwt., as in (iii.)

197. (xiii.) *To find the price of a pound, the price per hundred-weight being given.*

RULE. Multiply the given price by 3, and divide by 7, and reckon each shilling in the result as a farthing.

Ex. The price of 1 lb., at 17*s.* 9*d.* per cwt., is 7*f.* = $1\frac{3}{4}$*d.* (+). For to reckon each shilling as a farthing is to divide by 48; and to multiply by 3, divide by 7, and divide again by 48, is equivalent to dividing altogether by 112.

Ex. XIII.

Find the price of

			d.
(1.)	1 lb. at 9*s.* per cwt.	*Ans.*	$\frac{3}{4}$
(2.)	1 lb. at 14*s.* 6*d.* per cwt.	*Ans.*	$1\frac{1}{2}$
(3.)	1 lb. at 18*s.* 3*d.* per cwt.	*Ans.*	$1\frac{3}{4}$
(4.)	1 lb. at 19*s.* 7*d.* per cwt.	*Ans.*	2
(5.)	1 lb. at 23*s.* 5*d.* per cwt.	*Ans.*	$2\frac{1}{2}$
(6.)	1 lb. at 26*s.* 4*d.* per cwt.	*Ans.*	$2\frac{3}{4}$
(7.)	1 lb. at 31*s.* 8*d.* per cwt.	*Ans.*	$3\frac{1}{4}$
(8.)	1 lb. at 36*s.* 10*d.* per cwt.	*Ans.*	$3\frac{3}{4}$
(9.)	1 lb. at 43*s.* 2*d.* per cwt.	*Ans.*	$4\frac{1}{2}$
(10.)	1 lb. at 47*s.* 6*d.* per cwt.	*Ans.*	5
(11.)	1 lb. at 56*s.* 7*d.* per cwt.	*Ans.*	6
(12.)	1 lb. at 67*s.* 8*d.* per cwt.	*Ans.*	$7\frac{1}{4}$

198. (xiv.) *To find the interest on any sum for one year, at* 5 *per cent. per annum.*

Rule. Reckon a shilling for every pound in the given sum, and 3*d.* for every 5*s.* &c. as in (vii.)

Ex. The interest on £75 10*s.*, for one year, at 5 per cent., is £3 15*s.* 6*d.* For 5 per cent. is $\frac{1}{20}$ of the whole; and 1*s.* is £$\frac{1}{20}$, 3*d.* is $\frac{1}{20}$ of 5*s.*, &c.

Ex. XIV.

Find the interest for one year, at 5 per cent.,

		£	*s.*	*d.*		£	*s.*	*d.*			£	*s.*	*d.*		£	*s.*	*d.*
(1.)	On	7	15	0	*Ans.*		7	9	(2.)	On	15	10	0	*Ans.*		15	6
(3.)	On	23	5	0	*Ans.*	1	3	3	(4.)	On	31	15	0	*Ans.*	1	11	9
(5.)	On	42	2	6	*Ans.*	2	2	$1\frac{1}{2}$	(6.)	On	57	7	6	*Ans.*	2	17	$4\frac{1}{2}$
(7.)	On	68	12	6	*Ans.*	3	8	$7\frac{1}{2}$	(8.)	On	79	17	6	*Ans.*	3	19	$10\frac{1}{2}$
(9.)	On	88	7	6	*Ans.*	4	8	$4\frac{1}{2}$	(10.)	On	94	12	6	*Ans.*	4	14	$7\frac{1}{2}$
(11.)	On	134	17	6	*Ans.*	6	14	$10\frac{1}{2}$	(12.)	On	144	2	6	*Ans.*	7	4	$1\frac{1}{2}$

199. (xv.) *To find the interest on any sum for any number of months, at* 5 *per cent. per annum.*

Rule. Multiply the given sum by the number of months, and reckon a penny for every pound in the result, and $\frac{1}{4}$*d.* for every 5*s.*, &c.

Ex. The interest on £75 10*s.* for 7 months, at 5 per cent., is

$$7 \times 75\tfrac{1}{2}d. = 7 \times 6s.\ 3\tfrac{1}{2}d. = £2\ 4s.\ 0\tfrac{1}{2}d.$$

For 1*s.* is the interest on £1 for a year, and, therefore, 1*d.* is the interest on £1 for a month.

Ex. XV.

Find the interest, at 5 per cent. per annum,

		£	s.	d.					£	s.	d.
(1.)	On	7	15	0	for	3	months.	*Ans.*		1	$11\frac{1}{4}$
(2.)	On	15	10	0	for	4	months.	*Ans.*		5	2
(3.)	On	23	5	0	for	5	months.	*Ans.*		9	$8\frac{1}{4}$
(4.)	On	31	15	0	for	7	months.	*Ans.*		18	$6\frac{1}{4}$
(5.)	On	42	2	6	for	8	months.	*Ans.*	1	8	1
(6.)	On	57	7	6	for	9	months.	*Ans.*	2	3	$0\frac{1}{4}$
(7.)	On	68	12	6	for	10	months.	*Ans.*	2	17	$2\frac{1}{4}$
(8.)	On	79	17	6	for	11	months.	*Ans.*	3	13	$2\frac{1}{2}$
(9.)	On	88	7	6	for	$3\frac{1}{2}$	months.	*Ans.*	1	5	$9\frac{1}{4}$
(10.)	On	94	12	6	for	$5\frac{1}{2}$	months.	*Ans.*	2	3	$4\frac{1}{4}$
(11.)	On	134	17	6	for	$7\frac{1}{4}$	months.	*Ans.*	4	1	$5\frac{3}{4}$
(12.)	On	144	2	6	for	$9\frac{3}{4}$	months.	*Ans.*	5	17	1

200. (xvi.) *To find the interest on any sum for any number of years, at any rate per cent. per annum.*

Rule. Multiply the number of pounds in the given sum by the number of years, and the rate per cent., *after first doubling one of them.* Then the last figure of the result will give the odd pence, and the other figures the shillings, in the Answer.

Ex. To find the interest on £119, for 3 years, at $2\frac{1}{2}$ per cent.

$$\begin{array}{r} 119 \\ \underline{3} \\ 357 \\ 5 = 2 \times 2\tfrac{1}{2} \\ \overline{178,5} = \text{£}8\ 18s.\ 5d. \end{array}$$

To explain this very simple rule, we may leave out of consideration the multiplication by the number of years, the reason for which is evident; since the interest for 3 years must be 3 times the interest for one year. And, to find the interest for one year, we have to multiply by the rate and divide by 100, which will give the answer in pounds, or we may multiply by 2 × rate, and divide by 200, which will be the same thing, and will, therefore, also give the answer in pounds. But to divide by 200 is to divide by 20 and by 10; and we *have* divided by 20 when we took the result as shillings instead of pounds, and by 10 when we struck off the last figure of the result.

Strictly speaking, however, the 5 is not 5*d.* ($=\frac{5}{12}$ of a shilling), but $\frac{5}{10}s.=\frac{1}{2}s.=6d.$: and this process will bring out 1*d.* wrong in the result, if the pence are 5 or more.

If there should be shillings in the given sum, a separate calculation may be made for them; but the interest upon these will generally be so small, unless the time is long, or the rate very high, that in practice they may be altogether neglected.

Ex. XVI.

Find the interest

		£			£	s.	d.
(1.)	On	231,	for 2 years, at 3 per cent. per ann.	*Ans.*	13	17	2
(2.)	On	307,	for 3 years, at 2	*Ans.*	18	8	4
(3.)	On	429,	for 3 years, at 3½	*Ans.*	45	0	10
(4.)	On	517,	for 2 years, at 3¼	*Ans.*	33	12	1
(5.)	On	625,	for 2½ years, at 3	*Ans.*	46	17	5
(6.)	On	747,	for 3½ years, at 4	*Ans.*	104	11	6
(7.)	On	882,	for 3 years, at 2¼	*Ans.*	59	10	7
(8.)	On	914,	for 3 years, at 3¾	*Ans.*	102	16	5
(9.)	On	974,	for 1½ years, at 2½	*Ans.*	36	10	5
(10.)	On	1015,	for 2½ years, at 1½	*Ans.*	38	1	2
(11.)	On	1111,	for 2½ years, at 4¼	*Ans.*	118	0	8
(12.)	On	1853,	for 3¼ years, at 4¾	*Ans.*	286	1	1

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ANSWERS
TO
THE EXAMPLES.

PART I.

Ex. 1.

(1.) 12, 14, 19, 27.
(2.) 30, 39, 47, 50.
(3.) 61, 70, 88, 92.
(4.) 101, 110, 113, 155.
(5.) 200, 202, 212, 330.
(6.) 507, 663, 717, 880.
(7.) 1001, 1010, 1100, 1011.
(8.) 1101, 1110, 1111, 1605.
(9.) 1070, 2303, 3030, 4014.
(10.) 3212, 4118, 5607, 7190.
(11.) 1600, 2011, 8050, 9001.
(12.) 1014, 3000, 7006, 8523.
(13.) 20000, 43059, 68070, 89605.
(14.) 61000, 70008, 94620, 30211.
(15.) 13031, 30013, 31301, 52073.
(16.) 78011, 90704, 12005, 20102.
(17.) 500002, 600713, 810101, 905050.
(18.) 3500018, 14207573, 107204003, 1234567890.

Ex. 2.

(1.) Ten, thirteen, seventeen, twenty-three.

(2.) Thirty-five, forty, fifty-five, seventy-three.

(3.) One hundred, one hundred and one, one hundred and ten, one hundred and eleven.

(4.) Two hundred and seventy-six, three hundred and seventeen, three hundred and seventy-one, four hundred and one.

(5.) Four hundred and ten, four hundred and forty-four, five hundred and five, six hundred and eleven.

(6.) Seven hundred and seventeen, eight hundred and eighty, nine hundred and nine, nine hundred and seventy-one.

(7.) One thousand, one thousand and one, one thousand and ten, one thousand one hundred.

(8.) One thousand one hundred and ten, one thousand one hundred and one, one thousand and eleven, one thousand one hundred and eleven.

(9.) Two thousand and thirty-two, three thousand one hundred and five, four thousand and four, five thousand one hundred and thirty.

(10.) Six thousand and six, seven thousand six hundred and seven, eight thousand one hundred and eighteen, nine thousand and ninety.

(11.) Seven thousand, six thousand and five, eight thousand and thirty, nine thousand three hundred.

(12.) Four thousand seven hundred and sixty, three thousand nine hundred and seven, two thousand and eighty-one, six thousand seven hundred and eighty-nine.

(13.) Twelve thousand three hundred and forty-five, forty thousand three hundred and forty, sixty thousand and forty-five, three hundred and twelve thousand one hundred and forty-seven.

(14.) Seventy thousand seven hundred and seventeen, eighty-two thousand and ten, ninety-seven thousand five hundred and thirty-one, six hundred and five thousand one hundred and three.

(15.) Seven hundred and twenty-eight thousand and seven, eight hundred and one thousand and eleven, nine hundred and ten thousand and one, one million and twenty-three thousand seven hundred and fourteen.

(16.) Four hundred and fifty thousand one hundred and sixty, two hundred and seven thousand seven hundred and three, five hundred and sixty thousand six hundred and two, two millions one hundred and three thousand seven hundred and fifty-six.

(17.) Three millions and forty-six thousand three hundred and two, four millions four hundred and seventy-one thousand one hundred and seven, five millions six hundred and ten thousand and ninety-eight, seven millions two hundred and thirty-four thousand and ten.

(18.) Nine millions eight hundred and seventy-six thousand five hundred and forty-three, ninety-eight millions seven hundred and sixty-five thousand four hundred and thirty-two, nine hundred and eighty-seven millions six hundred and fifty-four thousand three hundred and twenty-one, nine thousand eight hundred and seventy-six millions five hundred and forty-three thousand two hundred and ten.

Ex. 3.

(1.) XIII, XIV, XV, XVI.
(2.) XVII, XVIII, XIX, XX.
(3.) XXV, XXIX, XXXIV, XXXVII.
(4.) XLVI, XLIX, LV, LVIII.
(5.) LXIII, LXVII, LXX, LXXVI.
(6.) LXXXIV, LXXXVIII, XCI, XCIX.
(7.) CIII, CLIV, CCXIV, CCXXVII.
(8.) CCCXXXVIII, CCCLXXXIII, CCCCXXVII, CCCCLXXII.
(9.) DIV, DLX, DCXVI, DCLXXVII.
(10.) DCCLXXXIX, DCCXCVIII, DCCCXV, DCCCLI.
(11.) DCCCCIX, DCCCCXLIV, MXXXVII, MLIV.
(12.) MCCXXXIV, MDLXVII, MDCCCLIII, MDCCCCXCIX.

Ex. 4.

(1.) 79. (2.) 80. (3.) 91. (4.) 115.
(5.) 105. (6.) 141. (7.) 813. (8.) 738.
(9.) 1386. (10.) 1657. (11.) 1494. (12.) 810.
(13.) 19380. (14.) 16046. (15.) 15531. (16.) 23871.
(17.) 10661. (18.) 18194. (19.) 15822. (20.) 18243.
(21.) 240868. (22.) 272737. (23.) 202771.
(24.) 219718. (25.) 2017300. (26.) 1920077.
(27.) 1097836. (28.) 1927842. (29.) 1909368.
(30.) 1601963. (31.) 2154904. (32.) 1263987.

Ex. 5.

(1.) 36. (2.) 40, 53, 93. (3.) 200. (4.) 5856, 4200.
(5.) 78. (6.) 1863. (7.) 49. (8.) 98. (9.) 65.
(10.) 124. (11.) 558. (12.) 332. (13.) 484.
(14.) 428. (15.) 282. (16.) 36273, 69686.
(17.) 72447. (18.) 1000. (19.) 2278. (20.) 939.
(21.) 550. (22.) 177. (23.) 2367. (24.) 2000.

Ex. 6.

(1.) 223234. (2.) 324264. (3.) 355422. (4.) 343154.
(5.) 125293. (6.) 421624. (7.) 316463. (8.) 133351.
(9.) 317304. (10.) 531415. (11.) 240933. (12.) 103227.
(13.) 67475. (14.) 456327. (15.) 182437. (16.) 454032.
(17.) 271456. (18.) 676231. (19.) 83252. (20.) 707322.
(21.) 5878578. (22.) 38523622. (23.) 18291029.
(24.) 79579358. (25.) 58690275. (26.) 9578344.

(27.) 6422929. (28.) 99887648. (29.) 27388074.
(30.) 8678423. (31.) 7889267. (32.) 75308643.

Ex. 7.

(1.) 310913, 255. (2.) 19. (3.) 1656, 508. (4.) 786.
(5.) 33, 11. (6.) 18. (7.) 28. (8.) 34.
(9.) 849. (10.) 59. (11.) 169. (12.) 209.
(13.) 389. (14.) 37. (15.) 899, 89.
(16.) 19, 191. (17.) 41. (18.) 46, 47; 83, 53.
(19.) Oct. 21, 1805. (20.) 259. (21.) 69.
(22.) 25, 31. (23.) 243. (24.) 150.

Ex. 8.

(1.) 468642. (2.) 306526. (3.) 528744.
(4.) 514172. (5.) 550588. (6.) 1072026.
(7.) 939750. (8.) 1721052. (9.) 1610622.
(10.) 2169416. (11.) 776388. (12.) 2687356.
(13.) 1831744. (14.) 3117100. (15.) 2387148.
(16.) 2328690. (17.) 1343975. (18.) 2581518.
(19.) 3283850. (20.) 4031904. (21.) 22712400.
(22.) 3662001. (23.) 2914734. (24.) 3969938.
(25.) 3355555. (26.) 5048336. (27.) 2013578.
(28.) 2765376. (29.) 6940512. (30.) 5608323.
(31.) 3021576. (32.) 4154283. (33.) 4119273.
(34.) 62005000. (35.) 3589155. (36.) 7473020.
(37.) 5758016. (38.) 7614804. (39.) 9268237.
(40.) 6765408. (41.) 4160695. (42.) 6730488.
(43.) 7202118. (44.) 8742432. (45.) 6240828.
(46.) 9195420. (47.) 9632634. (48.) 11588856.

Ex. 9.

(1.) 94, 376, 470. (2.) 278. (3.) 64. (4.) 390.
(5.) 227. (6.) 85. (7.) 324; 1019. (8.) 117.
(9.) 192. (10.) 1344; 1280. (11.) 143.
(12.) 3388. (13.) 1476. (14.) 240; 70.
(15.) 14. (16.) 144. (17.) 1078.
(18.) 420. (19.) 1275. (20.) 96.
(21.) 129. (22.) 557. (23.) 108.
(24.) Farmer *J* must pay to Farmer *B* £1.

Ex. 10.

(1.) 6086444. (2.) 5413170. (3.) 10468368.
(4.) 7325748. (5.) 14213367. (6.) 116179580.

(7.) 1623321600. (8.) 24265800. (9.) 13122999.
(10.) 10324608. (11.) 130236480. (12.) 15902568.
(13.) 21271536. (14.) 415409760. (15.) 36678852.
(16.) 4646046600. (17.) 19384155. (18.) 42219144.
(19.) 49964684. (20.) 38125242. (21.) 50065992.
(22.) 81654309. (23.) 48534288. (24.) 97183440.

Ex. 11.

(1.) 675. (2.) 20464. (3.) 12366. (4.) 1974.
(5.) 3888. (6.) 5110. (7.) 432. (8.) 2628.
(9.) 6648. (10.) 888. (11.) 2552; 464.
(12.) 733266. (13.) 788. (14.) 2016.
(15.) 88. (16.) 1028934; 3163338.
(17.) 435. (18.) 22. (19.) 508.
(20.) 294. (21.) 25056. (22.) 99960.
(23.) 6432. (24.) 107370.

Ex. 12.

(1.) 7465692. (2.) 114317010. (3.) 80974901.
(4.) 176135792. (5.) 22158146. (6.) 1362440700.
(7.) 28640516. (8.) 260775900. (9.) 322096660.
(10.) 22492649. (11.) 181424052. (12.) 29420822.
(13.) 4785230400. (14.) 74756752. (15.) 295313445.
(16.) 74268826. (17.) 2437271220. (18.) 2625925122.
(19.) 1244477520. (20.) 2215413970. (21.) 3036221280.
(22.) 4506647293. (23.) 5860980027. (24.) 3052222830

Ex. 13.

(1.) 169, 2197. (2.) 27702. (3.) 160063.
(4.) 67. (5.) 85; 59. (6.) 5472.
(7.) 97782. (8.) 118446. (9.) 1.
(10.) 24225. (11.) 27720000. (12.) 2405.
(13.) 1672; 41800. (14.) 173. (15.) 100170.
(16.) 32300. (17.) 1357. (18.) 9130; 1411.
(19.) 276948. (20.) 36120000. (21.) 1805000000.
(22.) 22; 9988. (23.) 14061600. (24.) 11891880.

Ex. 14.

(1.) 234103. (2.) 281937. (3.) 192288.
(4.) 382899. (5.) $376956\frac{1}{2}$. (6.) 223497.
(7.) $148567\frac{1}{2}$. (8.) 158935. (9.) $129884\frac{2}{3}$.
(10.) 160679. (11.) 52263. (12.) 190882.

(13.) $626181\frac{1}{4}$. (14.) 135207. (15.) $44463\frac{3}{4}$.
(16.) $177127\frac{2}{5}$. (17.) $634440\frac{4}{5}$. (18.) 1223807.
(19.) $52141\frac{2}{5}$. (20.) 164079. (21.) $49421\frac{5}{6}$.
(22.) 48950. (23.) $78535\frac{5}{6}$. (24.) 80496.
(25.) $98649\frac{5}{7}$. (26.) 95679. (27.) $125379\frac{1}{7}$.
(28.) 432768. (29.) $101543\frac{3}{8}$. (30.) 872936.
(31.) 123456. (32.) 411125. (33.) 86295.
(34.) $2345678\frac{9}{10}$. (35.) $137174\frac{1}{9}$. (36.) $56721\frac{3}{10}$.
(37.) 3972597. (38.) 3968361. (39.) 5214769.
(40.) 5728531. (41.) $3422952\frac{1}{11}$. (42.) $4731118\frac{1}{12}$.
(43.) $5955211\frac{8}{11}$. (44.) 4646840. (45.) 6012548.
(46.) 6094039. (47.) 7937497. (48.) 123456789.

Ex. 15.

(1.) 7. (2.) 67. (3.) 36; 24. (4.) 7.
(5.) 37. (6.) 273. (7.) 117; 91. (8.) 33; 24.
(9.) 18. (10.) 897. (11.) 48.
(12.) 19, and 3 over. (13.) 97. (14.) 16.
(15.) 32; 93. (16.) 1710. (17.) 29.
(18.) 3502. (19.) 27; 18; 6 (20.) 69.
(21.) 494. (22.) 87; 14. (23.) 29; 19.
(24.) 605; 46; 50.

Ex. 16.

(1.) 1974538. (2.) 2384163 (3.) 2167022.
(4.) 2645753. (5.) 1783618. (6.) $2033671\frac{7}{20}$.
(7.) $2812436\frac{13}{20}$. (8.) $2874865\frac{23}{30}$. (9.) 1575109.
(10.) 3057551. (11.) $1955866\frac{17}{40}$. (12.) $1342474\frac{47}{5}$.
(13.) $1367372\frac{6}{49}$. (14.) $1624957\frac{19}{45}$. (15.) $1160496\frac{13}{7}$.
(16.) $1184888\frac{31}{77}$. (17.) $125550\frac{79}{84}$. (18.) $455344\frac{47}{80}$.
(19.) $464243\frac{29}{96}$. (20.) $525289\frac{35}{108}$. (21.) $528937\frac{23}{120}$.
(22.) $583664\frac{91}{121}$. (23.) $518463\frac{7}{132}$. (24.) 248005.

Ex. 17.

(1.) $26693360\frac{1}{37}$. (2.) $2296870\frac{221}{430}$. (3.) $167399\frac{221}{5900}$.
(4.) $12995451\frac{45}{76}$. (5.) $1135234\frac{741}{870}$. (6.) $103963\frac{5821}{9500}$.
(7.) $923041\frac{451}{1070}$. (8.) $3903772\frac{5}{253}$. (9.) $1948036\frac{69}{507}$.
(10.) $492595\frac{1346}{2005}$. (11.) $281463\frac{654}{3509}$. (12.) $21103\frac{33921}{46800}$.

Ex. 18.

(1.) 11. (2.) 36; 3. (3.) 19. (4.) 22.
(5.) 18. (6.) 90; 5. (7.) 1. (8.) 15625.

(9.) 24. (10.) 331. (11.) 67266. (12.) 57.
(13.) 12345. (14.) 22221. (15.) Between 5 and 6 P.M.
(16.) 45. (17.) 118. (18.) 24 ; 8. (19.) 2.
(20.) 12. (21.) 57. (22.) 9; 114. (23.) 317.
(24.) 23, and 11 over.

Ex. 19.

(1.) 189562, 289474. (2.) 207824, 202607.
(3.) 183664, 156008. (4.) 315656, 403224.
(5.) 515511, 226818. (6.) 283294, $272938\frac{4}{5}$.
(7.) $593620\frac{5}{6}$, 259395. (8.) 349912, $570752\frac{4}{7}$.
(9.) 1850895, 1947056. (10.) 1499784, 4889570.
(11.) $8981089\frac{1}{3}$, 13830408. (12.) $15128990\frac{1}{5}$, $13410720\frac{1}{6}$.

Ex. 20.

(1.) 126. (2.) 2380. (3.) *A*, 26 ; *B*, 24.
(4.) 75. (5.) 354. (6.) 9.
(7.) 12, 9, 10, 5. (8.) 1617; 147. (9.) 360, 432.
(10.) 15 years. (11.) 2. (12.) 9.
(13.) 275. (14.) 98.
(15.) The latter by 1*d*. (16.) 74.
(17.) 632. (18.) 10000. (19.) 10.
(20.) 78. (21.) 234. (22.) 416.
(23.) 75. (24.) 1512, 1440.

Ex. 21.

(1.) £5; £16. (2.) £30; £4. (3.) £63; £308.
(4.) 19*s.*; 68. (5.) £30; 54. (6.) 32*s.*; 625.
(7.) 125*s.*; 20. (8.) 150*s.* (9.) £143; 77.
(10.) £180; 63. (11.) £33. (12.) £85.

Ex. 22.

(1.) 8. (2.) 21. (3.) 12. (4.) 16.
(5.) 32. (6.) 6. (7.) 6. (8.) 45.
(9.) 3. (10.) 6. (11.) 12800. (12.) 10.

Ex. 23.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(1.)	136	18	7.	(2.)	223	18	1.	(3.)	162	12	3.
(4.)	150	8	$6\frac{1}{2}$.	(5.)	175	17	$10\frac{3}{4}$.	(6.)	155	2	$10\frac{1}{4}$.
(7.)	404	17	$8\frac{1}{2}$.	(8.)	510	16	11.	(9.)	523	2	$8\frac{1}{2}$.
(10.)	430	0	4.	(11.)	468	18	$6\frac{1}{2}$.	(12.)	602	6	$0\frac{1}{2}$.

Ex. 24.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	17	3	4½.	(2.)	38	3	1¾.	(3.)	69	4	10.
(4.)	4	14	2.	(5.)	86	17	5½.	(6.)	2	10	5.
(7.)	2	11	2¼.	(8.)	24	10	3¾.	(9.)	105	8	11¾.
(10.)	26	19	7.	(11.)	112	19	8¼.	(12.)	1	0	10¼.
(13.)	119	19	2¾.	(14.)	188	9	2¾.	(15.)	268	12	9½.
(16.)	366	16	8¾.	(17.)	265	4	5¾.	(18.)	125	10	1½.
(19.)	109	15	1¾.	(20.)	577	17	9½.	(21.)	193	8	8¾.
(22.)	191	3	0¼.	(23.)	849	8	11.	(24.)	40	3	1¾.

Ex. 25.

	£	s.	d.		£	s.	d.	£	s.	d.		£	s.	d.
(1.)	10	19	6.	(2.)	1	2	4½;	19	17	1½.	(3.)	42	3	2.
(4.)		19	1.	(5.)		14	0.				(6.)		2	7.
(7.)			3½.	(8.)		16	4¾.				(9.)	*B* by	1	2.
(10.)	3	2	7¾.	(11.)		17	0½.				(12.)			1.

Ex. 26.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	75	2	4.	(2.)	94	1	10½.	(3.)	115	3	9.
(4.)	190	1	10½.	(5.)	293	14	10.	(6.)	203	16	11¼.
(7.)	306	13	5.	(8.)	401	17	8½.	(9.)	552	4	9.
(10.)	438	18	2¾.	(11.)	585	1	4½.	(12.)	697	4	3½.
(13.)	809	6	6.	(14.)	890	16	3¾.	(15.)	1119	0	4.
(16.)	1323	7	1½.	(17.)	1525	16	0½.	(18.)	1501	9	0¾.
(19.)	1716	16	3.	(20.)	1956	7	8¼.	(21.)	2160	11	3.
(22.)	2197	19	3.	(23.)	2311	12	6.	(24.)	2446	13	9.

Ex. 27.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	11	14	9½.	(2.)	2	2	9.	(3.)	21	17	1.
(4.)	7	10	4.	(5.)	18	16	3.	(6.)	4	14	7¾.
(7.)	3	1	0.	(8.)	96	3	4.	(9.)	52	1	0.
(10.)	177	14	6½.	(11.)	20	11	4.	(12.)	113	17	0.

Ex. 28.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	450	0	0.	(2.)	41	18	6.	(3.)		11	11½.
(4.)	3	7	1.	(5.)		7	9½. }	(6.)	2	5	0.
					1	0	10⅛. }				

	£	s.	d.		£	s.	d.		£	s.	d.
(7.)	2	15	6.	(8.)	2	11	9.	(9.)	3	7	3½.
(10.)	11	11	0.	(11.)	1	10	5.		1	12	8½.
	5	5	0.					(12.)	9	0	0.
	5	7	6.								

Ex. 29.

	£	s.	d.		£	s.	d.
(1.)	414	6	3.	(2.)	774	8	3.
(3.)	942	5	5.	(4.)	1840	15	7½.
(5.)	2912	17	2¼.	(6.)	3759	10	6.
(7.)	4538	15	1½.	(8.)	5264	4	4½.
(9.)	7381	8	3.	(10.)	11365	12	6.
(11.)	12784	9	6.	(12.)	14399	17	0.

Ex. 30.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	124	1	11.	(2.)	35	9	4.	(3.)	112	11	1½.
(4.)	60	12	5¼.	(5.)	919	6	10½.	(6.)	656	11	4½.
(7.)	214	17	6.	(8.)	211	4	5½.	(9.)	654	12	9.
(10.)	303	10	2.	(11.)	639	11	3.	(12.)	657	9	9½.

Ex. 31.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)		1	1.	(2.)	23	8	0.	(3.)	12	10	0.
(4.)	15	18	0.	(5.)	33	10	7½.	(6.)	9	18	8.
	9	6	0.								
(7.)	5	10	9¾.	(8.)	343	15	0.	(9.)	266	14	6¾.
					£3437500.						
(10.)		6	5.	(11.)	400	0	0.	(12.)	322	15	1½.

Ex. 32.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	3	18	4.	(2.)	6	10	1¾.	(3.)	5	0	10½.
(4.)	7	18	10¼.	(5.)	1	10	9¼.	(6.)	6	6	6¾.
(7.)	5	6	1½.	(8.)	5	8	4¾.	(9.)	8	18	8½.
(10.)	11	7	4¼.	(11.)	18	17	2½.	(12.)	67	17	9¾.

Ex. 33.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	1	9	10¾.	(2.)		2	2.	(3.)		3	11½.
(4.)	2	3	4.	(5.)		8	9.	(6.)		1	6.
					3	10	0.			4	0½.

	£	s.	d.		s.	d.		£	s.	d.
(7.)	2 1	8 18	0. 0.	(8.)	2	$3\frac{1}{2}$.	(9.)	6	6	$6\frac{1}{2}$.
(10.)	125 62	0 10	0. 0.	(11.)	7 3	6. 9.	(12.)		1	1.

Ex. 34.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	40	2	4.	(2.)	12	9	$7\frac{1}{4}$.	(3.)	11	17	$11\frac{1}{4}$.
(4.)	6	10	$6\frac{1}{2}$.	(5.)		5	$4\frac{3}{4}$.	(6.)	9	2	$5\frac{1}{2}$.
(7.)	88	10	$10\frac{1}{4}$.	(8.)	2	6	$10\frac{1}{2}$.	(9.)		14	$5\frac{3}{4}$.
(10.)	7	2	$2\frac{1}{4}$.	(11.)	8	1	$10\frac{1}{2}$.	(12.)		12	$5\frac{1}{2}$.

Ex. 35.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	72	12	8.	(2.)	88	11	$7\frac{1}{4}$.	(3.)	91	18	3.
(4.)	97	12	$6\frac{1}{4}$.	(5.)	109	4	$0\frac{3}{4}$.	(6.)	93	13	$0\frac{1}{4}$.
(7.)	92	2	$1\frac{1}{4}$.	(8.)	90	0	$0\frac{1}{4}$.	(9.)	84	15	3.
(10.)	68	6	3.	(11.)	78	4	$7\frac{1}{4}$.	(12.)	80	11	$2\frac{3}{4}$.

Ex. 36.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	21	19	$3\frac{3}{4}$.	(2.)	19	4	$0\frac{3}{4}$.	(3.)	1	15 11	$11\frac{1}{4}$. $11\frac{3}{4}$.
(4.)		18	10.	(5.)	2	1	9.	(6.)	343	3	$0\frac{1}{4}$.
(7.)	1	4	9.	(8.)	1	14	6.	(9.)	3	3	$3\frac{3}{4}$.
(10.)		2	11.	(11.)		2	1.	(12.)	1	7 18 9	6. 4. 2.

Ex. 37.

	£	s.	d.		£	s.	d.
(1.)	3	12	$9\frac{1}{4}$.	(2.)	45	10	$5\frac{1}{3}$.
(3.)	76	8	$3\frac{3}{4}$.	(4.)	125	16	$4\frac{1}{4}$.
(5.)	298	4	$4\frac{2}{3}$.	(6.)	144	16	$2\frac{1}{4}$.
(7.)	3697	10	$1\frac{1}{5}$.	(8.)	1443	19	$4\frac{1}{2}$.
(9.)	2525	1	$11\frac{3}{4}$.	(10.)	3325	16	1.
(11.)	12893	2	8.	(12.)	21209	10	$5\frac{1}{4}$.

Ex. 38.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	416	13	4.	(2.)	99 39	19 19	$9\frac{1}{2}$. 11.	(3.)	2 2	1 5	8. 10.

	£	s.	d.		£	s.	d.		£	s.	d.
(4.)	633	6	8.	(5.)		16	10½.	(6.)	1	4	11.
	366	13	4.		3	18	9.				
(7.)	25	4	0.	(8.)	376	17	6.	(9.)	12	19	7.
		4	4½.								
(10.)		8	4.	(11.)	4	7	1.	(12.)		3	5¼.

Ex. 39.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	2	13	9.	(2.)		10	6.	(3.)	40	19	0.
(4.)		8	9.	(5.)	63	15	0.	(6.)	1	4	4½.
(7.)	2	5	3¾.	(8.)	4	7	6.	(9.)	7	11	8.
(10.)	¾*d.*;	6	7½.	(11.)	318	3	0.	(12.)	93	15	0.
(13.)	93	0	6.	(14.)	150	3	4.	(15.)	66	3	0.
(16.)	160	13	0.	(17.)	141	11	0.	(18.)		16	4.
(19.)	1	1	4¼.	(20.)	37	15	5.	(21.)	4	5	11¼.
(22.)	1	5	1½.	(23.)	2	18	9.	(24.)	4	2	6.

Ex. 40.

	£	s.	d.		£	s.	d.
(1.)	2	15	4.	(2.)	8	15	10¼ (+).
(3.)	5	5	5 (+).	(4.)	6	13	0¼ (+).
(5.)	6	0	11¼ (+).	(6.)	9	12	1¾ (+).
(7.)	44	12	1¾ (+).	(8.)	21	5	7 (+).
(9.)	33	11	0.	(10.)	53	11	9½ (+).
(11.)	35	8	9.	(12.)	85	7	11½ (+).

Ex. 41.

	£	s.	d.		£	s.	d.
(1.)		13	3.	(2.)	1	17	2.
(3.)	104	16	3.	(4.)	115	1	4¼ (+).
(5.)	———— .			(6.)	2	19	11 (+).
					3	1	9 (+).
					6	1	8
(7.)	166	16	10. (+).	(8.)	224	5.	0.
(9.)		9	1½ (+).	(10.)	39	4	8.
(11.)	———— .			(12.)	11	2	1 (+).
					11	9	6 (+).
					11	17	1 (+).
					34	8	8

Ex. 42.

(1.) 61¼ bushels. (2.) 409 pounds. (3.) 366.

(4.) £292 10*s.* (5.) ox. 115 grains, nit. 385 grains.
(6.) £3 8*s.* 5*d.* (7.) 6*s.* 1½*d.* (8.) 4738.
(9.) £11 15*s.* (10.) 13*s.* 9*d.* (11.) £30.
(12.) 7*s.* 11¼*d.* (+).

Ex. 43.

(1.) 5280. (2.) 16848. (3.) 518520. (4.) 296160.
(5.) 3762. (6.) 3593. (7.) 1703. (8.) 7020.
(9.) 12212. (10.) 70090. (11.) 41601. (12.) 95999.

Ex. 44.

(1.) £6172 16*s.* (2.) £977 7*s.* 3*d.*
(3.) £360 1*s.* 7½*d.* (4.) 7613 *cr.* 9*d.*
(5.) £50080 1*fl.* (6.) 7937 *fl.* 6*d.*
(7.) £720 1*s.* 1½*d.* (8.) £700 7*s.* 10½*d.*
(9.) £3607 18*s.* 4*d.* (10.) £11137 11*s.* 9*d.*
(11.) £344 18*s.* 7¼*d.* (12.) £1599 2*s.* 3*d.*

Ex. 45.

(1.) £1041 13*s.* 4*d.* (2.) £5495 15*s.*
(3.) £76 12*s.* 6*d.* (4.) 5000; 9¾*d.*
(5.) £6 5*s.* (6.) £257 3*s.* 9*d.*
(7.) £20 16*s.* 8*d.* (8.) £43 0*s.* 5*d.*
(9.) £21 5*s.*; £225. (10.) £800000.
(11.) £37 6*s.* 8*d.* (12.) 20 times.
(13.) £10000; £9895 16*s.* 8*d.* (14.) £1048 19*s.*
(15.) 951*g.* 9*s.* (16.) 21.
(17.) 13. (18.) 441.
(19.) 40. (20.) 12.
(21.) 6. (22.) 5.
(23.) £5439583 6*s.* 8*d.* (24.) £5104 3*s.* 4*d.*

Ex. 46.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(1.)	80	10	0.	(2.)	154	13	4.	(3.)	9	5	0.
(4.)	163	16	0.	(5.)	293	16	8.	(6.)	5	7	6.
(7.)	110	8	4.	(8.)	406	2	0.	(9.)	64	3	4.
(10.)	719	6	8.	(11.)	340	8	4.	(12.)	79	7	6.

Ex. 47.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(1.)	390	10	0	(2.)	901	2	6.	(3.)	1141	4	0.

	£	s.	d.		£	s.	d.		£	s.	d.
(4.)	425	15	0.	(5.)	4672	16	0.	(6.)	1980	4	6.
(7.)	551	14	6.	(8.)	4173	7	0.	(9.)	2262	4	6.
(10.)	8150	17	0.	(11.)	7059	13	9.	(12.)	1087	18	3.

Ex. 48.

	£	s.	d.		£	s.	d.
(1.)	426	6	3½.	(2.)	677	6	9¼.
(3.)	284	4	3¾.	(4.)	2357	16	10.
(5.)	1151	19	0½.	(6.)	760	8	3¾.
(7.)	560	6	0.	(8.)	563	13	4.
(9.)	2637	15	0¾.	(10.)	3735	1	5½.
(11.)	2577	13	4¾.	(12.)	12886	8	10¼.

Ex. 49.

	£	s.	d.		£	s.	d.		£.	s.	d.
(1.)	186	19	6.	(2.)	111	7	6.	(3.)	29	8	1.
(4.)	279	13	9.	(5.)	87	17	8½.	(6.)	12	6	1½.
(7.)	29	2	3.	(8.)	117	8	4.	(9.)	81	16	10½.
(10.)	11	3	3.	(11.)	7	19	9½.	(12.)	27	18	1½.
	41	14	3.		8	1	4¾.				
					16	1	2¼				

Ex. 50.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)		6	5	(2.)		17	6¼	(3.)	10	19	0
		7	0½		1	0	0½		1	4	6½
		7	2¼		1	17	10¾		3	17	11
		11	11			10	8		4	11	0½
		5	6½			4	3		1	2	9
	1	18	1¼		4	10	4½		21	15	3

	£	s.	d.		£	s.	d.		£	s.	d.
(4.)	6	8	5	(5.)	1	0	2	(6.)	10	9	0
	5	3	4			10	0		1	3	4
	10	2	7			2	3			2	6
	4	7	0			18	9			7	8
	2	14	4½			14	0			6	6
	28	15	8½		3	5	2		12	9	0

	£	s.	d.
(7.)		14	7½
		5	1½
		7	4
		11	8
		12	4½
	2	11	1½

	£	s.	d.
(8.)		2	8
		5	1¼
		4	7
		11	3
		5	8
	1	9	3¼

	£	s.	d.
(9.)	3	7	6
		11	3
		10	6
		5	6½
	2	18	6
	7	13	3½

	£	s.	d.
(10.)	10	17	6
	13	16	6
		19	6
	1	5	6
		17	0
	27	16	0

	£	s.	d.
(11.)		1	10½
		13	6
		3	6
		7	0
		3	9
	1	9	7½

	£	s.	d.
(12.)		13	9
	5	4	6
		4	9½
		19	6
	1	7	9
	8	10	3½

ANSWERS

TO

THE EXAMPLES.

PART II.

Ex. 51.

(1.) 82880. (2.) 224000. (3.) 29952. (4.) 2616320
(5.) 5600. (6.) 93184. (7.) 7280. (8.) 2192.
(9.) 20336. (10.) 1152944. (11.) 57273. (12.) 36788.

Ex. 52.

(1.) 13 tons 18 cwt. 2 qr. 21 lbs.
(2.) 1 ton 11 cwt. 1 qr. 3 oz. (3.) 39 lbs. 1 oz.
(4.) 6 cwt. 2 qr. 23 lbs. 5 oz. (5.) 15 st. 9 lbs. 8 oz. 9 dr.
(6.) 10 qr. 3 lbs. 12 oz. 14 dr. (7.) 547 tons 16 cwt. 6 st.
(8.) 40 tons 14 cwt. 2 qr. 11 lbs. (9.) 8 qr. 14 lbs. 4 oz. 12 dr.
(10.) 227 st. 9 lbs. 10 oz. (11.) 18 cwt. 1 qr. 20 lbs. 9 oz.
(12.) 1 ton 10 cwt. 1 qr. 2 lbs. 3 oz. 3 dr.

Ex. 53.

(1.) 50 lbs. 1 oz. 14 dr. (2.) 10 qr. 26 lbs. 13 oz.
(3.) 50 cwt. 0 qr. 18 lbs. (4.) 10 st. 2 lbs. 12 oz.
(5.) 18 cwt. 0 qr. 4 lbs. (6.) 50 lbs. 11 oz. 11 dr.
(7.) 6 st. 6 lbs. 1 oz. 10 dr. (8.) 93 cwt. 2 qr. 8 lbs.
(9.) 54 tons 18 cwt. 17 lbs. (10.) 108 cwt. 2 qr. 12 lbs. 1 oz.
(11.) 54 tons 1 cwt. 2 qr. 24 lbs.
(12.) 8 st. 4 lbs. 15 oz. 7 dr.

Ex. 54.

(1.) 13 lbs. 4 oz. 15 dr. (2.) 1 st. 9 lbs. 15 oz.
(3.) 4 cwt. 1 qr. 22 lbs. (4.) 9 lbs. 12 oz.

(5.) 10 cwt. 2 qrs. 9 lbs.
(6.) 24 lbs. 4 oz. 14 dr.
(7.) 1 st. 11 lbs. 11 oz. 13 dr.
(8.) 27 cwt. 2 qr. 4 lbs. 9 oz.
(9.) 8 tons 2 cwt. 1 qr. 3 lbs.
(10.) 6 cwt. 1 qr. 10 lbs. 14 oz.
(11.) 15 tons 15 cwt. 3 qr. 24 lbs.
(12.) 1 st. 11 lbs. 6 oz. 7 dr.

Ex. 55.

(1.) 27 lbs. 7 oz. 4 dr.
(2.) 18 st. 2 lbs. 1 oz.
(3.) 109 cwt. 2 qr. 11 lbs. 13 oz.
(4.) 328 tons 14 cwt. 22 lbs.
(5.) 83 st. 11 lbs. 6 oz.
(6.) 310 cwt. 2 qr. 24 lbs.
(7.) 84 st. 1 lb. 14 oz.
(8.) 819 cwt. 1 qr. 15 lbs. 5 oz.
(9.) 188 cwt. 17 lbs.
(10.) 299 lbs. 11 oz. 5 dr.
(11.) 932 tons 6 cwt. 7 lbs.
(12.) 436 st. 13 lbs. 12 oz. 15 dr.

Ex. 56.

(1.) 13 lbs. 14 oz. 15 dr.
(2.) 63 st. 10 oz.
(3.) 7 cwt. 3 qr. 15 lbs. 7 oz. 12 dr.
(4.) 84 tons 18 cwt. 2 qr. 10 lbs.
(5.) 4 st. 12 lbs. 1 oz. 14 dr. (+).
(6.) 3 cwt. 8 lbs. 8 oz.
(7.) 1 lb. 13 oz. 14 dr.
(8.) 1 qr. 12 lbs. 5 oz.
(9.) 1 cwt. 3 qr. 8 lbs.
(10.) 1 lb. 6 oz. 8 dr. (×)
(11.) 5 cwt. 3 qr. 3 lbs.
(12.) 3 lbs. 3 dr.

Ex. 57.

(1). 226240; 4 tons 9 cwt. 1 qr. 5 lbs.
(2.) 1 ton 14 cwt. 3 qr. 14 lbs. 4 oz.
(3.) 12 tons 14 cwt. 26 lbs.; 2 tons 9 cwt. 1 qr. 10 lbs.; 28474, 5526.
(4.) 198 tons 2 cwt. 1 qr.
(5.) £13069 0*s.* 7*d.*
(6.) £1 1*s.* 1¼*d.*
(7.) 27 tons 18 cwt. 4 lbs.
(8.) 1 cwt. 2 qr. 3 lbs.
(9.) 13 oz.
(10.) 2 st. 7 lbs.
(11.) 16 lbs.
(12.) 38 tons 3 cwt. 20 lbs. 15 oz.

Ex. 58.

(1.) 51360.
(2.) 147360.
(3.) 8540.
(4.) 5832.
(5.) 75192.
(6.) 142577.
(7.) 41 lbs. 8 oz.
(8.) 20 oz. 16 dwt. 16 gr.
(9.) 2 lbs. 1 oz. 14 dwt. 9 gr.
(10.) 138 lbs. 10 oz. 13 dwt.
(11.) 190 lbs. 3 oz. 18 dwt.
(12.) 9 lbs. 10 oz. 6 dwt. 5 gr.

Ex. 59.

(1.) 43 lbs. 8 oz. 14 dwt. (2.) 18 oz. 5 dwt. 23 gr.
(3.) 43 lbs. 4 oz. 2 dwt. (4.) 2 lbs. 4 oz. 18 dwt.
(5.) 16 dwt. 3 gr. (6.) 3 lbs. 9 oz. 11 dwt. 6 gr.
(7.) 122 lbs. 3 oz. 11 dwt. (8.) 1 oz. 9 dwt. 13 gr.
(9.) 120 lbs. 7 oz. 17 dwt. 23 gr.
(10.) 42 lbs. 6 oz. 11 dwt. 15 gr.
(11.) 5 dwt. 17 gr.
(12.) 1 lb. 3 oz. 6 dwt. 19 gr.

Ex. 60.

(1.) 27 (+), 60. (2.) 21 lbs. 4 oz. 5 dwt.
(3.) 46 (+). (4.) 3 dwt. 15 gr. (+).
(5.) 114 lbs. 15 dwt.; £3437500.
(6.) £37 19*s.* 6*d.*
(7.) 1 lb. 3 oz. 7 dwt. 12 gr.; 600.
(8.) 168 gr. (+). (9.) £6 9*s.* 3*d.*
(10.) 1 lb. 10 oz. 16 dwt. 6 gr.; £5 14*s.* 7*d.*
(11.) 100 oz.
(12.) 1 lb. 11 oz. 5 dwt. 21 gr. (+).

Ex. 61.

(1.) 396000. (2.) 23760. (3.) 59620. (4.) 28380.
(5.) 43745. (6.) 193687.
(7.) 25 m. 3 fur. 175 yds. 2 ft.
(8.) 42 lea. 1 m. 6 fur. 160 yds.
(9.) 7 fur. 43 yds. 1 ft. 8 in. (10.) 71 m. 4 fur. 201 yds. 2 ft.
(11.) 6 m. 2 fur. 111 yds. 4 in.
(12.) 11 lea. 1 fur. 174 yds. 1 ft.

Ex. 62.

(1.) 39 lea. 0 m. 5 fur. (2.) 106 m. 2 fur. 23 po.
(3.) 61 fur. 36 P. 4 yds. 1 ft. 6 in. (4.) 76 po. 1 yd. 2 ft.
(5.) 82 yds. 1 ft. 9 in. (6.) 60 m. 7 fur. 186 yds.
(7.) 61 m. 1214 yds. 2 ft. (8.) 15 fur. 130 yds.
(9.) 33 lea. 1 m. 7 fur. 183 yds.
(10.) 84 m. 4 fur. 66 yds. 1 ft.
(11.) 50 m. 1422 yds. 2 ft. 1 in.
(12.) 25 fur. 186 yds. 7 in.

Ex. 63.

(1.) 5 lea. 2 m. 6 fur. (2.) 26 m. 5 fur. 33 po.
(3.) 3 fur. 35 po. 3 yds. 1 ft. 6 in. (4.) 2 po. 3 yds. 2 ft. 6 in.
(5.) 3 yds. 1 ft. 5 in. (6.) 3 m. 5 fur. 189 yds.
(7.) 16 m. 1678 yds. 2 ft. (8.) 3 fur. 148 yds. 2 ft.
(9.) 2 lea. 2 m. 7 fur. 176 yds. (10.) 3 m. 6 fur. 84 yds. 1 ft.
(11.) 6 m. 859 yds. 1 ft. 8 in. (12.) 8 fur. 157 yds. 1 ft. 11 in.

Ex. 64.

(1.) 151 yds. 2 ft. 8 in. (2.) 181 m. 7 fur. 80 yds.
(3.) 55 lea. 1200 yds. (4.) 71 fur. 30 po.
(5.) 54 lea. 6 fur. (6.) 380 yds.
(7.) 1230 m. (8.) 2366 po. 3 yds. 2 ft.
(9.) 577 yds. 1 ft. 7 in. (10.) 823 lea. 2 m. 467 yds.
(11.) 300 lea. 2 m. 7 fur. 21 po.
(12.) 883 fur. 29 po. 3 yds. 2 ft. 6 in.

Ex. 65.

(1.) 5 yds. 0 ft. 7 in. (2.) 2 m. 4 fur. 52 yds.
(3.) 63 yds. 1 ft. 2 in. (4.) 177 m. 4 fur. 102 yds.
(5.) 2 fur. 22 yds. 6 in.
(6.) 1 m. 6 fur. 137 yds. 1 ft. 6 in.
(7.) 1 yd. 2 ft. 3 in. (8.) 1 m. 1151 yds. 2 ft.
(9.) 144 yds. 2 ft. 7 in. (10.) 1 fur. 71 yds. 1 ft. 6 in.
(11.) 211 lea. 2 m. 1158 yds. (12.) 1 yd. 1 ft. 1 in.

Ex. 66.

(1.) 25000. (2.) 2 m. 400 yds.
(3.) 46640. (4.) 83 yds. 1 ft.
(5.) 26 ft. 8 in. (6.) 2 m. 480 yds.
(7.) 20 yds. 1 ft. 3 in.
(8.) 6 fur. 13 yds. 1 ft.; 1 m. 160 yds.
(9.) 84 ft. (10.) 11 ft.
(11.) 15 miles. (12.) 330 yds.

Ex. 67.

(1.) 94 yds. 1 na. (2.) 60 ells. 1 qr. 1 na.
(3.) 119 yds. 0 qr. 2 na. 1 in. (4.) 72 ells 3 qrs. 2 na. $0\frac{3}{4}$ in.

(5.) 15 yds. 1 qr. 1 na.
(6.) 30 ells 1 qr. 1 na.
(7.) 1 qr. 2 na. $1\frac{3}{4}$ in.
(8.) 2 ells 2 qr. 2 na. $1\frac{1}{2}$ in.
(9.) 110 yds. 0 qr. 1 na. $1\frac{1}{2}$ in.
(10.) 1 yd. 2 qrs. 2 na. $0\frac{1}{4}$ in.
(11.) 282 yds. 3 qr. 2 na.
(12.) 480 yds. 1 qr. 3 na. $1\frac{3}{4}$ in.

Ex. 68.

(1.) 1584 yds.
(2.) 15·62 yds.
(3.) 3 m. 1 fur.
(4.) 440.
(5.) 5 yds. 3 qrs.
(6.) 76 yds. 2 qrs. 3 na.
(7.) 27 yds. 3 na.
(8.) 31 yds. 1 qr.; 20.
(9.) £4 10*s.*
(10.) 1000.
(11.) 1122·66 yds.
(12.) 770 yds.

Ex. 69.

(1.) 2235.
(2.) 23679.
(3.) 8747.
(4.) 15508.
(5.) 140844.
(6.) 38543.
(7.) 37143.
(8.) 1125360.
(9.) 62A. 2R.
(10.) 77 sq. yds. 1 sq. ft. 64 sq. in.
(11.) 2A. 10P. 17 sq. yds. 4 sq. ft. 72 sq. in.
(12.) 1A. 2R. 15P. 2 sq. yds. 2 sq. ft. 100 sq. in.

Ex. 70.

(1.) 81 sq. yds. 7 sq. ft. 112 sq. in.
(2.) 86A. 3R. 39P.
(3.) 164 sq. yds. 3 sq. ft. 13 sq. in.
(4.) 62A. 2R. 3P.
(5.) 11R. 26P. 7 sq. yds. 1 sq. ft. 36 sq. in.
(6.) 109P. 6 sq. yds. 4 sq. ft. 73 sq. in.
(7.) 1 sq. yd. 7 sq. ft. 123 sq. in.
(8.) 18A. 2R. 32P.
(9.) 23 sq. yds. 1 sq. ft. 112 sq. in.
(10.) 35A. 2R. 18P.
(11.) 8 sq. yds. 1 sq. ft. 86 sq. in.
(12.) 136A. 2R. 33P.

Ex. 71.

(1.) 125A. 3R. 5P.
(2.) 401 sq. yds. 8 sq. ft. 35 sq. in.
(3.) 643A. 2R. 11P.
(4.) 404 sq. yds. 8 sq. ft. 28 sq. in.
(5.) 12A. 2R. 19P.
(6.) 31 sq. yds. 8 sq. ft. 120 sq. in.
(7.) 11A. 3R. 25P.
(8.) 3 sq. yds. 2 sq. ft. 50 sq. in.
(9.) 260A. 1R. 33P.
(10.) 2819 sq. yds. 5 sq. ft. 129 sq. in.
(11.) 1110A. 1R. 30P.
(12.) 508 sq. yds. 8 sq. ft. 41 sq. in.

Ex. 72.

(1.) 17A. 3R. 37P.; 11A. 3R. 38P.; 5A. 3R. 39P.
(2.) 32000000. (3.) 78A. 1R. 3P.
(4.) 3R. 31P. (5.) 1R. 17P.
(6.) 1A. 0R. 33P. (7.) 1R. 37P.
(8.) 3R. 12P. (9.) 33P.
(10.) 11A. 0R. 15P. (11.) 165A 3R. 30P.
(12.) 1A. 2R. 31P.

Ex. 73.

(1.) 8 sq. yds. 8 sq. ft.
(2.) 10 sq. yds. 1 sq. ft. 99 sq. in.
(3.) 70 sq. yds. 8 sq. ft. (4.) 19 sq. yds. 141 sq. in.
(5.) 58 sq. yds. 3 sq. ft.
(6.) 12 sq. yds. 5 sq. ft. 48 sq. in.
(7.) 96 sq. yds. 3 sq. ft.
(8.) 30 sq. yds. 4 sq. ft. 12 sq. in.
(9.) 82 sq. yds. (10.) 177 sq. yds. 5 sq. ft.
(11.) 8 sq. yds. 1 sq. ft. 15 sq. in.
(12.) 33 sq. yds. 1 sq. ft. 131 sq. in.

Ex. 74.

(1.) 85 sq. yds. 2 sq. ft. 90 sq. in. (2.) 380 sq. yds.
(3.) 1 sq. yd. 7 sq. ft. 48 sq. in.
(4.) 306 sq. yds. 2 sq. ft. 36 sq. in.
(5.) 90 sq. ft.; 99 sq. yds. 6 sq. ft. 108 sq. in.
(6.) 10A.
(7.) 232 sq. yds. 5 sq. ft. 9 sq. in.
(8.) 784 sq. yds. (9.) 21 sq. yds. 1 sq. ft.
(10) 42 sq. yds. 8 sq. ft. 24 sq. in.
(11.) 3R. 10P. 15½ sq. yds. (12.) 24 sq. ft.

Ex. 75.

(1.) 1A. 28P. (+). (2.) 2A. 1R. 17P. (+).
(3.) 4A. 3R. 39P. (+). (4.) 28A.
(5.) 1A. 32P. (6.) 32P. (+).
(7.) $78\frac{4}{7}$ sq. in. (8.) 1A. $188\frac{4}{7}$ sq. yds.
(9.) 640A. = 1 sq. mile. (10.) 3A. 17P.
(11.) 13 sq. ft. $92\frac{2}{7}$ sq. in.
(12.) 784 sq. yds.; 8A. 23P. 8¼ sq. yds.

Ex. 76.

(1.) 702. (2.) 722304. (3.) 1167400.
(4.) 3 c. yds. 5 c. ft. 1392 c. in.
(5.) 5 c. yds. 25 c. ft. 63 c. in.
(6.) 2 c. yds. 3 c. ft. 1505 c. in.
(7.) 23 c. yds. 17 c. ft. 620 c. in.
(8.) 194 c. yds. 14 c. ft. 392 c. in.
(9.) 85 c. yds. 5 c. ft. 277 c. in.
(10.) 18 c. yds. 14 c. ft. 1464 c. in.
(11.) 13 c. yds. 17 c. ft. 1113 c. in.
(12.) 2 c. yds. 9 c. ft. 1585 c. in.

Ex. 77.

	c. yds.	c. ft.	c. in.		c. yds.	c. ft.	c. in.
(1.)	22	10	933.	(2.)	193	26	329.
(3.)	375	1	147.	(4.)	504	3	1144.
(5.)	16	18	0.	(6.)	7	6	1296.
(7.)	36	22	1296.	(8.)	141	3	1431.
(9.)	26	17	389.	(10.)	6	18	1239 (+).
(11.)	19	20	460.	(12.)	61	8	160.

Ex. 78.

(1.) 16 c. yds. 9 c. ft. (2.) 1 c. yd. 8 c. ft.
(3.) 19 c. yds. 5 c. ft. (4.) 16 c. yds. 9 c. ft.
(5.) 6 c. yds. 18 c. ft. (6.) 77 c. yds. 21 c. ft.
(7.) 2 c. ft. (8.) 3 c. yds. 25 c. ft. 1536 c. in.
(9.) 2 c. ft. 144 c. in. (10.) 15 c. ft.
(11.) 6 c. yds. 22 c. ft. 1248 c. in.
(12.) 18 c. yds. 1 c. ft. 459 c. in.

Ex. 79.

(1.) 108 c. in. (2.) 36 c. ft.
(3.) 1728; 46656. (4.) 1 c. ft. 432 c. in.
(5.) 66 c. yds. 18 c. ft. (6.) 18 c. ft.; 10 cwt. 5 lbs.
(7.) 5 c. yds. 5 c. ft. 1080 c. in.
(8.) 5 c. yds. 23 c. ft. 1601 c. in.
(9.) 20 c. yds. 12 c. ft. 864 c. in.
(10.) 102700 c. yds. 16 c. ft. 1152 c. in.
(11.) 443520 c. yds. (12.) 8 c. ft. 960 c. in.

Ex. 80.

(1.) 296. (2.) 448. (3.) 920. (4.) 1152.
(5.) 1896. (6.) 1536. (7.) 359. (8.) 1535.
(9.) 445. (10.) 1531. (11.) 253. (12.) 1371.

Ex. 81.

(1.) 13 gals. 2 qts. 1 pt. (2.) 5 pks. 1 gal. 3 qts.
(3.) 18 bus. 3 pks. 1 gal. (4.) 23 co. 3 bus. 2 pks.
(5.) 23 qrs. 1 co. 2 bus. (6.) 42 co. 3 bus. 2 pks.
(7.) 27 pks. 1 gal. (8.) 31 bus. 2 pks. 1 gal.
(9.) 29 co. 3 pks. 1 gal. (10.) 61 qrs. 1 co. 3 bus. 3 pks.
(11.) 45 co. 3 pks. 1 gal. (12.) 27 qrs. 1 co. 2 bus. 3 pks.

Ex. 82.

(1.) 38 gals. 3 qts. (2.) 17 pks. 1 gal. 2 qts.
(3.) 51 bus. 1 pk. 1 gal. (4.) 54 co. 2 pks.
(5.) 89 qrs. 1 co. 2 bus. (6.) 72 co. 2 bus. 1 pk.
(7.) 8 pks. 1 gal. 2 qts. (8.) 28 bus. 1 pk. 1 gal.
(9.) 2 qts. 1 pt. (10.) 9 co. 2 bus. 3 pks.
(11.) 26 qrs. 3 bus. (12.) 5 co. 2 bus. 3 pks.

Ex. 83.

(1.) 22 gals. 1 pt. (2.) 67 co. 3 bus. 1 pk.
(3.) 3 bus. 3 pks. 1 gal. (4.) 1 qr. 2 bus.
(5.) 1 pk. 3 qts. (6.) 2 bus. 1 gal. 1 pt.
(7.) 720 pks. 3 qts. (8.) 4717 qrs. 1 pk.
(9.) 340 pks. 1 gal. 2 qts. (10.) 1 bus. 2 pks. 1 gal. 1 qt.
(11.) 2040 gals. 2 qts. 1 pt.
(12.) 12973 qrs. 1 co. 3 bus. 1 pk. 1 gal. 1 pt.

Ex. 84.

(1.) 15. (2.) 4 gals. 3 qts.
(3.) 13 gals. 2 qts. 1 pt. (4.) 265½ qrs.
(5.) 15 qrs. 3 bus. 3 pks. 2 qts. (6.) 14 bus. 3 pks. 1 gal.
(7.) 58 qrs. 1 bus. (8.) 6 qrs. 1 co. 1 bus.
(9.) 4 qrs. 1 bus. 3 pks. (10.) 7*s*.
(11.) 25. (12.) 67½ gallons.

Ex. 85.

(1.) 4745. (2.) 840. (3.) 648.
(4.) 43200. (5.) 131040. (6.) 1040688000.
(7.) 19155. (8.) 6962400. (9.) 2805120.
(10.) 172140. (11.) 6106620. (12.) 104373300.

Ex. 86.

(1.) 338 yrs. 86 days. (2.) 1786 w. 3 d. 5 hrs.
(3.) 529 mo. 3 w. 5 d. 11 hrs. (4.) 74 hrs. 28 min. 57 sec.
(5.) 49 w. 4 d. 5 hrs. 20 sec.
(6.) 3 d. 12 hrs. 51 min. 43 sec.
(7.) 701 mo. 3 w. 3 d. 6 hrs. (8.) 37 w. 2 d. 9 hrs. 43 min.
(9.) 1 d. 16 hrs. 39 min. 17 sec.
(10.) 41 yrs. 252 d. 22 hrs.
(11.) 16 mo. 5 d. 23 hrs. 25 min.
(12.) 7 d. 19 hrs. 10 min. 56 sec.

Ex. 87.

(1.) 24 hrs. 7 min. 20 sec. (2.) 88 d. 19 hrs. 23 min.
(3.) 38 hrs. 4 min. 6 sec. (4.) 52 w. 0 d. 5 hrs.
(5.) 54 mo. 0 w. 3 d. (6.) 50 w. 4 d. 4 hrs.
(7.) 5 w. 5 d. 17 hrs. (8.) 9 mo. 0 w. 4 d.
(9.) 2 w. 6 d. 12 hrs. (10.) 2 yrs. 226 d. 19 hrs.
(11.) 6 d. 22 hrs. 55 min. (12.) 4 hrs. 56 min. 28 sec.

Ex. 88.

(1.) 16 hrs. 35 min. 8 sec. (2.) 141 d. 3 hrs. 4 min.
(3.) 1 w. 2 d. 7 hrs. 20 min.
(4.) 1 mo 1 w. 3 d. 21 hrs. 36 min.
(5.) 127 d. 17 hrs. 25 min.
(6.) 12 d. 34 min. 10 sec. (7.) 1917 hrs. 1 min. 24 sec.
(8.) 1686 w. 1 d. 3 hrs. 45 min.
(9.) 152 mo. 3 w. 1 d.
(10.) 25 d. 15 hrs. 24 min. 18 sec.
(11.) 2384 hrs. 37 min. 20 sec.
(12.) 1313 d. 5 hrs. 24 min. 48 sec.

Ex. 89.

(1.) 1 h. 41 min. 2 sec.; 10 h. 3 min. 58 sec.
(2.) 94656000 miles.
(3.) 29 d. 12 hrs. 44 min. 3 sec.

(4.) 25 d. 8 hrs. 45 min. (5.) 6 d. 22 hrs. 40 min.
(6.) 378691200. (7.) 1139580.
(8.) 11 d. 7 hrs. (+); 41 d. 10 hrs. (+); 12 yrs. 266 d. 14 hrs. (+).
(9.) 300 days. (10.) 328 yrs. 280 d.
(11.) 556 yrs. 185 d.*
(12.) 3 yrs. 80 d. 11 hrs. 48 min. 20 sec.

Ex. 90.

(1.) 9. (2.) 11. (3.) 9. (4.) 13.
(5.) 15. (6.) 3. (7.) 27. (8.) 43.
(9.) 12. (10.) 113. (11.) 79. (12.) 26.

Ex. 91.

(1.) 14080. (2.) 8. (3.) 19.
(4.) 13. (5.) 10. (6.) 8.
(7.) 21; 28. (8.) 30. (9.) 9.
(10.) 5. (11.) 64. (12.) 56.
(13.) 64. (14.) 64. (15.) 3 hours.
(16.) 584. (17.) 40. (18.) 92.
(19.) 9. (20.) 24. (21.) 192.
(22.) 27 cub. ft. (23.) 12 sec. (24.) 480.

Ex. 92.

(1.) 60 yards. (2.) £1 8*s.* 2*d.* (3.) £9 6*s.* 8*d.*
(4.) 133 miles. (5.) 432 yards. (6.) £50.
(7.) £3. (8.) 55000.
(9.) 43 cwt. 3 qrs. 14 lbs. (10.) £4 14*s.* 6*d.*
(11.) 85 gallons. (12.) 3*s.* 4*d.*

Ex. 93.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(1.)	1221	17	6.	(2.)	29	9	0.	(3.)	10	19	9.
(4.)	410	0	6.	(5.)	75	5	$8\frac{3}{4}$.	(6.)	13	11	$10\frac{1}{2}$.
(7.)	61	19	4.	(8.)	22	2	$10\frac{1}{2}$.	(9.)	74	8	6.
(10.)	39	19	1.	(11.)	10	9	3.	(12.)	100	7	0.

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* It is worth observing that the Answers to (9), (10). (11), are, roughly, 300 *days*, 330 *years*, and 550 *years*,—where every day of the year in the third Answer is equivalent to the whole 330 years of the second Answer. We thus get an idea of the enormous distances of the fixed stars.

Ex. 94.

(1.) £104 14*s*. 9*d*.
(2.) £2918 4*s*. 4*d*.
(3.) £19 2*s*. 1½*d*.
(4.) 138A. 3R. 31P.
(5.) £2740 8*s*. 6½*d*.
(6.) £2030 10*s*. 6¼*d*.
(7.) £455 1*s*. 10*d*.
(8.) £596 1*s*. 10½*d*.
(9.) £10 1*s*. 6*d*.
(10.) 2 m. 730 yds.
(11.) ———.
(12.) £4 8*s*. 6*d*.

Ex. 95.

	cwt.	qrs.	lbs.		cwt.	qrs.	lbs.		cwt.	qrs.	lbs.
(1.)	61	1	4	(2.)	13	3	10	(3.)	80	1	7
(4.)	19	0	11	(5.)	43	0	0	(6.)	44	0	1
(7.)	80	1	4	(8.)	32	2	25				
(9.)	10 cwt. 3 lbs. 2 oz.										
(10.)	35	3	21	(11.)	32	0	15	(12.)	67	0	3

Ex. 96.

(1.) 4 ft.
(2.) 8 yds. 8 in.
(3.) 1 ft.
(4.) 88 yds.
(5.) 6 yds. 2 ft.
(6.) 54 yds.
(7.) 8 miles 2 fur.
(8.) 8 ft.
(9.) 16 ft.
(10.) 2 ft. 4 in.
(11.) 15 ft.
(12.) 42 yds.; 5*s*. 3*d*.

Ex. 97.

(1.) 9 ft.
(2.) 2 yds.
(3.) 10 miles.
(4.) 2 ft. 6 in.
(5.) 5 ft.; 5 in.
(6.) 11 ft.
(7.) 27 yds.
(8.) 16 ft.
(9.) 3 c. ft. 216 in.
(10.) 2 ft.
(11.) 1 ft. 6 in.
(12.) 2 m. 1280 yds.

Ex. 98.

	£	*s*.	*d*.		£	*s*.	*d*.		£	*s*.	*d*.
(1.)	6	5	0.	(2.)	8	13	3.	(3.)	8	6	8.
(4.)	2	3	4.	(5.)	7	17	6.	(6.)	4	4	6.
(7.)	91	17	6.	(8.)	15	15	0.	(9.)	81	0	0.
(10.)	3	3	0.	(11.)	50	6	10½.	(12.)	1	1	8.
(13.)	7	7	0.	(14.)	6	8	0.	(15.)	1	2	9.
(16.)	6	14	7.	(17.)	11	2	0.	(18.)	8	2	0.
(19.)	1	1	4.	(20.)	1	16	1.	(21.)	27	10	0.
(22.)	9	18	0.	(23.)	8	8	0.	(24.)		17	1(+)

Ex. 99.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*	
(1.)	26	5	0.	(2.)	5	8	0.	(3.)	11	8	0.	
(4.)	3	0	9.	(5.)	11	0	0.	(6.)	11	4	3.	(+)

Ex. 100.

(1.) 1824 sq. ft.
(2.) 4570 sq. yds.
(3.) 6032 sq. rods.
(4.) 741 sq. yds. 2 ft.
(5.) 7 rods 2 qrs. 60 ft.
(6.) 6 rods 48 ft.
(7.) 23 rods 2 qrs. 8 ft.
(8.) £44 16*s.* 2¾*d.* (+)
(9.) £17 4*s.* 8*d.* (+)
(10.) £4 16*s.* 6*d.* (+)
(11.) 9 rods 52 ft.
(12.) 7 rods 3 qrs. 11 ft; £11 13*s.* 8½*d.* (+)

ANSWERS

TO

THE EXAMPLES.

PART III.

Ex. 101.

(1.) 35. (2.) 64. (3.) 56. (4.) 65.
(5.) 101. (6.) 143. (7.) 3. (8.) 11.
(9.) 133. (10.) 15. (11.) 112. (12.) 27.

Ex. 102.

(1.) 144. (2.) 360. (3.) 54. (4.) 60.
(5.) 210. (6.) 600. (7.) 180. (8.) 1080.
(9.) 360. (10.) 1008. (11.) 1800. (12.) 600.

Ex. 103.

(1.) 1, 2, 3, 4, 5, 6, 8, 9, 10, 12, 15, 18, 20, 24, 30, 36, 40, 45, 60, 72, 90, 120, 180, 360:
1, 2, 4, 5, 8, 10, 16, 20, 25, 40, 50, 80, 100, 200, 400.
(2.) 40; 3600. (3.) 38; 2736.
(4.) 17, 34, 51, 68, 102, 204.
(5.) 1547. (6.) 5040. (7.) £21; 10*s.*
(8.) 180. (9.) 18. (10.) 252.
(11.) 8 yards; 150. (12.) 180.

Ex. 104.

(1.) $\frac{21}{3}$; $\frac{35}{5}$. (2.) $\frac{81}{9}$; $\frac{63}{7}$.
(3.) $\frac{121}{11}$; $\frac{143}{13}$. (4.) $\frac{156}{13}$; $\frac{180}{15}$.
(5.) $\frac{8}{3}$; $\frac{15}{4}$; $\frac{24}{5}$. (6.) $\frac{34}{3}$; $\frac{93}{7}$; $\frac{123}{8}$.
(7.) $\frac{270}{13}$; $\frac{350}{11}$; $\frac{653}{15}$. (8.) $\frac{1113}{11}$; $\frac{779}{6}$; $\frac{1727}{12}$.
(9.) 7; $7\frac{4}{5}$; $5\frac{4}{7}$. (10.) $15\frac{3}{8}$; 8; $16\frac{5}{13}$.
(11.) $83\frac{3}{16}$; 40; $37\frac{1}{27}$. (12.) $21\frac{5}{17}$; 19; $15\frac{15}{23}$.

Ex. 105.

(1.) $\frac{135}{56}$; $\frac{45}{14}$; $\frac{45}{8}$; $\frac{45}{7}$. (2.) $\frac{45}{112}$; $\frac{15}{56}$; $\frac{9}{56}$; $\frac{5}{56}$.
(3.) $\frac{108}{25}$; $\frac{972}{125}$; $\frac{1296}{125}$; $\frac{108}{5}$. (4.) $\frac{108}{625}$; $\frac{12}{125}$; $\frac{9}{125}$; $\frac{108}{3125}$.
(5.) $\frac{200}{231}$; $\frac{100}{77}$; $\frac{400}{231}$; $\frac{500}{231}$; $\frac{100}{33}$; $\frac{100}{21}$.
(6.) $\frac{50}{231}$; $\frac{100}{693}$; $\frac{25}{231}$; $\frac{20}{231}$; $\frac{100}{1617}$; $\frac{100}{2541}$.

Ex. 106.

(1.) $\frac{9}{20}$. (2.) $\frac{5}{6}$. (3.) $\frac{9}{11}$. (4.) $\frac{4}{15}$.
(5.) $\frac{5}{8}$. (6.) $\frac{2}{3}$. (7.) $\frac{23}{35}$. (8.) $\frac{13}{17}$.
(9.) $\frac{10}{23}$. (10.) $\frac{15}{28}$. (11.) $\frac{19}{29}$. (12.) $\frac{31}{41}$.

Ex. 107.

(1.) $\frac{7}{11}$. (2.) $\frac{13}{17}$. (3.) $\frac{19}{23}$. (4.) $\frac{23}{26}$. (5.) $\frac{26}{29}$. (6.) $\frac{29}{31}$.

Ex. 108.

(1.) 119 ; 289 ; 1309. (2.) $\frac{1}{8}$; $\frac{7}{8}$.
(3.) 26 ; 78. (4.) 89; 131.
(5.) 11, and there would be nearly a whole cake over; 37.
(6.) 91 ; 9, with a fraction over.
(7.) $\frac{5}{12}$; $\frac{1}{3}$. (8.) $\frac{1}{22}$; $\frac{7}{66}$. (9.) $\frac{3}{7}$; $\frac{4}{7}$.
(10.) $\frac{2}{5}$; $\frac{3}{5}$. (11.) $\frac{5}{8}$; $\frac{3}{8}$. (12.) $\frac{2}{9}$; $\frac{4}{9}$; $\frac{1}{3}$.

Ex. 109.

(1.) $3\frac{1}{3}$. (2.) $8\frac{1}{4}$. (3.) $15\frac{1}{9}$. (4.) $24\frac{1}{6}$.
(5.) $\frac{5}{6}$. (6.) $\frac{11}{12}$. (7.) $\frac{19}{20}$. (8.) $\frac{29}{30}$.
(9.) $16\frac{2}{3}$; $46\frac{4}{5}$. (10.) $1\frac{7}{10}$; $\frac{109}{110}$.
(11.) $185\frac{7}{9} \div 11 = 16\frac{8}{9}$. (12.) $9 \times \frac{11}{12} = 8\frac{1}{4}$.

Ex. 110.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)	5	11	$6\frac{1}{8}$.	(2.)	22	18	$4\frac{4}{5}$.	(3.)	33	8	$6\frac{2}{3}$.
(4.)	12	1	$3\frac{2}{3}$.	(5.)	155	8	4.	(6.)	126	10	9.
(7.)	113	17	$4\frac{5}{7}$.	(8.)	321	13	$5\frac{1}{8}$.	(9.)	580	1	$3\frac{5}{6}$.
(10.)	264	18	$10\frac{4}{5}$.	(11.)	395	16	0.	(12.)	104	7	3.

Ex. 111.

	£	s.	d.		£	s.	d.		£	s.	d.
(1.)		17	$6\frac{1}{19}$.	(2.)		12	$2\frac{13}{18}$.	(3.)		18	$9\frac{9}{32}$.
(4.)	1	8	$10\frac{17}{30}$.	(5.)	1	11	$3\frac{25}{32}$.	(6.)	1	4	$5\frac{77}{80}$.
(7.)		4	$8\frac{87}{160}$.	(8.)		6	$10\frac{7}{24}$.	(9.)		7	$11\frac{137}{220}$.
(10.)		2	$8\frac{9}{88}$.	(11.)		4	$10\frac{17}{32}$.	(12.)		5	$7\frac{89}{100}$.

Ex. 112.

(1.) $\frac{5}{6}$. (2.) 4. (3.) $\frac{3}{10}$. (4.) 7.
(5.) $\frac{1}{2}$. (6.) $1\frac{1}{6}$. (7.) $13\frac{1}{3}$. (8.) 1.
(9.) $23\frac{2}{5}$. (10.) $\frac{1}{10}$. (11.) $62\frac{1}{2}$. (12.) 420.

Ex. 113.

(1.) $\frac{35,\ 63,\ 75}{105}$. (2.) $\frac{135,\ 144,\ 160}{180}$.
(3.) $\frac{8,\ 6,\ 4,\ 3}{24}$. (4.) $\frac{54,\ 80,\ 99,\ 104}{144}$.
(5.) $\frac{8,\ 12,\ 10,\ 7}{32}$. (6.) $\frac{12,\ 63,\ 40,\ 33}{72}$.
(7.) $\frac{27,\ 36,\ 21,\ 10}{243}$. (8.) $\frac{18,\ 25,\ 28,\ 27}{60}$.
(9.) $\frac{88,\ 78,\ 68,\ 57}{144}$. (10.) $\frac{72,\ 15,\ 20,\ 27}{450}$.
(11.) $\frac{225,\ 280,\ 330,\ 351}{1080}$. (12.) $\frac{140,\ 135,\ 132,\ 130}{240}$.

Ex. 114.

(1.) $2\frac{2}{11}$. (2.) $11\frac{1}{5}$. (3.) $12\frac{1}{8}$. (4.) $7\frac{8}{9}$.
(5.) $8\frac{43}{90}$. (6.) $7\frac{41}{72}$. (7.) $4\frac{1}{10}$. (8.) $11\frac{1}{4}$.
(9.) $13\frac{47}{120}$. (10.) $29\frac{1}{4}$. (11.) $13\frac{11}{24}$. (12.) $11\frac{31}{96}$.

Ex. 115.

(1.) $\frac{4}{11}$. (2.) $\frac{1}{5}$. (3.) $\frac{13}{60}$. (4.) $3\frac{5}{6}$. (5.) $1\frac{1}{2}$. (6.) $1\frac{7}{10}$.
(7.) $6\frac{3}{8}$. (8.) $\frac{1}{2}$. (9.) $3\frac{14}{15}$. (10.) $\frac{17}{30}$. (11.) $\frac{5}{8}$. (12.) $6\frac{9}{40}$.

Ex. 116.

	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(1.)	64	7	$0\frac{1}{24}$.	(2.)	59	14	$1\frac{5}{36}$.	(3.)	70	6	$11\frac{13}{15}$.
	cwt.	qrs.	lbs.		yds.	ft.	in.		A.	R.	P.
(4.)	72	2	$20\frac{2}{15}$	(5.)	104	1	$6\frac{7}{40}$.	(6.)	146	2	$6\frac{17}{18}$.
	£	*s.*	*d.*		£	*s.*	*d.*		£	*s.*	*d.*
(7.)	7	2	$0\frac{5}{8}$.	(8.)	10	7	$7\frac{19}{20}$.	(9.)	9	16	$1\frac{19}{24}$.
	c. yds.	c. ft.	c. in.						qrs.	bus.	gals.
(10.)	5	20	$827\frac{14}{15}$.					(11.)	7	7	$5\frac{29}{48}$.
	days	hrs.	min.								
(12.)	15	15	$59\frac{17}{24}$.								

Ex. 117.

(1.) $2\frac{1}{12}$. (2.) $1\frac{1}{12}$. (3.) $1\frac{1}{20}$. (4.) $1\frac{5}{6}$.
(5.) $\frac{3}{5}$. (6.) $\frac{11}{48}$. (7.) $\frac{1}{8}$. (8.) 399.
(9.) $\frac{1}{1260}$. (10.) $\frac{19}{20}$. (11.) $\frac{1}{5}$, $\frac{1}{10}$, $\frac{3}{10}$. (12.) $4\frac{2}{45}$.

Ex. 118.

(1.) 5. (2.) 4. (3.) $\frac{45}{88}$. (4.) $1\frac{1}{11}$.
(5.) $\frac{4}{9}$. (6.) $\frac{5}{7}$. (7.) $2\frac{11}{12}$. (8.) $\frac{32}{75}$.
(9.) $14\frac{6}{11}$. (10.) $\frac{2}{25}$. (11.) $\frac{16}{175}$. (12.) $1\frac{3}{5}$.

Ex. 119.

(1.) 12. (2.) $\frac{3}{8}$. (3.) $\frac{3}{20}$. (4.) $\frac{2}{9}$.
(5.) $\frac{2}{3}$. (6.) $1\frac{1}{2}$. (7.) $1\frac{1}{3}$. (8.) $5\frac{5}{11}$.
(9.) $7\frac{1}{2}$. (10.) $\frac{2}{9}$. (11.) $\frac{7}{25}$. (12.) $\frac{9}{10}$.

Ex. 120.

(1.) $\frac{3}{10}$. (2.) 30. (3.) 5*s.* $6\frac{2}{5}$*d.*
(4.) $103\frac{3}{4}$; $3\frac{1}{3}$. (5.) 405. (6.) 5 ft. 4 in.
(7.) $\frac{3}{8}$. (8.) 18; $1\frac{1}{2}$. (9.) 6; 4 yds.
(10.) 6; $5\frac{1}{3}$; $2\frac{14}{17}$. (11.) 256. (12.) 7; $1\frac{3}{8}$.

Ex. 121.

(1.) 14*s.* $0\frac{3}{4}$*d.* (2.) £1 16*s.*
(3.) £5 7*s.* $1\frac{5}{7}$*d.* (4.) £29 3*s.* 4*d.*
(5.) £7 11*s.* 8*d.* (6.) £24 11*s.* $4\frac{4}{5}$*d.*
(7.) 11 cwt. 3 qrs. 26 lbs.
(8.) 18 tons 2 cwt. 1 qr. 23 lbs. 8 oz.
(9.) 8A. 1R. $39\frac{2}{7}$P. (10.) 66A. 2R. $16\frac{4}{5}$P.
(11.) 3 c. yds. 21 c. ft. $33\frac{3}{5}$ c. in.
(12.) 65 c. yds. 16 c. ft. $295\frac{1}{3}$ c. in.

Ex. 122.

(1.) £2 8*s.* 3*d.* (2.) £1 2*s.* (3.) 18*s.* $6\frac{3}{4}$*d.*
(4.) 13*s.* $6\frac{1}{4}$*d.* (5.) 5 cwt. 1 qr. 6 lbs.
(6.) $6\frac{6}{7}$. (7.) $4\frac{4}{7}$. (8.) $6\frac{3}{4}$; 9.
(9.) 36; 18; $6\frac{2}{3}$. (10.) 4. (11.) 12. (12.) 60.

Ex. 123.

(1.) $\frac{1}{6}$. (2.) $\frac{1}{8}$. (3.) $6\frac{2}{3}$. (4.) $\frac{4}{75}$.
(5.) $\frac{4}{7}$. (6.) $3\frac{2}{3}$. (7.) $2\frac{5}{6}$. (8.) 3.
(9.) $3\frac{4}{5}$. (10.) $4\frac{5}{6}$. (11.) $5\frac{6}{7}$. (12.) $6\frac{7}{8}$.

Ex. 124.

(1.) $\frac{1}{2}$.
(2.) $2\frac{1}{9}$.
(3.) $3\frac{4}{5}$.
(4.) $\frac{5}{17}$.
(5.) $1\frac{4}{7}$.
(6.) $2\frac{3}{5}$.
(7.) $16\frac{5}{16}$.
(8.) $\frac{11}{15}$.
(9.) $2\frac{1}{7}$.
(10.) 14.
(11.) $\frac{1}{11}$.
(12.) $7\frac{1}{7}$.

Ex. 125.

(1.) $3\frac{1}{2}$.
(2.) 23.
(3.) 5.
(4.) $2\frac{1}{20}$.
(5.) $3\frac{1}{3}$.
(6.) $1\frac{2}{3}$.
(7.) $3\frac{3}{8}$.
(8.) 1. [illegible]
(9.) $\frac{1}{2}$.
(10.) $5\frac{1}{2}$.
(11.) 5.
(12.) $3\frac{1}{2}$.

Ex. 126.

(1.) $8\frac{8}{9}$.
(2.) $\frac{144}{175}$. $\frac{192}{175}$.
(3) £7 4*s.*
(4.) 16.
(5.) $\frac{1}{8}$.
(6.) $\frac{1}{20}$; £500, £400, £200.
(7.) $\frac{11}{18}$; $\frac{7}{18}$.
(8.) $3\frac{1}{3}$; 6.
(9.) 2*s.*
(10.) £4 3*s.* $8\frac{1}{2}$*d.*
(11.) 32 yds.
(12.) $6\frac{6}{7}$.
(13.) £2 8*s.* 9*d.*
(14.) 5*s.* 8*d.*
(15.) 2R. 21P. 25 sq. yds.
(16.) $49\frac{11}{13}$.
(17.) $1\frac{7}{15}$; $3\frac{1}{18}$.
(18.) $27\frac{3}{7}$ ft.
(19.) 2 inches.
(20.) $7\frac{1}{6}$*d.*
(21.) 1*s.* $4\frac{2}{3}$*d.*
(22.) 136 sq. ft.; $20\frac{4}{27}$.
(23.) 21 ft. 4 in.
(24.) £630.

Ex. 127.

(1.) ·9, 23·5, ·76.
(2.) ·03, ·017, ·0039.
(3.) ·01, ·011, ·000111.
(4.) ·0002, 12·4, 7·05.
(5.) ·117, ·0081, 7·3.
(6.) ·013, 30·1, ·00123.
(7.) ·570057.
(8.) 3·105023.
(9.) 703·070037.
(10.) 1·011069.
(11.) 20·301102.
(12.) 1·770077.

Ex. 128.

(1.) $\frac{59}{1000}$, $\frac{3}{10000}$, $4\frac{3}{10}$.
(2.) $\frac{9}{40}$, $\frac{3}{400}$, $13\frac{1}{40}$.
(3) $\frac{1}{32}$, $7\frac{3}{8}$, $\frac{5}{8}$.
(4.) $\frac{33}{80}$, $\frac{64}{125}$, $1\frac{3}{125}$.
(5.) $40\frac{24}{25}$, $1\frac{7}{8}$, $\frac{4}{625}$.
(6.) $\frac{8}{625}$, $\frac{1}{800}$, $\frac{32}{125}$.
(7.) 7·3; 3170.
(8.) ·073; ·00317.
(9.) 23; ·00023.
(10.) 10170; ·0001017.
(11.) 2602·5, 260250, 26025000.
(12.) 13·94, ·1394, ·001394.

Ex. 129.

(1.) 20·23388. (2.) 15·71105. (3.) 21·60848.
(4.) 44·96035. (5.) 71·65377. (6.) 356·27889.
(7.) 6·214. (8.) 2·07253. (9.) ·08885.
(10.) ·01241. (11.) 21·17627. (12.) 1·62107.

Ex. 130.

(1.) 30. (2.) 30. (3.) ·03.
(4.) 3. (5.) ·00023. (6.) 29·75.
(7.) 7·503. (8.) 19·6. (9.) ·0024.
(10.) ·000024. (11.) ·1061208. (12.) ·0000663.

Ex. 131.

(1.) 7·5; 7·5; 750000; ·075.
(2.) 32·16; 321·6. (3.) ·03321; 3321.
(4.) 23·1; ·000231. (5.) 2345; ·002345.
(6.) ·0301; 30100. (7.) 122·21; 122210.
(8.) ·016; 16. (9.) 112; ·00112.
(10.) 23·846, &c. (11.) 12941·17, &c.
(12.) ·008487, &c.

Ex. 132.

(1.) ·12. (2.) ·26. (3.) 5·875.
(4.) 3·088. (5.) ·16. (6.) 7·8125.
(7.) ·8203125. (8.) ·768. (9.) 12·15.
(10.) 2·109375. (11.) ·1625. (12.) 31·85.

Ex. 133.

(1.) $\cdot\dot{8}$. (2.) $\cdot 61\dot{6}$. (3.) $1\cdot 2\dot{2}\dot{7}$.
(4.) $\cdot 1\dot{0}\dot{9}$. (5.) $1\cdot 6\dot{4}2857\dot{1}$. (6.) $\cdot 9\dot{1}\dot{8}$.
(7.) $\cdot\dot{9}9\dot{0}$. (8.) $3\cdot 006\dot{8}\dot{1}$. (9.) $\cdot\dot{9}5238\dot{0}$.
(10.) $7\cdot\dot{9}2307\dot{6}$. (11.) $1\cdot\dot{0}\dot{6}$. (12.) $4\cdot 5\dot{7}6923\dot{0}$.

Ex. 134.

(1.) 1. (2.) $2\frac{1}{33}$. (3.) $3\frac{1}{66}$. (4.) $\frac{3}{110}$.
(5.) $\frac{1}{275}$. (6.) $5\frac{101}{1110}$. (7.) $1\frac{89}{1980}$. (8.) $3\frac{65}{88}$.
(9.) $\frac{27}{88}$. (10.) $17\frac{11}{36}$. (11.) $\frac{7}{240}$. (12.) $4\frac{6}{7}$.

Ex. 135.

(1.) 36·3617.
(2.) 61·4361.
(3.) 6·1686.
(4.) ·8568.
(5.) 3·9346.
(6.) 65·4647.
(7.) 2·8$\dot{4}$.
(8.) ·3$\dot{1}$.
(9.) ·$\dot{2}\dot{8}$.
(10.) ·$\dot{0}\dot{9}$.
(11.) 1·52$\dot{7}$.
(12.) 1·4$\dot{7}\dot{2}$.

Ex. 136.

(1.) 15*s.*
(2.) 2*s.* 7½*d.*
(3.) £6 19*s.* 6*d.*
(4.) £3 9*s.* 4½*d.*
(5.) 5*s.* 9¾*d.*
(6.) 3*s.* 10½*d.*
(7.) £1 3*s.* 3*d.*
(8.) £5 9*s.* 3¾*d.*
(9.) £19 6*s.* 8*d.*
(10.) £14 6*s.* 11½*d.*
(11.) £15 6*s.* 8*d.*
(12.) £4 3*s.* 0¾*d.*

Ex. 137.

(1.) 19 cwt. 2 qrs.
(2.) 2 fur. 22 yds.
(3.) 4A. 3R. 16P.
(4.) 84 c. ft. 648 in.
(5.) 1 gal. 2 qts.
(6.) 3 d. 4 h. 8 min. 24. sec.
(7.) 5 cwt. 2 qrs. 11 lbs.
(8.) 8 yds. 9 in.
(9.) 14 sq. yds. 1 ft. 18 in.
(10.) 5 c. yds. 18 ft. 972 in.
(11.) 16 qrs. 4 bus.
(12.) 21 yrs. 118 dys. 15 hrs.

Ex. 138.

(1.) ·1875.
(2.) ·0328125.
(3.) ·375.
(4.) ·575.
(5.) 3·375.
(6.) ·590625.
(7.) ·75.
(8.) 27·5.
(9.) ·778125.
(10.) 1·$\dot{3}$.
(11.) 1·71875.
(12.) 8·2097$\dot{2}$.

Ex. 139.

(1.) ·28515625.
(2.) ·13125.
(3.) ·125.
(4.) ·$\dot{2}$9$\dot{6}$.
(5.) ·40625.
(6.) 1·076875.
(7.) ·03625.
(8.) ·0008$\dot{8}\dot{3}$.
(9.) ·525.
(10.) 12·196.
(11.) 10·75.
(12.) 1·30$\dot{5}$7142$\dot{8}$.

Ex. 140.

(1.) 5·070619.
(2.) 11·557.
(3.) ·2386.
(4.) 20·81 ; 79 c. ft. 328·32 in.; 20 c. ft. 1399·68 in.
(5.) ·125 ; ·1$\dot{3}$; ·2 ; ·$\dot{3}$; ·208$\dot{3}$.
(6.) ·$\dot{6}$; 1·$\dot{6}$.
(7.) ·00132$\dot{5}\dot{7}$.
(8.) ·2 ; ·$\dot{2}$8571$\dot{4}$.
(9.) 3·2*d.*
(10.) 1·05 ; ·$\dot{9}$5238$\dot{0}$.
(11.) 198; 171; 81.
(12.) 2 ft. 4·4 in.
(13.) 2·08.

(14.) £1 19*s.* $4\frac{1}{2}$*d.* (15.) 216; 240; 132.
(16.) 123·274 (+). (17.) 500.
(18.) ·001267 (+); 2750 yds.; 2748 yds. 2 ft. 8 in. (+).
(19.) It = 3·141592, &c. (20.) 18 yds. 2 ft. 6 in. (+).
(21.) ·55881 sq. in. (22.) 34·65925 c. in.; 20 oz. (+).
(23.) 365 d. 5 h. 48 min. 47 sec. (+).
(24.) 24·2218 days (in defect); 3·1128 days (in excess); 1·128 days (in excess).

Ex. 141.

(1.) 479. (2.) 958. (3.) 143·7. (4.) 3353.
(5.) ·507. (6.) 1014. (7.) 15·21. (8.) 3549.
(9.) 1·4142, &c.; ·44721, &c.
(10.) 5·4772, &c.; ·17320, &c.
(11.) 69 yds. 1 ft. $8\frac{1}{2}$ in. (+). (12.) 39 yds. 9 in. (+).

Ex. 142.

(1.) 253. (2.) 517. (3.) 7·59. (4.) 126·5.
(5.) 714. (6.) 1428. (7.) 1·034. (8.) 676.
(9.) 2·714; ·1259. (10.) 6·694; ·6694.
(11.) 2 ft. $1\frac{4}{5}$ in. (+). (12.) 5 ft. $5\frac{1}{5}$ in. (+).

Ex. 143.

(1.) 12. (2.) 144. (3.) 1728. (4.) 2943.
(5.) 19006. (6.) 20735. (7.) *t.* (8.) 84.
(9.) 6*e*4. (10.) 10*t*5. (11.) 525*t.* (12.) 5953.

Ex. 144.

	sq. ft.	sq. in.		sq. ft.	sq. in.		sq. ft.	sq. in.
(1.)	20	90.	(2.)	80	12.	(3.)	105	87.
(4.)	117	110.	(5.)	725	10.	(6.)	1215	4.
(7.)	36	$54\frac{3}{4}$.	(8.)	566	45.	(9.)	1589	$76\frac{1}{2}$.
(10.)	19	$54\frac{3}{4}$.	(11.)	17	$137\frac{1}{18}$.	(12.)	54	$35\frac{2}{9}$.

Ex. 145.

(1.) 21 sq. ft. $139\frac{1}{2}$ sq. in. (2.) 30 sq. ft. $61\frac{1}{2}$ sq. in.
(3.) £2 3*s.* 9*d.* (4.) £9 14*s.* (+).
(5.) £4 18*s.* $6\frac{3}{4}$*d.* (+) (6.) £17 2*s.* $5\frac{1}{4}$*d.* (+).
(7.) 13 ft. 1 in. (8.) ———.
(9.) 33 yds. 1ft. 8in. (+). (10.) $170\frac{2}{3}$ yds.
(11.) £28 2*s.* $7\frac{1}{2}$*d.* (+). (12.) $301\frac{1}{3}$ yds.
(13.) $58\frac{1}{3}$ c. ft. (14.) 26 c. ft. 702 c. in.

(15.) 35 c. ft. 373 c. in.
(16.) 6 ft. $5\frac{5}{6}$ in.
(17.) 103 ft. $4\frac{1}{4}$ in.
(18.) $13153066\frac{2}{3}$ c. ft.
(19.) 10 ft. $4\frac{1}{2}$ in. (+).
(20) 62 c. ft. 1207 c. in.
(21.) £31 16*s*. 9*d*. (+).
(22.) £3 11*s*. $11\frac{1}{2}$*d*. (+).
(23.) 888 c. ft. 1027 c. in.
(24.) 3 ft. $4\frac{1}{4}$ in.

Ex. 146.

(1.) $1\frac{1}{2}$.
(2.) $2\frac{1}{7}$.
(3.) $1\frac{1}{15}$.
(4.) 3.
(5.) $\frac{1}{4}$.
(6.) $1\frac{7}{8}$.
(7.) $7\frac{1}{2}$.
(8.) $2\frac{1}{9}$.
(9.) 3.
(10.) 10.
(11.) $12\frac{3}{5}$.
(12.) $6\frac{3}{4}$.

Ex. 147.

(1.) 5*s*.
(2.) £73 6*s*. 8*d*.
(3.) 2*s*. 4*d*.
(4.) 208 ft.
(5.) 5*s*. 5*d*.
(6.) £53 16*s*. 3*d*.
(7.) £11 11*s*. $1\frac{1}{3}$*d*.
(8.) The 15*s*. hat; 12*s*.
(9.) $7\frac{1}{2}$*d*.
(10.) 72 ft.
(11.) 7A. 2R. 20P.
(12.) 5 ft. 10 in.

Ex. 148.

(1.) 5 weeks.
(2.) £31 9*s*. 4*d*.
(3.) 12 men.
(4.) 27 men.
(5.) 60*s*.
(6.) 27 men.
(7.) 6 hours.
(8.) 208 acres.
(9.) 34 days.
(10.) 18 men.
(11.) 9 men.
(12.) 5 days.

Ex. 149.

(1.) 3 for 1*d*.
(2.) £4 3*s*. 4*d*.
(3.) $20\frac{5}{6}$.
(4.) $13\frac{3}{4}$.
(5.) £1 6*s*. 8*d*.
(6.) 4*s*. 6*d*.
(7.) $21\frac{1}{3}$.
(8.) 40.
(9.) $11\frac{1}{4}$*d*.
(10.) $5\frac{5}{8}$.
(11.) 16.
(12.) 7*s*. 4*d*.; 60.

Ex. 150.

(1.) 49, 147, 245.
(2.) 135, 45, 27.
(3.) £135, £195, £255.
(4.) 3 cwt. 3 qrs., 2 qrs., 3 qrs.
(5.) £12 10*s*., £8 6*s*. 8*d*., £6 5*s*., £5.
(6.) £5, £10, £13 10*s*.
(7.) 14 cwt. 1 qr. 4 lbs.; 1 cwt. 2 qrs. 8 lbs.
(8.) £1 2*s*. 6*d*.; 18*s*. 9*d*.
(9.) £18, £17 10*s*
(10.) 8 dwts. $9\frac{9}{11}$ grs.
(11.) £21 12*s*., £22 10*s*., £48 12*s*.
(12.) £51 12*s*., £81.

Ex. 151.

(1.) £1400. (2.) £10100. (3.) £5200.
(4.) £2231. (5.) £3457 18*s.* (6.) £6097 1*s.* $11\frac{1}{4}$*d.*
(7.) £96. (8.) £79 16*s.* (9.) £2738.
(10.) £583 11*s.* $10\frac{1}{2}$*d.* (11.) £78 5*s.* (12.) £6 5*s.* loss.

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